

UNIT 15: MOLECULAR THEORY OF GASES

MCQ's KEY:

KEY

1. b	2. a	3. a	4. a	5. a
6. a	7. c	8. d	9. c	10. c

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Section (B): CRQs (Short Answered Questions)

15.1. Why the earth is not in thermal equilibrium with the sun?

Ans) Earth and sun are not in equilibrium because they don't form an isolated system.

15.2. Describe the relationship between temperature and kinetic energy of molecules.

Ans) The relationship between temperature and the kinetic energy of molecules is direct and proportional. As the temperature of a substance increases, the average kinetic energy of its molecules also increases. This means that at higher temperatures, molecules move faster, and at lower temperatures, they move more slowly. Temperature is essentially a measure of the average kinetic energy of the molecules in a substance.

15.3. It is observed that when mercury in glass thermometer is put in a flame, the column of mercury first descends and then rises. Explain.

Ans) Due to expansion of the glass, mercury first falls but later on rises due to larger coefficient of expansion for mercury than that of glass.

15.4. What is standard temperature, pressure?

Ans) Standard temperature and pressure (STP) refers to the nominal conditions in the atmosphere at sea level. These conditions are 0 degrees Celsius and 1 atmosphere (atm) of pressure.

15.5. A thermometer is placed in direct sun light. What will it read the temperature?

Ans) If a thermometer is placed in direct sun light, it will measure a much higher temperature than that of the air.

15.6. The pressure in a gas cylinder containing hydrogen will leak more quickly than if it is containing oxygen Why?

Ans) As the hydrogen is lighter than oxygen i.e., its molecular mass and density is less than that of oxygen therefore rate of diffusion of hydrogen gas is greater than oxygen. That is why the pressure in a gas cylinder containing hydrogen will leak more quickly than the gas cylinder containing oxygen.

15.7. When a sealed thermos bottle full of coffee is shaken, what are the changes occur?

Ans) When the bottle is shaken, the coffee will experience some internal movement, which may cause some molecules to collide and exchange kinetic energy. However, this effect is small, and the overall temperature of the coffee will remain nearly constant.

15.9. How does the Kinetic theory account for the following observed facts:

(a) A gas exerts pressure

(b) The pressure of a gas depends upon its temperature.

Ans) The Kinetic Theory explains the behavior of gases through the motion of their particles:

(a) A gas exerts pressure: According to the Kinetic Theory, gas particles are in constant random motion. When these particles collide with the walls of a container, they exert a force on the walls. The collective force of these collisions per unit area is what we observe as gas pressure.

(b) The pressure of a gas depends upon its temperature: The temperature of a gas is a measure of the average kinetic energy of its particles. As the temperature increases, the particles move faster, resulting in more frequent and forceful collisions with the container walls. This increase in the frequency and intensity of collisions leads to an increase in pressure.

15.10. Calculate the average speed of an air molecule at room temperature (20°C) and compare it to the speed of sound in air (330 m/s).

Data:

$$T = 20^{\circ}\text{C} = 20 + 273 = 293 \text{ K}, v_{\text{avg}} = ?, k = 1.38 \times 10^{-23} \text{ J/K}$$

Solution:

$$K_{\text{avg}} = \frac{3}{2} kT$$

$$\frac{1}{2} m v_{\text{avg}}^2 = \frac{3}{2} kT$$

$$v_{\text{avg}} = \sqrt{\frac{3kT}{m}}$$

For air, $m = 28.96 \text{ amu}$ & $1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$

$$v_{\text{avg}} = \sqrt{\frac{3(1.38 \times 10^{-23})(293)}{28.96 \times 1.66 \times 10^{-27}}} = 502.32 \text{ m/s}$$

In comparison, molecule of air is moving faster than sound because $v_{\text{avg}} = 502.32 \text{ m/s} > v_{\text{sound}} = 330 \text{ m/s}$.

Section (C): ERQs (Long Answered Questions)

15.1. What is temperature? Explain the scales of temperature in detail.

Ans) **Temperature:** Temperature is a measure of the average translational kinetic energy of the molecules of body.

Scales of Temperature: There are three scales of temperature which are commonly used these days.

- i. Centigrade or Celsius scale
- ii. Fahrenheit scale
- iii. Kelvin or absolute scale

(i) Centigrade or Celsius Scale (Anders Celsius): In the Celsius scale the freezing point of water or Melting point of ice is marked 0°C and boiling point is 100°C . The interval between these two points is divided into hundred equal parts. Each part thus represents one degree Celsius (1°C).

(ii) Fahrenheit Scale (Daniel Fahrenheit): In Fahrenheit scale the fixed points are marked 32°F and 212°F respectively and the interval between the fixed points is divided into 180 equal parts. Each part represents 1°F .

(iii) Kelvin Scale (James Lord Kelvin): In this scale the melting point of ice is 273K and the boiling point of water is 373K . The temperature is given in units called Kelvin instead of degrees. The lowest temperature is OK known as absolute zero.

Empirical Formulae: In order to derive empirical formulae among Centigrade, Fahrenheit and Kelvin scales, let the three thermometers be placed in a bath tub and the mercury in each thermometer rises to the same level as shown in figure below. We arrive at the relation.

$$\frac{\text{Temp. on one scale} - F.P.}{B.P. - F.P} = \frac{\text{Temp. on 2nd scale} - F.P.}{B.P. - F.P}$$
$$\frac{T_c - 0}{100} = \frac{T_F - 32}{180} = \frac{T_K - 273}{100}$$

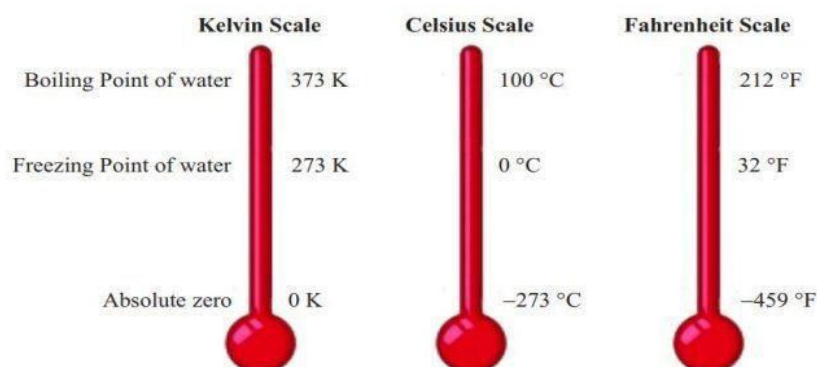


Figure: Scales of temperature mentioning the different temperatures

15.2. Define and explain Boyle's law, Charles's and Avogadro's law.

Ans) **Boyle's law:**

Definition: Boyle's law states that "Volume V of given mass of a gas is inversely proportional to the pressure P , provided the temperature T of the gas remains constant."

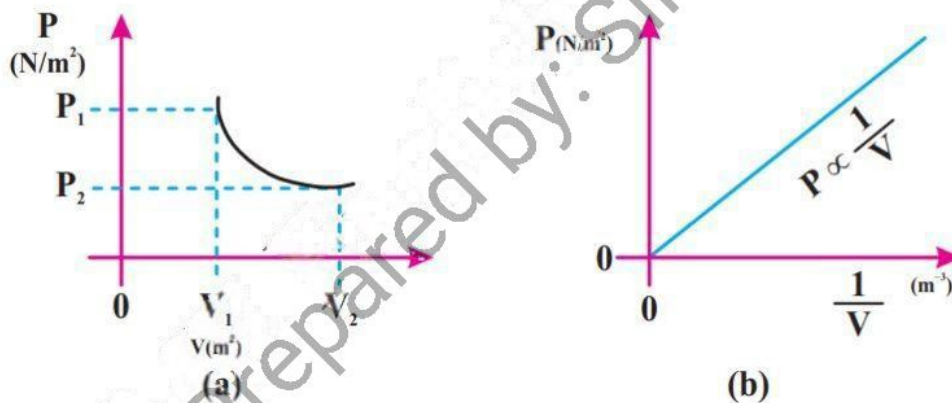
Explanation: Boyle's law can be written as.

$$V \propto \frac{1}{P} \text{ (at constant temperature)}$$

$$PV = \text{constant}$$

We can also represent Boyle's law on a graph, as shown in figure (a). The graph plotted between P and V at constant temperature is a curve called hyperbola showing the inverse relation between them for two different states, while figure (b) graph of P plotted against $\frac{1}{V}$ is a straight line passing through the origin, showing direct proportionality. Boyle's law can be written as:

$$P_1V_1 = P_2V_2 = \text{Constant}$$



Charles's Law:

Definition: This law states that "The volume V of a given mass of a gas is directly proportional to the absolute temperature T at constant pressure P ".

Explanation: Charles law can be written as

$$V \propto T \text{ (at constant pressure)}$$

Or

$$\frac{V}{T} = \text{constant}$$

If V_1 and V_2 are the volumes of the gas at temperature T_1 and T_2 respectively then for two different states, Charles's law is represented as

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} = \text{constant}$$

From the graph at 0°C the gas still has a volume V_0 . The graph between volume and temperature is a straight line. If the graph is extrapolated backward, it cuts the temperature axis at -273°C . This graph shows that volume of a gas is zero. Kelvin selected this temperature (-273°C) as the zero called absolute zero (0K).

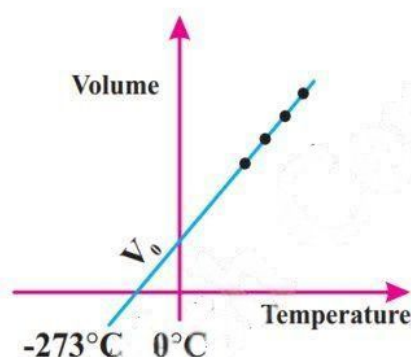


Figure: Shows relation between temperature and volume

Avogadro's law:

Definition: Avogadro's law which states that "equal volume of all gases contains the same number of molecules at the same temperature and pressure". Thus, the volume of a gas is directly proportional to number of moles of the gas at constant temperature and pressure.

Explanation: In symbol, we can write as

$$V \propto n \text{ (at constant temperature and pressure)}$$

$$\frac{V}{n} = \text{constant}$$

Where V is volume and n is the number of moles of a gas. Thus 1dm^3 (or cm^3 , m^3) of oxygen contains the same number of molecules as 1dm^3 or 1cm^3 , 1m^3 etc of hydrogen or of any other gas, provided the volumes are measured under the same conditions of temperature and pressure.

It can be written as

$$\frac{V_1}{n_1} = \frac{V_2}{n_2} = \text{constant}$$

When V_1 and V_2 are volumes of gas and n_1 and n_2 are amount of gas.

15.3. Derive general gas law by making use of gas laws.

Ans) **General Gas Law:** In order to derive general gas law, we make use of Boyle's law, Charles's law and Avogadro's law. An interrelation among the physical quantities e.g. pressure, volume, temperature and amount of matter of a given sample of gas is termed as "equation of state" of gas or General gas law.

According to Boyle's law:

$$V \propto \frac{1}{P} \text{ (when } n \text{ number of moles and temperature } T \text{ are kept constant)}$$

According to Charles's law:

$$V \propto T \text{ (when } n \text{ and pressure } P \text{ are kept constant)}$$

According to Avogadro's law:

$$V \propto n \text{ (when } T \text{ and } P \text{ are kept constant)}$$

Consider for a moment that none of the variables are to be kept constant, then all the above three relationships can be joined together.

$$\begin{aligned} V &\propto \frac{nT}{P} \\ V &= \text{constant} \times \frac{nT}{P} \\ V &= \frac{RnT}{P} \end{aligned}$$

Where R is constant of proportionality and is called General gas constant or universal gas constant and does not depend on the quantity of gas in the sample. If P is measured in Nm^{-2} , V in m^3 and T in Kelvin then the value of universal gas constant is $R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$.

Above equation is written as:

$$PV = nRT$$

15.4. Describe the molecular movement causes the pressure exerted by gas, derive pressure equation.

Ans) **Pressure of gas:** The pressure exerted by a gas is merely the momentum transferred to the walls of the container per second per unit area due to the continuous collisions of molecules of the gas.

Pressure equation: In order to calculate the pressure of an ideal gas from Kinetic Theory. Let us consider a cube having side length L whose walls are perfectly elastic contains N number of molecules each of mass m as shown in figure.

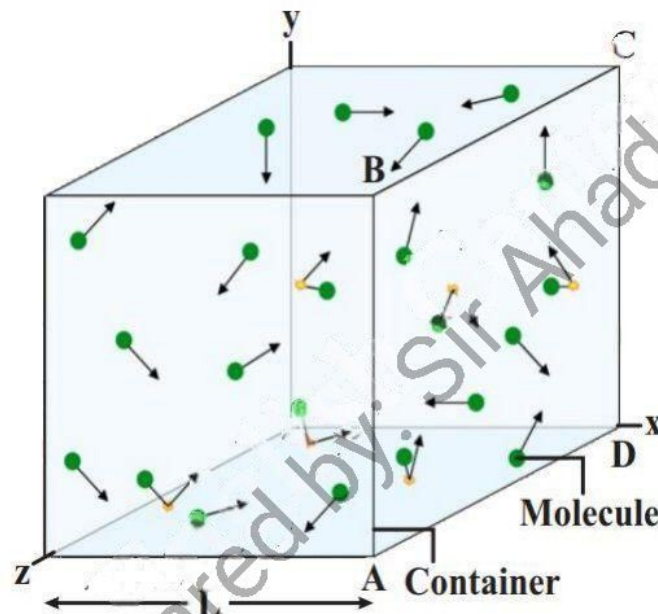


Figure shows gas molecules moving in random directions within container

Consider a single molecule of mass m moving with velocity V_1 parallel to x -axis. It moves back and forth, colliding at regular intervals with the ends of the box and thereby contributing to the pressure of the gas. A molecule which has a velocity V_1 can be resolved into three rectangular components V_{1x} , V_{1y} and V_{1z} parallel to three co-ordinates axes x , y and z .

A molecule which collides with the face $ABCD$ of the cube, it will rebound elastically in opposite direction, such that x -component of the velocity V_{1x} , is reversed, the V_{1y} and V_{1z} remain unaffected. Therefore, the momentum before collision is mV_{1x} and after collision is $-mV_{1x}$ causing a change of momentum.

$$\text{Change in momentum} = P_i - P_f = mV_{1x} - (-mV_{1x}) = mV_{1x} + mV_{1x}$$

$$\text{Change in momentum} = 2mV_{1x} \dots\dots\dots(1)$$

After recoil the molecule travels to opposite face and collides with it, rebounds and travels back to the face $ABCD$ after covering a distance $2L$. The time Δt between two successive collisions with face $ABCD$ is:

$$\Delta t = \frac{2L}{v_{1x}} \dots \dots (2)$$

Now we can find the force that this one molecule exerts on face $ABCD$, using Newton's 2nd law of motion. This says that the rate of change of momentum of the molecule is equal to force applied by the wall. According to Newton's 3rd law of motion, force F_1 exerted the molecule on face $ABCD$ is equal but opposite.

$$\begin{aligned} \text{Force} = F_1 &= \frac{\text{change in momentum}}{\text{time taken}} = \frac{\Delta P}{\Delta t} \\ F_1 &= \frac{2(mV_{1x})}{\frac{2L}{V_{1x}}} = \frac{mV_1^2 x}{L} \end{aligned}$$

Similarly, the forces due to all other molecules can be determined. Thus, the total x- directed F due to N number of molecules of the gas moving with velocities $V_1, V_2, V_3, \dots, V_n$ is:

$$\begin{aligned} F &= F_1 + F_2 + F_3 + \dots + F_n \\ F &= \frac{mV_1^2 x}{L} + \frac{mV_2^2 x}{L} + \frac{mV_3^2 x}{L} + \dots + \frac{mV_n^2 x}{L} \end{aligned}$$

As pressure is normal force per unit area, hence P on the face perpendicular to x-axis is:

$$\begin{aligned} P &= \frac{F}{A} = \frac{F}{L^2} \\ P &= \left(\frac{mV_1^2 x}{L} + \frac{mV_2^2 x}{L} + \frac{mV_3^2 x}{L} + \dots + \frac{mV_n^2 x}{L} \right) \\ P &= \frac{m}{L^3} (V_1^2 x + V_2^2 x + V_3^2 x \dots + V_n^2 x) \dots \dots (3) \end{aligned}$$

The number of molecules in unit volume n , is $\frac{N}{L^3}$. Where N is the total number of molecules. Therefore:

$$\begin{aligned} L^3 &= \frac{N}{n_v} \text{ and substituting this value in eq: (3)} \\ P &= mn_v \left(\frac{V_1^2 x + V_2^2 x + V_3^2 x \dots + V_n^2 x}{N} \right) \dots \dots (4) \end{aligned}$$

Where mn_v is the mass per unit volume which we call density ρ and $\frac{v_1^2x+v_2^2x+v_3^2x...+v_n^2x}{N}$ is the average value of $\overline{V_x^2}$ for all the molecules in the container, we call this average square velocity $\overline{V_x^2}$. The square root of $\overline{V_x^2}$ is referred as V_{rms} . Eq: (4) can be written as:

$$P = \rho \overline{V_x^2}$$

The terms $\overline{V_x^2}$ is only one component of the total velocity. Since $\overline{V^2} = \overline{V_x^2} + \overline{V_y^2} + \overline{V_z^2}$ on the average $\overline{V_x^2} = \overline{V_y^2} = \overline{V_z^2}$ due to randomness of the molecular motion.

$$\overline{V^2} = 3\overline{V_x^2} \text{ and } \overline{V_x^2} = \frac{1}{3}\overline{V^2}$$

Substituting this value into the above equation, we find that:

$$P = \frac{1}{3}\rho\overline{V^2}$$

15.5. Interpret mathematically that temperature is a measure of average translational K.E of the molecules of a gas.

Ans) As we know that:

$$P = \frac{1}{3}mn_v\overline{V^2} \dots\dots(1) \text{ since } \rho = mn_v$$

Since n_v represents the number of molecules per unit volume $n_v = \frac{N}{V}$. Equation (1) can be written as

$$P = \frac{1}{3}m\frac{N}{V}\overline{V^2} \dots\dots(2)$$

Now we can compare the equation (2) with this ideal gas equation $PV = nRT$. The left-hand sides are the same. So, the two right-hand sides must also be equal.

$$\frac{1}{3}Nm\overline{V^2} = nRT$$

Substituting $n = \frac{N}{N_A}$ and multiplying both sides by $\frac{3}{2}$, we obtain the relation.

$$\frac{3}{2} \times \frac{1}{3}Nm\overline{V^2} = \frac{3}{2} \times \frac{N}{N_A}RT$$

$$\frac{1}{2}m\overline{V^2} = \left(\frac{3}{2}\right) \times \frac{R}{N_A}T$$

Since $\frac{R}{N_A} = k$ (Boltzmann constant)

$$k = 1.38 \times 10^{-23} \text{ J/Molecule}$$

$$\text{Hence } \frac{1}{3} m \overline{V^2} = \frac{3}{2} KT$$

$$K.E_{avg} = \frac{3}{2} KT$$

The mean translational Kinetic energy of a molecule of an ideal gas is proportional to the absolute temperature.

Section (D): Numerical:

15.1. The freezing point of mercury is -39°C . Convert it into $^{\circ}\text{F}$ and the comfort level temperature of 20°C into Kelvin.

Data:

$$T_{Cm} = -39^{\circ}\text{C}, T_{Fm} = ?$$

$$T_{Cc} = 20^{\circ}\text{C}, T_{Kc} = ?$$

Solution:

$$T_{Fm} = \frac{9}{5} T_{Cm} + 32 = \frac{9}{5} (-39) + 32$$

$$T_{Fm} = -38.2^{\circ}\text{F}$$

$$T_{Kc} = T_{Cc} + 273 = 20 + 273$$

$$T_{Kc} = 293 \text{ K}$$

15.2. The boiling point of liquid nitrogen is -321°F . Change it into equivalent Kelvin temperature.

Data:

$$T_{FN_2} = -321^{\circ}\text{F}$$

$$T_{KN_2} = ?$$

Solution:

$$\frac{T_{FN_2} - 32}{180} = \frac{T_{KN_2} - 273}{100}$$

$$\frac{-321 - 32}{180} = \frac{T_{KN_2} - 273}{100}$$

$$T_{KN_2} = 76.888 \text{ K} \approx 77 \text{ K}$$

15.3. Calculate the volume occupied by a gram-mole of a gas at 0°C and a pressure of 1.0 atmosphere.

Data:

$$V = ?, n = 1 \text{ moles}, R = 0.0821 \text{ L.atm/mol.K}$$

$$P = 1 \text{ atm}, T = 0^\circ\text{C} = 0 + 273 = 273\text{K}$$

Solution:

$$V = \frac{nRT}{P} = \frac{(1)(0.0821)(273)}{1}$$

$$V = 22.4 \text{ liters/mole}$$

15.4. An air storage tank whose volume is 112 liters contains 3 kg of air at a pressure of 18 atmospheres. How much air would have to be forced into the tank to increase the pressure to 21 atmospheres, assuming no change in temperature?

Data:

$$V_1 = 112 \text{ liters}, m_1 = 3 \text{ kg}, P_1 = 18 \text{ atm}$$

$$\Delta m = ?, P_2 = 21 \text{ atm}$$

$$V_1 = V_2 \text{ (constant)}, T_1 = T_2 \text{ (constant)}$$

Solution:

$$P_1 V_1 = n_1 R T_1 \dots\dots(1)$$

$$P_2 V_2 = n_2 R T_2 \dots\dots(2)$$

Dividing equation (1) by equation (2)

$$\frac{P_1 V_1}{P_2 V_2} = \frac{n_1 R T_1}{n_2 R T_2}$$

$$\frac{P_1}{P_2} = \frac{n_1}{n_2}$$

$$\therefore n = \frac{m}{M}$$

$$\frac{P_1}{P_2} = \frac{\frac{m_1}{M}}{\frac{m_2}{M}}$$

$$\frac{P_1}{P_2} = \frac{m_1}{m_2}$$

$$\frac{18}{21} = \frac{3}{m_2}$$

$$m_2 = 3.5 \text{ kg}$$

$$\Delta m = m_2 - m_1 = 3.5 - 3 = 0.5 \text{ kg}$$

15.5. A balloon contains 0.04 m^3 of air at a pressure of 120 kPa . Calculate the pressure required to reduce its volume to 0.025 m^3 at constant temperature.

Data:

$$V_1 = 0.04 \text{ m}^3, P_1 = 120 \text{ kPa}$$

$$V_2 = 0.025 \text{ m}^3, P_2 = ?$$

Solution:

$$P_1 V_1 = P_2 V_2$$

$$(120)(0.04) = P_2(0.025)$$

$$P_2 = 192 \text{ kPa} \approx 1.9 \times 10^5 \text{ Pa}$$

15.6. The molar mass of nitrogen gas N_2 is 28 g mol^{-1} . For 100 g of nitrogen, calculate

(a) the number of moles

(b) the volume occupied at room temperature (20°C) and pressure of $1.01 \times 10^5 \text{ Pa}$.

Data:

$$M = 28 \text{ g mol}^{-1}, m = 100 \text{ g}$$

$$(a) n = ?$$

$$(b) V = ?, T = 20^\circ\text{C} = 20 + 273 = 293 \text{ K}, P = 1.01 \times 10^5 \text{ Pa},$$

$$R = 8.314 \text{ J/mole}^{-1}\text{K}^{-1}$$

Solution:

(a)

$$n = \frac{m}{M} = \frac{100}{28}$$

$$n = 3.57 \text{ mole}$$

(b)

$$PV = nRT$$

$$(1.01 \times 10^5)V = (3.57)(8.314)(293)$$

$$V = 0.086 \text{ m}^3$$

15.7. A sample of a gas contains 3.0×10^{24} atoms. Calculate the volume of the gas at a temperature of 300 K and a pressure of 120K Pa.

Data:

$$\text{No. of atoms} = 3.0 \times 10^{24} \text{ atoms}$$

$$V = ?, T = 300 \text{ K}, P = 120 \text{ K Pa}$$

Solution:

$$n = \frac{\text{No. of atoms}}{\text{Avogadro's number}}$$

$$n = \frac{3.0 \times 10^{24}}{6.022 \times 10^{23}} = 4.98 \text{ moles}$$

$$PV = nRT$$

$$(1.20 \times 10^5)V = (4.98)(8.314)(300)$$

$$V = 0.104 \text{ m}^3$$

15.8. Calculate the root mean square speed of hydrogen molecules at 0°C and 1.0 atm pressure. Assuming hydrogen to be an ideal gas. The density of hydrogen is $8.99 \times 10^{-2} \text{ kg/m}^3$.

Data:

$$V_{rms} = ?, T = 0^\circ\text{C}, P = 1 \text{ atm} = 1 \times 1.01 \times 10^5 = 1.01 \times 10^5 \text{ kPa},$$

$$\rho = 8.99 \times 10^{-2} \text{ kg/m}^3$$

Solution:

$$P = \frac{1}{3}\rho\overline{V^2}$$

$$V_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3(1.01 \times 10^5)}{8.99 \times 10^{-2}}} \rightarrow V_{rms} = 1835.86 \text{ ms}^{-1}$$

15.9. Calculate the root mean square speed of hydrogen molecule at 500K (mass of proton = 1.67×10^{-27} kg and $k = 1.38 \times 10^{-23}$ J /molecule – K)

Data:

$$V_{rms} = ?, T = 500K, \text{mass of proton} = 1.67 \times 10^{-27} \text{ kg}$$

$$k = 1.38 \times 10^{-23} \text{ J /molecule – K}$$

Solution:

$$V_{rms} = \sqrt{\frac{3kT}{m}}$$

$$m = 2 \times 1.67 \times 10^{-27} \text{ kg} = 3.34 \times 10^{-27} \text{ kg}$$

$$V_{rms} = \sqrt{\frac{3(1.38 \times 10^{-23})(500)}{3.34 \times 10^{-27}}}$$

$$V_{rms} = 2489.49 \text{ ms}^{-1}$$

15.10. (a) Determine the average value of the Kinetic energy of the particles of an ideal gas at 10°C and at 40°C.

(b) What is the Kinetic energy per mole of an ideal gas at these temperatures?

Data:

$$(a) K.E_{avg} = ? \text{ at } 10^\circ\text{C and } 40^\circ\text{C}$$

$$(b) K.E = ? \text{ at } 10^\circ\text{C and } 40^\circ\text{C}$$

Solution:

$$K.E_{avg} = \frac{3}{2}kT$$

$$(a) K.E_{avg1} = \frac{3}{2}(1.38 \times 10^{-23})(10 + 273)$$

$$K.E_{avg1} = 5.86 \times 10^{-21} \text{ J}$$

$$K.E_{avg2} = \frac{3}{2}(1.38 \times 10^{-23})(40 + 273)$$

$$K.E_{avg2} = 6.48 \times 10^{-21} \text{ J}$$

KEY

UNIT: 16

1. a	2. b	3. a	4. b	5. c
6. c	7. b	8. a	9. d	10. a

Section (B): CRQs (Short Answered Questions):

1. Explain the concept of the first law of thermodynamics in your own words.

Ans) The first law of thermodynamics, also known as the law of energy conservation, states that energy cannot be created or destroyed, only transferred or transformed. In a thermodynamic system, the change in internal energy is equal to the heat added to the system minus the work done by the system. This means that any energy added as heat must either increase the system's internal energy or be used to do work or be used in both and the total energy remains constant.

$$\Delta U = \Delta Q - \Delta W$$

Where

- ΔU is the change in internal energy of the system.
- ΔQ is the heat added to the system.
- ΔW is the work done by the system.

2. How does the first law of thermodynamics relate to the conservation of energy?

Ans) The first law of thermodynamics is essentially a statement of the conservation of energy principle, specifically tailored for thermodynamic systems. It states that energy cannot be created or destroyed, only transferred or converted from one form to another.

Mathematically, the first law of thermodynamics is expressed as:

$$\Delta U = \Delta Q - \Delta W$$

Where

- ΔU is the change in the internal energy of the system.
- ΔQ is the heat added to the system.
- ΔW is the work done by the system.

This equation shows that any change in a system's internal energy is equal to the heat added to the system minus the work done by the system on its surroundings. This reflects the conservation of energy because it accounts for all the energy entering and leaving the system.

In essence, the first law ensures that the total energy in an isolated system remains constant, affirming that energy is conserved in every process, regardless of how it's transformed or transferred.

3. Distinguish between the work done by a system and the heat exchanged with the surroundings in the context of the first law.

Ans) In the context of the first law of thermodynamics

- Work is the organized energy transfer when a system does mechanical tasks, like moving a piston. It affects the system's internal energy, decreasing it when work is done by the system, and increasing it when work is done on the system.
- Heat is the disorganized energy transfer due to temperature differences. It increases the system's internal energy when added and decreases it when lost.

4. Give an example daily life that illustrates the principles of the first law of thermodynamics.

Ans) A simple daily life example of the first law of thermodynamics is boiling water in a kettle:

- Heat is added to the water from the stove (energy input).
- The water's internal energy increases, causing its temperature to rise.
- Some of the energy is used to convert water into steam, doing work by expanding the steam and pushing against the surrounding air but total amount of energy remains constant.

5. Explain the role of the system and its surroundings in the context of the first law of thermodynamics.

Ans) Energy can be exchanged between the system and its surroundings through heat and work. The first law states that any change in the system's internal energy is due to these energy exchanges with the surroundings, ensuring total energy is conserved.

6. How does heat capacity relate to the amount of energy required to change the temperature of a substance?

Ans) Heat capacity is a measure of the amount of energy required to change the temperature of a substance by a certain amount. Specifically, it is the amount of heat energy needed to raise the temperature of a substance by 1 degree Celsius (or 1 Kelvin).

Relationship:

$$\Delta Q = C\Delta T$$

- ΔQ is the amount of heat energy added or removed.
- C is the heat capacity of the substance.
- ΔT is the change in temperature.

Explanation:

- A substance with a high heat capacity requires more energy to change its temperature by a given amount compared to a substance with a low heat capacity.
- For example, water has a high heat capacity, so it takes a lot of energy to warm it up or cool it down, whereas metals generally have lower heat capacities and heat up or cool down more quickly with the same amount of energy.

Section (C): ERQs (Long Answered Questions):

1. Provide the mathematical expression of the first law of thermodynamics and explain each term.

Ans) The first law of thermodynamics states that, "The change in internal energy of a system equals the net heat transfer into the system minus the net work done by the system".

In equation form, the first law of thermodynamics is,

$$\Delta U = \Delta Q - \Delta W$$

Explanation of Each Term:

1. ΔU (Change in Internal Energy): This term represents the change in the internal energy of the system, which includes the total kinetic and potential energy of the particles within the system. If ΔU is positive, the internal energy of the system has increased. If it is negative, the internal energy has decreased.
2. ΔQ (Heat exchange): ΔQ is the amount of heat energy transferred to the system. Heat can enter or leave the system depending on the conditions. If ΔQ is positive, heat is added to the system, increasing its internal energy. If ΔQ is negative, heat is removed from the system.
3. ΔW (Work done): ΔW represents the work done. Work can involve processes like expansion or compression of gases. If ΔW is positive, the system has done work on the surroundings, decreasing its internal energy. If ΔW is negative, work is done on the system, increasing its internal energy.

2. Describe what happens to the internal energy of a system in an adiabatic process, and why.

Ans) According to the first law of thermodynamics;

$$\Delta Q = \Delta U + \Delta W$$

$$0 = \Delta U + \Delta W$$

$$\Delta U = -\Delta W$$

Thus, an increase in the internal energy of the system in an adiabatic process is equal to the work done on the system.

Above equation can be written as;

$$\Delta W = -\Delta U$$

This means that if the system does the work, then in adiabatic process, the work is done at the cost of internal energy.

Types of Heat Capacity:

1. **Specific Heat Capacity (c):** Specific heat capacity tells us how much heat is required to raise the temperature of one kilogram of the substance by one degree Celsius.

Mathematically,

$$c = \frac{Q}{m\Delta T}$$

Where;

Q is the heat energy absorbed or released,

m is the mass of the substance,

c is the specific heat, and

ΔT is the change in temperature.

2. **Molar Specific Heat Capacity (C_m):** Molar heat capacity tells us how much heat is required to raise the temperature of one kilogram of the substance by one degree Celsius. Mathematically,

$$C_m = \frac{Q}{n\Delta T}$$

Where

Q is the heat energy absorbed or released,

n is the number of moles,

C_m is the molar specific heat

ΔT is the change in temperature.

Significance of heat capacity in Thermodynamics:

1. **Thermal Energy Storage:** Heat capacity is directly related to a substance's ability to store thermal energy. A substance with a high heat capacity can absorb more heat for a given change in temperature compared to a substance with a low heat capacity.
2. **Temperature Changes:** Substances with high heat capacities will experience smaller temperature changes for a given amount of heat compared to those with low heat capacities.
3. **Implications for Heat Transfer:**
 - In engineering applications and various thermodynamic processes, understanding heat capacity helps in designing systems for heating, cooling, and thermal management. For instance, in thermal insulation, materials with low heat capacity are used to minimize heat transfer.
 - In chemical reactions, knowing the heat capacities of reactants and products helps in predicting the temperature changes and the amount of heat required or released during the reaction.

Heat Capacity and Thermal Energy Storage:

- **High Heat Capacity:** A substance with a high heat capacity can absorb a large amount of heat without a significant rise in temperature. This means it can store more thermal energy for a given temperature change. For example, water has a high specific heat capacity ($4.18 \text{ J/g}^\circ\text{C}$), which allows it to absorb and store substantial amounts of heat energy, making it an effective heat reservoir.
- **Low Heat Capacity:** A substance with a low heat capacity absorbs less heat for the same temperature change, meaning it stores less thermal energy. Metals like aluminum or copper have lower heat capacities compared to water, so they heat up and cool down more quickly.

Implications for Temperature Changes:

For a given amount of heat added or removed (Q), the temperature change (ΔT) of a substance is inversely related to its heat capacity.

$$\Delta T = \frac{Q}{C}$$

So, a high heat capacity results in a smaller temperature change for a given amount of heat. Conversely, a low heat capacity results in a larger temperature change.

Section (D): Numerical:

1. A gas undergoes isothermal expansion at a constant temperature of 300 K. If the gas absorbs 500 J of heat during the process, calculate the work done by the gas.

Data:

$$T = 300 \text{ K}, \Delta Q = 500 \text{ J}, \Delta W = ?$$

Solution:

$$\Delta U = \Delta Q - \Delta W$$

Since, process is isothermal expansion, therefore, $\Delta U = 0$

$$0 = \Delta Q - \Delta W$$

$$\Delta W = 500 \text{ J}$$

Types of Heat Capacity:

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Where;

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2. **Molar Specific Heat Capacity (C_m):** Molar heat capacity tells us how much heat is required to raise the temperature of one kilogram of the substance by one degree Celsius. Mathematically,

$$C_m = \frac{Q}{n\Delta T}$$

Where

Q is the heat energy absorbed or released,

n is the number of moles,

C_m is the molar specific heat

ΔT is the change in temperature.

Significance of heat capacity in Thermodynamics:

1. **Thermal Energy Storage:** Heat capacity is directly related to a substance's ability to store thermal energy. A substance with a high heat capacity can absorb more heat for a given change in temperature compared to a substance with a low heat capacity.
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 - In chemical reactions, knowing the heat capacities of reactants and products helps in predicting the temperature changes and the amount of heat required or released during the reaction.

5. A gas undergoes a cyclic process, starting at point A with a volume of 0.02 m^3 , going to B (isochoric heating), then to C (isothermal expansion), and finally back to A. If the heat added during isothermal expansion is 1000 J and the heat rejected during isochoric heating is 500 J , calculate the net work done by the system.

Data:

$$\Delta Q = 1000 \text{ J},$$

$$\Delta U = 500 \text{ J} \text{ (heat rejected during isochoric heating is the change in internal energy)}$$

$$V_1 = 0.02 \text{ m}^3, \Delta W = ?$$

Solution:

$$(1000) = (500) + \Delta W$$

$$\Delta W = 500 \text{ J}$$

6. A gas expands from 0.03 m^3 to 0.06 m^3 against a constant pressure of 100 kPa . Calculate the work done in both a reversible and an irreversible process, and compare the results.

Data:

$$V_1 = 0.03 \text{ m}^3, V_2 = 0.06 \text{ m}^3, P = 100 \text{ kPa}, \Delta W_{\text{reversible}} = ?$$

Solution:

$$\Delta W_{\text{reversible}} = P\Delta V$$

$$\Delta W_{\text{reversible}} = P(V_2 - V_1)$$

$$\Delta W_{\text{reversible}} = 100 \times 10^3 (0.06 - 0.03)$$

$$\Delta W_{\text{reversible}} = 3000 \text{ J}$$

Since complete data is not available that's why we are unable to calculate irreversible work.

7. A 50 g piece of copper at 100°C is placed in 200 g of water at 20°C. If the final temperature of the system is 30°C, calculate the specific heat capacity of copper. (Specific heat capacity of water = 4.18 J/g°C).

Data:

$$m_c = 50 \text{ g}, T_c = 100^\circ\text{C}, m_w = 200 \text{ g}, T_w = 20^\circ\text{C}, T_s = 30^\circ\text{C}, c_c = ?,$$

$$c_w = 4.18 \text{ J/g}^\circ\text{C}$$

Solution:

$$Q = mc\Delta T$$

$$\text{Heat lost by copper} = \text{Heat gained by water}$$

$$50 \times c_c \times (100 - 30) = 200 \times 4.18 \times (30 - 20)$$

$$c_c = 2.34 \text{ J/g}^\circ\text{C}$$

8. How much heat is required to raise the temperature of 1 kg of lead from 25°C to 100°C? (Specific heat capacity of lead = 0.128 J/g°C).

Data:

$$Q = ?, m = 1 \text{ kg}, T_1 = 25^\circ\text{C}, T_2 = 100^\circ\text{C}, c_L = 0.128 \text{ J/g}^\circ\text{C} = 0.128 \times 10^3 \text{ J/kg}^\circ\text{C}$$

Solution:

$$Q = mc_L\Delta T = mc_L(T_2 - T_1)$$

$$Q = (1)(0.128 \times 10^3)(100 - 25)$$

$$Q = 9600 \text{ J}$$

UNIT 17: SECOND LAW OF THERMODYNAMICS

MCQ'S

KEY

1. c	2. a	3. c	4. d	5. c
6. a	7. a	8. a	9. d	10. d

Section (B): CRQs (Short Answered Questions):

1. What are some factors that affect the efficiency of automobile engines?

Ans) Some factors that affect the efficiency of automobile engines:

- The efficiency of the automobile engine cannot exceed the Carnot efficiency: it is limited by the temperature of burning fuel and the temperature of the environment into which the exhaust is dumped.
- The engine block cannot be allowed to go over a certain temperature.
- Any practical engine has friction, incomplete burning of fuel, and limits set by timing and energy transfer by heat.

2. What happens to the temperature of a room in which an air conditioner is left running on table in the middle of the room?

Ans) Temperature of the room increases, as heat absorbed from the room by the air conditioner is expelled in the same room.

3. Under what conditions can heat be added to a system without changing its temperature?

Ans) Heat can be given to a substance without raising its temperature when a substance is changing its physical state, that is phase changes for example from solid to liquid and liquid to gas.

4. Is it possible to cool a room by keeping the refrigerator door open?

Ans) No, keeping a refrigerator door open cannot effectively cool a closed room.

5. When does the entropy of a system decrease?

Ans) The entropy of a system decreases only when it interacts with some other system whose entropy increases in the process.

6. Is it possible, according to the 2nd law of thermodynamics to construct an engine that is free from thermal pollution?

Ans) It is not possible to construct a heat engine which is free from thermal pollution.

7. Can heat be completely converted to work?

Ans) No, we can not completely convert heat to work.

8. Define that why entropy has often been called as "time arrow".

Ans) Entropy is often called the 'time's arrow' because it tells us the direction in which natural processes occur.

Section (C): ERQs (Long Answered Questions):

1. Give the two statements of the second law of the thermodynamics. Elaborate the concept of entropy and state the second law of thermodynamics in terms of this concept.

Ans) **Statements of the second law of the thermodynamics:**

Kelvin Statement: According to this statement "It is impossible to construct an engine, operating continuously in a cycle that can take heat from a source and converts completely into work".

Clausius Statement: "It is impossible to cause heat to flow from a cold body to a hot body without the expenditure of energy".

Entropy: Entropy was first expressed by German physicist and mathematician, Rudolf Clausius in 1865 into the study of thermodynamics to give a quantitative basis for the 2nd law of thermodynamics. The disorderness of the system is known as entropy. Entropy is also defined as

"The measure of a system's thermal energy per unit temperature that is unavailable for doing useful work"

We have discussed several processes that proceed naturally in the direction of increasing disorder. Irreversible heat flow increases disorder because the molecules are initially sorted into hotter and cooler regions, this sorting is lost when the system comes to thermal equilibrium. Adding heat to a body increases its disorder because it increases average molecular speed and therefore the randomness of molecular motion. Free expansion of a gas increases its disorder because the molecules have more randomness of position after the expansion than before.

Now, the second law of thermodynamics is

“The entropy of the universe during any process either remains constant or increases.”

2. Explain the working principle of heat engine and also derive the formula for its efficiency.

Ans) **Working principle of heat engines:** Any device that transforms heat into mechanical energy (work) is called heat engine. The essentials of a heat engine are the furnace or hot body, the working substance and a condenser or cold body.

The simplest kind of engine to discuss is one in which the working substance undergoes a cyclic process, that is, a sequence of processes that eventually leaves the substance in the same state in which it started. In a steam turbine the water is recycled and used over and over. Internal combustion engines do not use the same air again but we can still analyze them in terms of cyclic processes that approximate their actual operation.

All the heat engines absorb heat from a source at a relatively low temperature, perform some mechanical work, and discard some heat at a low temperature. The engine treats extra heat as waste. It gets rid of this waste heat through the exhaust pipes and the cooling system in internal combustion engines.

In a heat engine, Q_1 is positive but Q_2 is negative, representing heat leaving the working substance.

We can represent the energy transformation in a heat engine by the flow diagram below.

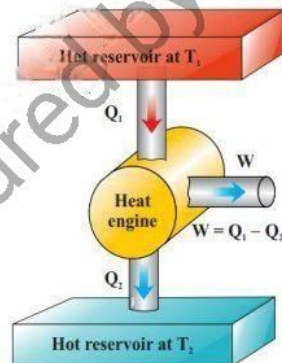


Fig: Heat Engine

The engine itself is represented by the circle. The amount of heat supplied Q_1 to the engine by the hot reservoir is proportional to the cross section of the incoming pipeline at the top of the diagram. The cross section of the outgoing pipeline at the bottom is proportional to the magnitude $|Q_2|$ of the heat discarded in the exhaust.

The branch line to the right represents the portion of the heat supplied that the engine converts to mechanical work W .

When an engine repeats the same cycle over and over, Q_1 and Q_2 represent the quantities of heat absorbed and rejected by the engine during one cycle. The net heat absorbed per cycle is

$$\Delta Q = Q_1 - Q_2$$

Let Q_1 be the heat absorbed by an engine from a high temperature reservoir and Q_2 be the heat rejected by the engine to low temperature reservoir or sink. The rest of heat energy is converted into useful work i.e.

$$W = Q_1 - Q_2$$

Efficiency of heat engine:

“The thermal efficiency " η " of a cycle heat engine is defined to the ratio of the network 'W' done by the engine in each cycle to the heat absorbed Q_1 in each cycle”.

$$\text{Efficiency}(\eta) = \frac{\text{output}}{\text{input}} = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

For 100% efficiency $Q_2 = 0$ i.e. no heat is rejected to low temperature reservoir (sink) and whole heat absorbed is converted into work. But experimental facts show that no cyclic engine can achieve an efficiency of 100%.

3. Describe the concept of reversible and irreversible process.

Ans) **Reversible Process:** A reversible process is an idealized or theoretical concept in thermodynamics where a system undergoes changes in such a way that the process can be reversed without leaving any net change in either the system or its surroundings. In other words, both the system and surroundings are restored to their initial states after the process is reversed.

Key Characteristics of Reversible Processes:

- i. **Quasi-static:** The process happens infinitely slowly, ensuring that the system is always in thermodynamic equilibrium with its surroundings.
- ii. **No Entropy Generation:** There is no increase in the total entropy of the system and its surroundings during a reversible process.
- iii. **Maximum Efficiency:** Reversible processes are the most efficient processes, often serving as the benchmark for real processes.

Examples:

- **Isothermal Expansion/Compression:** An ideal gas undergoing isothermal expansion or compression in a piston-cylinder assembly, where the process is carried out slowly enough to maintain thermal equilibrium.
- **Adiabatic Expansion/Compression:** An adiabatic process that is also reversible, like the slow compression or expansion of an ideal gas in an insulated cylinder.

- iv. In the final process $D \rightarrow A$ (fig. d), the base of the cylinder is replaced by a non-conducting wall and the gas is compressed adiabatically by increasing the load on the piston. The temperature of the gas increases from T_2 to T_1 , and volume decreases from V_4 to V_1 and work done by the piston on the gas is W_{CD} .

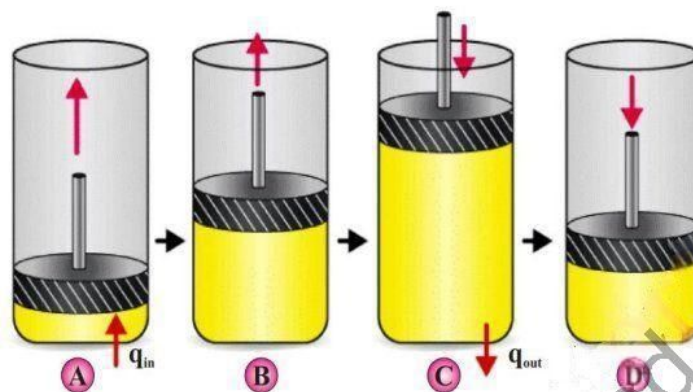


Fig: Carnot Engine cycle

Efficiency of Carnot engine is less than 100%: The efficiency of Carnot engine in percentage is

$$\text{Percentage efficiency } (\eta) = \left(1 - \frac{T_2}{T_1}\right) \times 100\%$$

Thus, the efficiency of Carnot engine depends on the temperature of hot and cold reservoirs. The larger the temperature difference of two reservoirs the greater is the efficiency but it can never be one of 100% unless cold reservoir is at absolute zero temperature ($T_2 = 0K$)

5. Describe that refrigerator is a heat engine operating in reverse as that of an ideal heat engine and find its efficiency.

Ans) **Refrigerator is reverse heat engine:** In a heat engine, the direction of energy transfer is from the hot reservoir to the cold reservoir, which is the natural direction. The role of heat engine is to process the energy from the hot reservoir so as to do useful work. What we wanted to transfer energy from the cold reservoir to the hot reservoir, because that is not the natural direction of energy transfer. We must put some energy into a device to be successful. Devices that perform this task are called refrigerator.

For example, houses in summer are cooled using refrigerators called air conditioners. The air conditioner transfers energy from the cool room in the home to the warm air outside.

Efficiency of a Refrigerator: The effectiveness of a refrigerator is described in terms of a number called the coefficient of performance (COP).

The COP is similar to the thermal efficiency for a heat engine in that it is a ratio of what you gain (energy transferred to or from a reservoir) to what you give (work input).

Entropy Decrease (Negative Change in Entropy): Conversely, when heat is removed from a system, the energy of the molecules decreases, leading to less vigorous molecular motion. This reduction in energy results in a more ordered state within the system, decreasing the system's entropy.

- **Negative ΔQ :** When heat is removed from the system (i.e. $\Delta Q < 0$), the change in entropy ΔS is negative. This indicates a decrease in disorder within the system.

7. Explain that increase in entropy means degradation of energy.

Ans) **Increase in Entropy Means Degradation of Energy:** Suppose a quantity of heat Q_1 in a reservoir at temperature T_1 . Let the temperature of the coldest available reservoir be T_0 . A Carnot engine working between the temperatures T_1 and T_0 can absorb heat Q_1 at temperature T_1 and do useful work W_1 given by

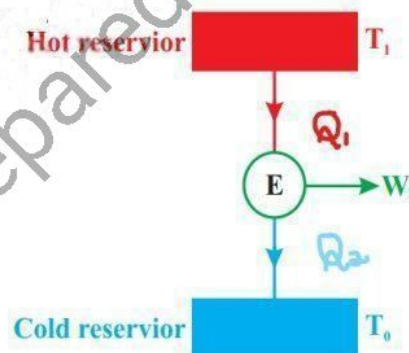
$$W_1 = Q_1 - Q_2$$

The efficiency of Carnot engine is

$$\eta = \frac{W_1}{Q_1} = 1 - \frac{T_0}{T_1}$$

$$W_1 = Q_1 \left(1 - \frac{T_0}{T_1} \right) \quad \dots (1)$$

Equation (1) gives maximum available energy which can be converted into useful work, when heat Q_1 is stored in the reservoir at temperature T_1 .

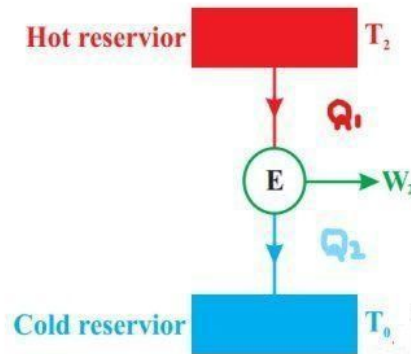


Consider an irreversible process, in which heat Q_1 flows from the reservoir at temperature T_1 , to another reservoir at a lower temperature T_2 . A Carnot engine working between the temperatures T_2 and T_0 can now take heat Q_1 from the reservoir at temperature T_2 and do useful work W_2 given by:

$$\text{Efficiency} = \eta = \frac{W_2}{Q_1} = 1 - \frac{T_0}{T_2}$$

$$W_2 = Q_1 \left(1 - \frac{T_0}{T_2}\right) \dots \dots (2)$$

Equation (2) gives the maximum available energy which can be converted into useful work. When heat Q_1 is stored in the reservoir at lower temperature T_2 .



As $T_2 < T_1$, we see from equation (1) and (2) that W_2 is less than W_1 , i.e. available energy decreases with the increase of entropy during an irreversible process. Since all natural processes are irreversible, we conclude that the energy of the universe is continuously becoming unavailable for useful work. This is called the degradation of energy. From the equation (1) and (2), we get

$$W_1 - W_2 = Q_1 \left(\frac{T_0}{T_2} - \frac{T_0}{T_1} \right)$$

$$W_1 - W_2 = Q_1 \left(\frac{Q_1}{T_2} - \frac{Q_1}{T_1} \right) T_0 \dots \dots (3)$$

Now $\frac{Q}{T_1}$ = decrease in entropy of the reservoir at temperature T_1 and $\frac{Q}{T_2}$ = increase in entropy of the reservoir at temperature T_2 .

Thus $\left(\frac{Q_1}{T_2} - \frac{Q_1}{T_1} \right)$ is increase in entropy of the universe. Also $(W_1 - W_2)$ is the amount of energy which has been degraded or made unavailable for useful work. Hence equation (3) shows that the increase of unavailable energy is equal to the increase in entropy of the universe multiplied by the temperature of the coldest available reservoir. Using eq: (3) an entropy increases, so unavailable energy also increases, so the useful energy decreases. It is called degradation of energy.

section (D) Numerical:

1. A Carnot engine takes 2000 J of heat from a reservoir at 500 K does some work, and discards some heat to a reservoir at 350 K. How much heat is discarded, how much work does the engine do, and what is the efficiency?

Data:

$$Q_1 = 2000 \text{ J}, T_1 = 500 \text{ K}, T_2 = 350 \text{ K}, Q_2 = ?, W = ?, \eta = ?$$

Solution:

$$\eta = 1 - \frac{T_2}{T_1}$$

$$\eta = 1 - \frac{350}{500}$$

$$\eta = 0.3 \times 100\% = 30\%$$

$$\eta = \frac{W}{Q_1}$$

$$0.3 = \frac{W}{2000}$$

$$W = 600 \text{ J}$$

$$W = Q_1 - Q_2$$

$$600 = 2000 - Q_2$$

$$Q_2 = 1400 \text{ J}$$

2. One kilogram of ice at 0°C is melted and converted to water at 0°C. Compute its change in entropy.

Data:

$$m = 1 \text{ kg}, T = 0^\circ\text{C} = 0 + 273 = 273 \text{ K}, \Delta S = ?$$

Solution:

Assuming that the melting is done reversibly. The heat of fusion of water is $L_f = 3.34 \times 10^5 \text{ J/kg}$.

$$\Delta Q = m \times L_f$$

$$\Delta Q = 1 \times 3.34 \times 10^5 = 3.34 \times 10^5 \text{ J}$$

$$\Delta S = \frac{3.34 \times 10^5}{273} = 1223.443 \text{ J/K}$$

3. In a high-pressure steam turbine engine, the steam is heated to 600°C and exhausted at about 90°C . What is the highest possible efficiency of any engine that operates between these two temperatures?

Data:

$$T_1 = 600^{\circ}\text{C} = 600 + 273 = 873 \text{ K}, T_2 = 90^{\circ}\text{C} = 90 + 273 = 363 \text{ K}, \eta = ?$$

Solution:

We are going to calculate Carnot efficiency

$$\eta = 1 - \frac{T_2}{T_1}$$

$$\eta = 1 - \frac{363}{873} = 0.584$$

$$\eta = 0.584 \times 100\% = 58.4 \%$$

4. Temperature difference between the surface water and bottom water in Manchester Lake might be 5°C . Assuming the surface water to be at 20°C . What highest efficiency a steam engine could have if it operates between these two temperatures?

Data:

$$\Delta T = 5^{\circ}\text{C} = 5 + 273 = 278 \text{ K}, T_1 = 20^{\circ}\text{C} = 20 + 273 = 293 \text{ K}, \eta = ?$$

Solution:

We are going to calculate Carnot efficiency

$$\eta = 1 - \frac{T_2}{T_1}$$

$$\therefore \Delta T = T_1 - T_2$$

$$5 = 20 - T_2$$

$$T_2 = 15^{\circ}\text{C} = 288 \text{ K}$$

$$\eta = 1 - \frac{288}{293}$$

$$\eta = 0.0171$$

$$\eta = 0.0171 \times 100\%$$

$$\eta = 1.71 \%$$

5. A heat engine works at the rate of 500 kW. The efficiency of the engine is 30%. Calculate the loss of heat per hour.

Data:

$$\dot{W} = 500 \text{ kW}, \eta = 30\%/100\% = 0.3, Q_2 = ?$$

Solution:

$$\eta = \frac{W}{Q_1}$$

$$0.3 = \frac{500 \times 10^3}{Q_1}$$

$$\dot{Q}_1 = 1.67 \times 10^6 \text{ W}$$

$$\dot{W} = \dot{Q}_1 - \dot{Q}_2$$

$$500 \times 10^3 = 1.67 \times 10^6 - \dot{Q}_2$$

$$\dot{Q}_2 = 1.67 \times 10^6 - 500 \times 10^3$$

$$\dot{Q}_2 = 1.17 \times 10^6 \text{ W}$$

$$Q_2 = 1.17 \times 10^6 \times 3600 \text{ (loss of heat per hour)}$$

$$Q_2 = 1.17 \times 10^6 \times 3600 = 4.2 \times 10^9 \text{ J}$$

6. A heat engine performs work of 0.4166 watts in one hour and rejects 4500 J of heat to the sink. What is the efficiency of engine?

Data:

$$\dot{W} = 0.4166 \text{ watts}, t = 1 \text{ hr} = 1 \times 3600 = 3600 \text{ s}, Q_2 = 4500 \text{ J}, \eta = ?$$

Solution:

$$W = \dot{W} \times t = 0.4166 \times 3600 = 1499.76 \text{ J}$$

$$W = Q_1 - Q_2$$

$$1499.76 = Q_1 - 4500$$

$$Q_1 = 5999.76 \text{ J}$$

$$\eta = \frac{W}{Q_1}$$

$$\eta = \frac{1499.76}{5999.76}$$

$$\eta = 0.249 \rightarrow \eta = 0.249 \times 100\% = 24.9\%$$

7. A Carnot engine operates between the temperatures 850K and 300K the engine performs 1200 J of work in each cycle, which takes 0.25 sec.

- (a) What is the efficiency of this engine?
- (b) What is the average power of this engine?
- (c) How much energy is extracted as heat from the high temperature reservoir?
- (d) How much energy is delivered as heat to the low temperature reservoir?

Data:

$$T_1 = 850 \text{ K}, T_2 = 300 \text{ K}, W = 1200 \text{ J}, t = 0.25 \text{ sec}$$

$$(a) \eta = ?$$

$$(b) \dot{W} = ?$$

$$(c) Q_1 = ?$$

$$(d) Q_2 = ?$$

Solution:

(a)

$$\eta = 1 - \frac{T_2}{T_1}$$

$$\eta = 1 - \frac{300}{850}$$

$$\eta = 0.647$$

$$\eta = 0.647 \times 100\% = 64.7 \%$$

(b)

$$\dot{W} = \frac{W}{t}$$

$$\dot{W} = \frac{1200}{0.25}$$

$$\dot{W} = 4800 \text{ W} = 4.8 \text{ kW}$$

(c)

$$\eta = \frac{W}{Q_1}$$

$$0.647 = \frac{1200}{Q_1} \rightarrow Q_1 = 1854.71 \text{ J} \approx 1855 \text{ J}$$

(d)

$$W = Q_1 - Q_2$$

$$1200 = 1854.71 - Q_2$$

$$Q_2 = 654.71 \text{ J} \approx 655 \text{ J}$$

8. A Carnot engine absorbs 52kJ as heat and exhaust 36kJ as heat in each cycle. Calculate:

(a) The engine efficiency

(b) The work done per cycle in kilojoules.

Data:

$$Q_1 = 52 \text{ kJ}, Q_2 = 36 \text{ kJ},$$

$$(a) \eta = ?$$

$$(b) W = ?$$

Solution:

(b)

$$W = Q_1 - Q_2$$

$$W = 52 - 36$$

$$W = 16 \text{ kJ}$$

(a)

$$\eta = \frac{16}{52}$$

$$\eta = 0.3076$$

$$\eta = 0.3076 \times 100\% = 30.76\%$$

UNIT: 18 MAGNETIC FIELD

KEY

1. b	2. b	3. a	4. a	5. d
6. b	7. a	8. a	9. c	10. c

Section (B) CRQs (Short Answered Questions):

1. Charge particles are fired in vacuum tube hit a fluorescence screen. Will it be possible to know whether they positive or negative?

Ans) Yes it is possible to know whether they positive or negative.

2. What is a solenoid, and how does it differ from a simple coil of wire?

Ans) Solenoids: Solenoids are basically coils of wire. These generate a magnetic field which exerts a force over a metallic element. This happens when we apply the electric current to the solenoids.

Difference between simple coil and solenoid: A solenoid is a type of coil with a long, tightly wound cylindrical shape that produces a uniform magnetic field inside, often used in applications requiring precise control of magnetic effects. In contrast, a simple coil has a more varied magnetic field and is typically used for generating inductance or in circuits.

3. Can a solenoid generate a magnetic field without any current flowing through it? Why or why not?

Ans) No, a solenoid cannot generate a magnetic field without current flowing through it because the magnetic field is produced by the movement of electric charges through the wire. Without current, there are no moving charges to create the magnetic field.

4. Explain why a toroid is often preferred over a straight solenoid when designing certain types of electrical components.

Ans) A toroid is often preferred over a straight solenoid because it confines the magnetic field within its core, minimizing external electromagnetic interference and improving efficiency. The continuous, closed-loop shape of a toroid ensures a more uniform and controlled magnetic field, making it ideal for applications like transformers and inductors where precise magnetic properties are crucial.

5. What role does the Ampere's circuital law play in understanding the magnetic field inside a toroid?

Ans) Ampere's circuital law helps in understanding the magnetic field inside a toroid by relating the magnetic field strength around a closed loop to the current passing through the loop. For a toroid, it shows that the magnetic field is constant along the circular path inside the toroid and

directly proportional to the current and the number of turns in the winding, allowing for precise calculations of the field strength within the core.

6. What is a galvanometer, and what is its primary function in an electrical circuit?

Ans) **Galvanometer:** A galvanometer is a device that is used to detect a small electric current or measure its magnitude.

The function of a galvanometer in a circuit:

- The galvanometer is a piece of electromagnetic equipment that detects electric currents.
- This is an electromagnetic instrument that measures the electric current in a circuit.

Section (C): FRQs (Long Answered Questions):

1. Can a galvanometer measure both DC and AC currents? Explain any limitations it might have with AC measurements.

Ans) **DC Measurements:** For direct current (DC) measurements, a galvanometer is quite effective. The current flows through a coil in a magnetic field, creating a torque that moves a needle or pointer. The amount of deflection is directly proportional to the current, allowing for accurate readings of steady, unidirectional currents.

AC Measurements: No, we cannot use galvanometer to measure alternating current because in AC, the direction of current keeps changing frequently, and so pointer will not be able to deflect.

Limitations:

- Response to AC:** A traditional galvanometer, designed primarily for DC, struggles with the alternating nature of AC. The needle may not move in response to the rapidly changing direction of AC, causing it to show a lower average deflection or inaccurate readings.
- Frequency Dependence:** The frequency of the AC signal affects the galvanometer's ability to provide accurate readings. Higher frequencies cause the galvanometer to average the current over time, potentially leading to significant discrepancies between the actual AC current and the measured value.
- Design Considerations:** Some galvanometers are specifically designed to handle AC measurements by incorporating features such as rectifiers to convert AC to DC before measurement. However, standard galvanometers without these modifications are generally not suitable for accurate AC measurement.

2. Why should an ammeter ideally have a very low resistance compared to the circuit it is measuring?

Ans) An ammeter should ideally have very low resistance compared to the circuit it is measuring to ensure accurate current measurement and minimal impact on the circuit. Here's why:

- i. **Minimize Voltage Drop:** A low-resistance ammeter ensures that the voltage drop across it is minimal. If the ammeter had high resistance, it would cause a significant voltage drop in the circuit, altering the current flow and leading to inaccurate measurements.
- ii. **Negligible effect on the total resistance of the circuit:** To measure the current accurately, the ammeter should ideally not affect the circuit. Low resistance ensures that the ammeter has a negligible effect on the total resistance of the circuit, thus allowing it to measure the true current without influencing the circuit operation.
- iii. **Preventing Circuit Disruption:** High resistance in the ammeter would create an additional resistance in the circuit, potentially changing the circuit's behavior and reducing the current flowing through it. A low-resistance ammeter prevents such disruptions, ensuring that the circuit operates as intended while the current is being measured.

3. What is the potential risk of using a voltmeter with a high internal resistance in a circuit? How can this risk be mitigated?

Ans) **Potential Risk:** Using a voltmeter with high internal resistance generally reduces the impact on a circuit, but it can still present potential risks in specific situations:

- i. **Loading Effect in High-Impedance Circuits:** In circuits with extremely high impedance, even a high-resistance voltmeter can cause a small but significant loading effect. This means the voltmeter might alter the circuit's behavior slightly by adding its own resistance to the circuit, leading to a slight decrease in the measured voltage and potentially inaccurate readings.
- ii. **Impact on Sensitive Measurements:** For very sensitive or precision measurements, the high internal resistance of the voltmeter might still affect the circuit. If the circuit impedance is comparable to or higher than the voltmeter's resistance, it can cause measurable deviations in voltage readings.

Mitigation:

- i. **Select Appropriate Equipment:** Choose a voltmeter with a resistance that is several orders of magnitude higher than the circuit's impedance to minimize the loading effect.
- ii. **Consider Measurement Conditions:** Be aware of the circuit's impedance and the accuracy requirements of your measurement. In cases where precise voltage readings are crucial, ensure that the measurement setup and equipment are suitable for the circuit characteristics.

4. Describe the basic working principle of a voltmeter. How does it measure voltage across a component in a circuit?

Ans) **Working principle:** The working principle of a voltmeter is based on Ohm's Law, where it measures the potential difference across a component by being connected in parallel with high resistance, ensuring minimal impact on the circuit.

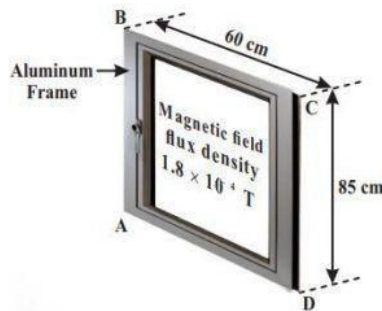
Working: A voltmeter measures the voltage across a component in a circuit by utilizing its high internal resistance and parallel connection to the component. Here's a detailed explanation of the process:

- i. **High Internal Resistance:** A voltmeter is designed with very high internal resistance, typically in the range of megaohms (e.g., 10 megohms or more). This high resistance ensures that it draws minimal current from the circuit, thereby not significantly affecting the circuit's operation or altering the voltage being measured.
- ii. **Parallel Connection:** To measure the voltage across a component, the voltmeter is connected in parallel with that component. This means the voltmeter's terminals are attached to the two points where the voltage is to be measured. The parallel connection allows the voltmeter to measure the potential difference between these two points directly.
- iii. **Voltage Measurement Mechanism:**
 - **Analog Voltmeters:** In an analog voltmeter, the voltage causes a small current to flow through the meter's internal circuit. This current moves a needle across a scale. The position of the needle on the scale corresponds to the voltage across the component.
 - **Digital Voltmeters:** In a digital voltmeter, the voltage is converted into a digital signal by an internal analog-to-digital converter (ADC). The ADC translates the voltage into a numerical value, which is then displayed on the digital screen.
- iv. **Display of Voltage:** The voltmeter's display shows the measured voltage, reflecting the potential difference between the two points where it is connected. This voltage reading represents the electrical potential difference across the component.

Section (D): Numerical:

1. An aluminum window has a width of 60 cm and length of 85 cm as shown in the figure.

- When the window is closed the magnetic flux density is $1.8 \times 10^{-4} \text{ T}$ is normal to window.
- Calculate the magnetic flux through the window.



Data:

$$w = 60 \text{ cm}, l = 85 \text{ cm}, B = 1.8 \times 10^{-4} \text{ T}, \theta = 0^\circ (\text{normal}), \phi = ?$$

Solution:

$$A = 60 \times 85 = 5100 \text{ cm}^2 = 5100 \times 10^{-4} \text{ m}^2$$

$$\phi = BA \cos(\theta)$$

$$\phi = (1.8 \times 10^{-4})(5100 \times 10^{-4}) \cos(0)$$

$$\phi = 9.18 \times 10^{-5} \text{ Wb}$$

2. The poles of a horse shoe magnet measures $8 \text{ cm} \times 3.2 \text{ cm}$, the magnetic flux density between the magnet poles is 80 mT . Outside of the magnet the magnetic flux density is zero. Calculate the magnetic flux between the poles of a magnet.

Data:

$$A = 8 \text{ cm} \times 3.2 \text{ cm} = 25.6 \times 10^{-4} \text{ m}^2, B = 80 \text{ mT} = 80 \times 10^{-3} \text{ T}, \phi = ?$$

Solution:

$$\phi = BA \cos \theta$$

We assume area and magnetic flux density are parallel, therefore $\theta = 0^\circ$

$$\phi = (80 \times 10^{-3})(25.6 \times 10^{-4}) \cos(0)$$

$$\phi = 2,048 \times 10^{-4} \text{ Wb}$$

5. A moving coil galvanometer has resistance of $50\ \Omega$ and it gives full scale deflection at 4mA current. A voltmeter is made using this galvanometer and a $5\text{ k}\Omega$ resistance. Calculate the maximum voltage that can be measured using this voltmeter.

Data:

$$R_g = 50\ \Omega, I_g = 4\text{mA} = 4 \times 10^{-3}\text{A}, R_x = 5\text{ k}\Omega = 5 \times 10^3\ \Omega, V = ?$$

Solution:

$$V = I_g(R_g + R_x)$$

$$V = 4 \times 10^{-3}(50 + 5 \times 10^3)$$

$$V = 20.2\text{ V}$$

6. Compute the magnitude of the magnetic field of a long, straight wire carrying a current of 1A at distance of 1 m from it. Compare it with Earth's magnetic field.

Data:

$$B = ?, I = 1\text{A}, r = 1\text{ m}$$

Compare it with Earth's magnetic field = ?

Solution:

Magnetic field due to a long wire

$$B = \frac{\mu_0 I}{2\pi r}$$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\therefore \mu_0 = 4\pi \times 10^{-7}$$

$$B = \frac{(4\pi \times 10^{-7})(1)}{(2\pi)(1)}$$

$$B = 2 \times 10^{-7}\text{ T}$$

$$\text{Earth's magnetic field} = 5 \times 10^{-5}\text{ T}$$

If we divide them

$$\frac{2 \times 10^{-7}}{5 \times 10^{-5}} = \frac{1}{250}$$

So the magnetic field is 250 times less strong than earth's magnetic field.

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7. Find the current in a long straight wire that would produce a magnetic field twice the strength of the Earth's at a distance of 5.0cm from the wire. (Magnetic field of Earth = $5.0 \times 10^{-5} T$)

Data:

$$I = ?, B = 2B_e = 2(5.0 \times 10^{-5}) = 1 \times 10^{-4} T, r = 5.0 \text{ cm} = 5.0 \times 10^{-2} \text{ m}$$

Solution:

$$1 \times 10^{-4} = \frac{(4\pi \times 10^{-7})I}{(2\pi)(5.0 \times 10^{-2})}$$

$$I = 25 \text{ A}$$

8. What is flux density at a distance of 0.1 m in air from along straight conductor carrying a current of 6.5 A. Calculate the force per unit length on a similar parallel conductor at a distance of 0.1m from the first and carrying a current of 3A.

Data:

$$B = ?, r = 0.1 \text{ m}, I_1 = 6.5 \text{ A}, \frac{F}{L} = ?, I_2 = 3 \text{ A}$$

Solution:

$$B = \frac{\mu_0 I_1}{2\pi r}$$

$$\therefore \mu_0 = 4\pi \times 10^{-7}$$

$$B = \frac{(4\pi \times 10^{-7})(6.5)}{(2\pi)(0.1)}$$

$$B = 13 \times 10^{-6} T = 13 \times 10^{-6} \frac{\text{Weber}}{\text{m}^2}$$

$$F = B I_2 L \sin \theta$$

We assume length and magnetic flux density are perpendicular, therefore $\theta = 90^\circ$

$$\frac{F}{L} = (13 \times 10^{-6})(3)\sin 90$$

$$\frac{F}{L} = 39 \times 10^{-6} \frac{N}{m}$$

UNIT: 20 AC CIRCUIT

MCQ'S

KEY

1. a	2. a	3. b	4. a	5. d
6. c	7. d	8. a	9. c	10. b

Section (B): CRQs (Short Answered Questions):

1. Define RMS voltage and explain its significance in AC circuits.

Ans) **Definition of RMS Voltage:** RMS (Root Mean Square) voltage is a measure of the effective value of an alternating current (AC) voltage. It represents the equivalent DC voltage that would deliver the same power to a resistive load.

Significance in AC Circuits:

- **Power Calculation:** RMS voltage is crucial for calculating the power in AC circuits.
- **Comparison with DC:** RMS voltage allows for a direct comparison between AC and DC circuits by expressing AC voltage in terms of an equivalent DC voltage.
- **Safety and Standards:** Electrical standards and safety guidelines typically use RMS values to specify voltages and currents because they reflect the actual potential for delivering power.

2. Explain the difference between peak voltage, RMS voltage, and average voltage in AC circuits.

Ans)

Parameter	Peak Voltage (V_{peak})	RMS Voltage (V_{RMS})	Average Voltage (V_{avg})
Definition	Maximum instantaneous voltage in the waveform.	Effective value of AC voltage equivalent to DC voltage in terms of power.	Average of the instantaneous voltage over a full cycle.
Formula	It has no formula.	$V_{RMS} = \frac{V_{peak}}{\sqrt{2}}$	For sinusoidal: $V_{avg} = \frac{2 \times V_{peak}}{\pi}$
Cycle Average	Value of the highest point in one cycle.	Represents effective power-carrying capability.	Zero for a full sinusoidal cycle; mean absolute value is used.
Practical Use	Useful for understanding maximum values.	Used for power calculations and effective voltage measurement.	Less commonly used for power calculations; more for general average.

3. Define alternating current (AC) and explain how it differs from direct current (DC).

Ans) **Alternating current (AC):** Alternating current (AC) is an electric current that periodically reverses direction and changes its magnitude continuously with time.

Differentiate between alternating current (AC) and direct current (DC):

Characteristic	Alternating Current (AC)	Direct Current (DC)
Definition	Electric current that reverses direction periodically.	Electric current that flows in one constant direction.
Direction of Flow	Changes direction periodically (back and forth).	Flows in a single, unchanging direction.
Frequency	Has a frequency (e.g., 50 Hz or 60 Hz, depending on the region).	Frequency is zero (no oscillation).
Source	Generated by power plants (e.g., using generators).	Generated by batteries, solar cells, and DC generators.

4. Define reactance in AC circuits and differentiate between capacitive and inductive reactance.

Ans) **Reactance in AC circuits:** Reactance is the measure of the opposition to the flow of alternating current caused by the inductance and capacitance in a circuit rather than by resistance.

Differentiate between capacitive and inductive reactance:

Characteristic	Capacitive Reactance	Inductive Reactance
Definition	Opposition to the change in voltage in a circuit due to capacitance.	Opposition to the change in current in a circuit due to inductance.
Formula	$X_C = \frac{1}{2\pi fC}$	$X_L = 2\pi fL$
Depends on	Inversely proportional to frequency (f) and capacitance (C).	Directly proportional to frequency (f) and inductance (L).
Frequency Relationship	Decreases as frequency increases.	Increases as frequency increases.

5. Describe the behavior of capacitors and inductors in AC circuits.

Ans) **Behavior of Capacitors in AC Circuits:** In AC circuits, a capacitor causes the current to lead the voltage by 90 degrees, meaning that the current reaches its peak before the voltage does. The opposition a capacitor provides to AC, known as capacitive reactance, decreases as the frequency increases. This is described by the formula $X_C = \frac{1}{2\pi fC}$, where f is the frequency and C is the capacitance. As a result, capacitors allow more current to pass through at higher frequencies. Capacitors store energy in the form of an electric field between their plates, releasing it when the voltage changes. At high frequencies, capacitors behave almost like short circuits, allowing AC to flow through with minimal resistance.

Behavior of Inductors in AC Circuits: In an AC circuit, an inductor causes the current to lag behind the voltage by 90 degrees, meaning that the voltage reaches its peak before the current does. The inductive reactance, which is the opposition an inductor provides to AC, increases with frequency, as described by the formula $X_L = 2\pi fL$, where f is the frequency and L is the inductance. Therefore, inductors resist the flow of current more as the frequency increases. Inductors store energy in the form of a magnetic field around their coils, releasing it when the current changes. At high frequencies, inductors act almost like open circuits, significantly impeding the flow of AC.

6. Explain the phase relationship between voltage and current in capacitive and inductive AC loads.

Ans) When capacitors or inductors are involved in an AC circuit, the current and voltage do not peak at the same time. The fraction of a period difference between the peaks expressed in degrees is said to be the phase difference. The phase difference is ≤ 90 degrees. It is customary to use the angle by which the voltage leads the current. This leads to a positive phase for inductive circuits since current lags the voltage in an inductive circuit. The phase is negative for a capacitive circuit since the current leads the voltage.

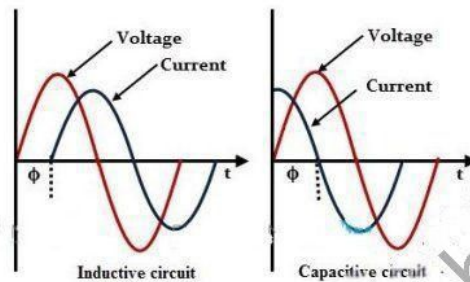


Fig: Phase relationship between voltage and current

7. What is resonance in AC circuits? Discuss its conditions and applications.

Ans) **Resonance in AC Circuits:** Resonance in AC circuits occurs when the inductive reactance (X_L) and capacitive reactance (X_C) are equal in magnitude but opposite in phase, effectively canceling each other out. When this happens, the circuit's impedance is minimized, and the circuit behaves purely resistively. The result is a maximum current flow at a specific frequency known as the resonant frequency.

Conditions for Resonance: For resonance to occur in an AC circuit, particularly in an RLC circuit (a circuit containing a resistor, inductor, and capacitor), the following condition must be met:

$$X_L = X_C$$

Applications of Resonance:

1. **Tuning Circuits:** Resonance is used in radio and television receivers to select specific frequencies from a broad range of signals. Tuning circuits resonate at the desired frequency, filtering out all other signals.
2. **Filters:** Resonant circuits are used in band-pass and band-stop filters to allow or block specific frequency ranges. This is particularly useful in communication systems.
3. **Impedance Matching:** Resonance can be employed to match the impedance of different parts of a circuit, maximizing power transfer. This is important in applications like antenna systems and audio engineering.

4. **Inductive Heating:** Resonance is used in inductive heating applications, such as in induction cooktops, where the resonant frequency is used to efficiently transfer energy and generate heat.
5. **Oscillators:** Resonant circuits are fundamental in designing oscillators, which generate AC signals at a specific frequency. These oscillators are widely used in clocks, radios, and computers.

8. Discuss the role of transformers in AC circuits and explain how they work.

Ans) **Role of Transformers:** Transformers are essential in adjusting voltage levels in AC circuits, enabling efficient long-distance power transmission. They step up voltage to minimize losses and step it down for safe consumer use, making them vital in power distribution systems.

How Transformers Work: Transformers operate based on the principle of electromagnetic induction, as described by Faraday's Law. When an alternating current flows through the primary winding, it generates a changing magnetic field within the transformer's core. This fluctuating magnetic field induces a voltage in the secondary winding. The voltage induced depends on the ratio of the number of turns in the primary and secondary coils, known as the turns ratio. If the secondary coil has more turns than the primary, the transformer is a step-up transformer, increasing the voltage. Conversely, if the primary coil has more turns, it's a step-down transformer, reducing the voltage. This process ensures efficient energy transfer and voltage regulation across various applications.

9. Explain the behavior of an RLC series circuit in terms of impedance, resonance, and phase angle.

Ans) **Impedance in an RLC series circuit:** In an RLC series circuit, the total impedance (Z) is a combination of resistance (R), inductive reactance (X_L), and capacitive reactance (X_C). The impedance is given by:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

The impedance depends on the frequency of the AC supply. At low frequencies, X_C dominates, leading to higher impedance. At high frequencies, X_L dominates, also resulting in higher impedance. The impedance is minimized at the resonant frequency, where $X_L = X_C$.

Resonance in an RLC Series Circuit: Resonance occurs in an RLC series circuit when the inductive reactance and capacitive reactance are equal:

$$X_L = X_C$$

At the resonant frequency f_r , the impedance of the circuit is purely resistive and is at its minimum value:

$$Z_{\text{resonance}} = R$$

At this point, the current in the circuit is at its maximum, and the circuit can oscillate at the resonant frequency without external input, depending on losses.

Phase Angle in an RLC Series Circuit: The phase angle (ϕ) in an RLC series circuit represents the phase difference between the total voltage and the current. It is determined by the relative values of X_L and X_C :

- If $X_L > X_C$, the circuit is inductive, and the phase angle is positive ($\phi > 0$), meaning the current lags behind the voltage.
- If $X_C > X_L$, the circuit is capacitive, and the phase angle is negative ($\phi < 0$), meaning the current leads the voltage.
- At resonance, $X_L = X_C$, and the phase angle is zero ($\phi = 0$), meaning the current and voltage are in phase.

10. Describe the operation and applications of a transformer in AC circuits.

Ans) **Operation of a Transformer:** A transformer operates based on the principle of electromagnetic induction to transfer electrical energy between two or more circuits. It consists of two main components: primary and secondary coils (windings) wound around a common core.

- AC Voltage Application:** An alternating current (AC) is applied to the primary coil. This creates a time-varying magnetic field around the coil.
- Magnetic Induction:** The varying magnetic field generated by the primary coil induces a voltage in the secondary coil through the core, which is usually made of ferromagnetic material to enhance magnetic coupling.
- Voltage Transformation:** The voltage induced in the secondary coil is proportional to the turns ratio of the coils. If N_p and N_s are the number of turns in the primary and secondary coils, respectively, the voltage ratio is given by:

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

Where V_p is the primary voltage and V_s is the secondary voltage. This allows the transformer to either step up (increase) or step down (decrease) the voltage according to the turns ratio.

Applications of a Transformer:

- Voltage Regulation:** Transformers are used to adjust voltage levels in power distribution systems. They step up the voltage for efficient long-distance transmission and step it down to usable levels for end-user applications.
- Isolation:** Transformers provide electrical isolation between different parts of a circuit, which enhances safety and prevents interference between high-voltage and low-voltage systems.

- iii. **Impedance Matching:** Transformers are used to match impedances between different circuit components, such as in audio systems, where they ensure maximum power transfer between stages of the circuit.
- iv. **Signal Processing:** In electronics, transformers are used in signal processing applications to convert and filter signals. For instance, in radio and television receivers, transformers help in tuning and signal amplification.

Section (C): FRQs (Long Answered Questions):

1. Explain the concept of phasor in the context of AC voltage. How are phasors used to represent sinusoidal voltages?

Ans) **Vector Representation of an Alternating quantity:** To solve AC problems effectively, it is beneficial to represent a sinusoidal quantity, such as voltage or current, using a line of specific length that rotates in a counterclockwise direction with the same angular velocity as the sinusoidal quantity. This rotating line is commonly referred to as a phasor.

Let's consider a line denoted as OA, referred to as a phasor, which accurately represents, to scale, the maximum value of an alternating quantity, such as electromotive force (emf). In this representation, OA equals the maximum emf value and rotates counterclockwise at an angular velocity of radians per second around the point O, as illustrated in Figure below. An arrowhead is placed at the outer end of the phasor, serving both to indicate the assumed direction of movement and to specify the precise length of the phasor, especially when multiple phasors coincide.

In Figure below, OA represents the phasor after it has rotated through an angle θ , equivalent to ωt , from its initial position when the emf was at its zero value. The projection of OA on the Y-axis, denoted as OB, equals OA multiplied by $\sin \theta$, which is equivalent to $E_{\max} \sin \omega t$, representing the instantaneous value of the emf, denoted as 'e', at that particular moment. Therefore, the projection of OA on the vertical axis accurately portrays, to scale, the instantaneous value of the emf.

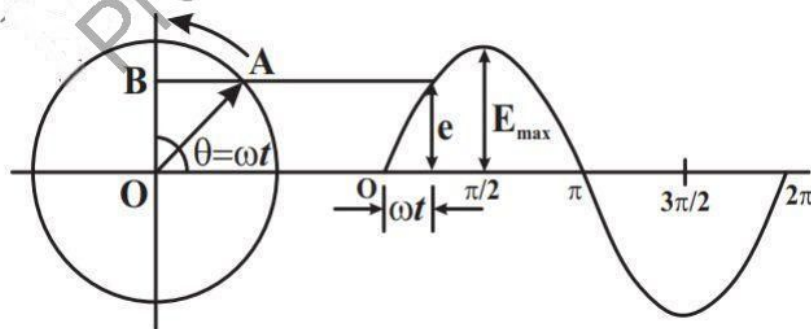


Fig: The propagation of sinusoidal wave and its vector (phasor) representation

2. Describe the concept of impedance in AC circuits. How does impedance differ from resistance, and what are the units of impedance?

Ans) **Key Aspects of Impedance:** Impedance is a fundamental concept in AC (alternating current) circuits, representing the total opposition that a circuit offers to the flow of alternating current. It is a combination of resistance (from resistors) and reactance (from inductors and capacitors). Impedance takes into account both the magnitude of opposition and the phase shift between voltage and current.

It is a complex quantity that includes both resistance and reactance. The formula for impedance is typically expressed as:

$$Z = R + jX$$

Where

- Z is the impedance (a complex number).
- R is the resistance (real part).
- X is the reactance (imaginary part), which can be either inductive or capacitive.
- j is the imaginary unit (equivalent to the square root of -1).

Difference between impedance and resistance:

i. Definition:

- **Impedance:** Impedance (denoted as Z) is a measure of the opposition that a circuit presents to the flow of alternating current (AC). It includes both resistance and reactance (the opposition due to capacitors and inductors).
- **Resistance:** Resistance (denoted as R) is the opposition to the flow of direct current (DC) in a circuit and depends only on the material and shape of the conductor.

ii. Nature:

- **Impedance:** Impedance is a complex quantity consisting of both real (resistance) and imaginary (reactance) components. It is frequency-dependent and affects AC circuits.
- **Resistance:** Resistance is a purely real quantity and applies to both AC and DC circuits, but it is independent of frequency.

iii. Formula:

- **Impedance:**

$$Z = R + jX$$

- **Resistance:**

$$R = \frac{V}{I}$$

Where R is resistance, V is voltage, and I is current.

iv. **AC vs. DC:**

- **Impedance:** Relevant to AC circuits where both resistance and reactance (due to capacitors and inductors) are present. Impedance affects the amplitude and phase of the current.
- **Resistance:** Applies to both AC and DC circuits, but in AC circuits, it only contributes to the real part of impedance. In DC circuits, only resistance matters since reactance is zero at zero frequency.

v. **Frequency Dependence:**

- **Impedance:** Varies with the frequency of the AC signal. Higher frequencies increase reactance for inductors and decrease it for capacitors.
- **Resistance:** Constant regardless of the frequency of the current.

Units of Impedance: Impedance is measured in ohms (Ω), just like resistance. However, since it's a complex quantity, it reflects both the resistive and reactive components of the circuit.

3. How does the reactance of an inductor and a capacitor change with frequency in an AC circuit? Provide an explanation based on the fundamental formulas?

Ans) The reactance of an inductor and a capacitor in an AC circuit changes with the frequency of the alternating current, and this relationship is governed by fundamental formulas for inductive and capacitive reactance.

- i. **Inductive Reactance (X_L):** Inductive reactance (X_L) is the opposition that an inductor offers to the change in current in an AC circuit. It increases with the frequency of the AC signal.

The formula for inductive reactance is:

$$X_L = 2\pi fL$$

Where:

- X_L is the inductive reactance (in ohms),
- f is the frequency of the AC signal (in hertz),
- L is the inductance (in henries).

Explanation:

- As frequency (f) increases, the inductive reactance increases linearly. This means that the higher the frequency, the more the inductor resists the flow of alternating current.
- At low frequencies, the reactance of an inductor is small, allowing more current to flow. As the frequency increases, the inductor opposes the current flow more strongly, causing the current to lag behind the voltage.

- ii. **Capacitive Reactance (X_C):** Capacitive reactance (X_C) is the opposition that a capacitor offers to the change in voltage in an AC circuit. It decreases with increasing frequency.

The formula for capacitive reactance is:

$$X_C = \frac{1}{2\pi fC}$$

- X_C is the capacitive reactance (in ohms),
- f is the frequency of the AC signal (in hertz),
- C is the capacitance (in farads).

Explanation:

- As the frequency (f) increases, capacitive reactance decreases. This means that at high frequencies, the capacitor offers less opposition to the current flow.
- At low frequencies, the capacitive reactance is high, blocking much of the current. As the frequency increases, the capacitor allows more current to flow, and the current leads the voltage.

4. Discuss the concept of resonance in RLC circuits. What conditions lead to resonance and how does it affect the behavior of the circuit?

Ans) **Resonant Frequency:** For a certain frequency the capacitive and inductive reactance becomes equal, $X_C = X_L$. This frequency is called resonant frequency f_r , and circuit is said to be in resonance state.

Condition lead to resonance: Resonance occurs when

$$\text{Capacitive Reactance} = \text{Inductive Reactance}$$

$$X_C = X_L$$

Behavior of the circuit due to resonance: From figure below note that X_C and X_L are in opposite direction. Therefore, at resonant frequency they cancel each effect in circuit. Now opposition to the current flow is solely offered by resistor, resulting in maximum current to flow through the circuit, moreover either side of resonant frequency the current in the circuit decreases, as shown in the figure.

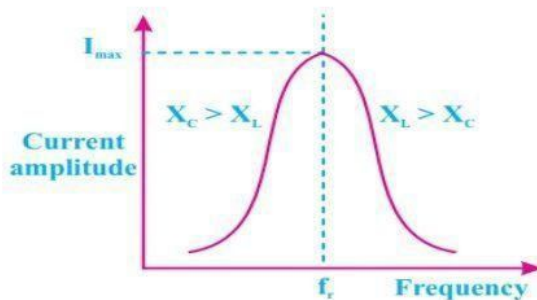


Figure: The resonant frequency

5. Compare and contrast series and parallel resonance in RLC circuits. What are the key differences between the two resonance configurations?

Ans) Series and parallel resonance are two important configurations of RLC circuits, each having unique characteristics and behaviors. Here's a comparison of the two:

i. **Circuit Configuration:**

- **Series Resonance:** In a series resonance circuit, the resistor (R), inductor (L), and capacitor (C) are connected in series, meaning the same current flows through all three components.
- **Parallel Resonance:** In a parallel resonance circuit, the resistor, inductor, and capacitor are connected in parallel, meaning the voltage across each component is the same, but the current is divided among them.

ii. **Impedance at Resonance:**

- **Series Resonance:** At resonance, the total impedance of the circuit is at its minimum and is equal to the resistance (R) alone, as the inductive and capacitive reactances cancel each other out. The circuit behaves like a purely resistive circuit.
- **Parallel Resonance:** At resonance, the total impedance of the circuit is at its maximum. The impedance becomes theoretically infinite in an ideal circuit (due to zero net reactance), but in practical circuits, it is large, leading to minimal current through the circuit.

iii. **Current Behavior:**

- **Series Resonance:** At resonance, the current is maximum since the impedance is minimized. This makes series resonance circuits useful in applications where high current at a specific frequency is needed.
- **Parallel Resonance:** At resonance, the current drawn from the source is minimum, as the impedance is maximized. However, large circulating currents flow between the inductor and capacitor within the parallel branches.

iv. **Voltage Behavior:**

- **Series Resonance:** The voltage across individual components can be much larger than the supply voltage, particularly across the inductor or capacitor due to high current at resonance. This is called voltage magnification.
- **Parallel Resonance:** The voltage across the parallel circuit components is approximately equal to the source voltage, but the current through the inductor and capacitor can be large (current magnification), while the total current from the source remains low.

v. **Application:**

- **Series Resonance:** Commonly used in applications requiring frequency selection, such as tuned circuits in radio receivers and filters, where high current is desired at a specific resonant frequency.
- **Parallel Resonance:** Used in tank circuits or LC oscillators, where high impedance at resonance limits current, making it suitable for generating stable frequencies in oscillators or filtering out signals in power supply systems.

6. Explain the transient response of an RLC circuit when initially connected to an AC source. What happens to the currents and voltages in the circuit?

Ans) When an RLC circuit is initially connected to an AC source, the transient response refers to the behavior of the currents and voltages before the circuit reaches its steady-state operation. This response occurs because the circuit contains reactive elements (inductors and capacitors) that store and release energy, which causes the system to take time to settle into a steady oscillatory state. Here's an overview of what happens during the transient response in an RLC circuit:

- Initial Condition:** When the circuit is first connected to an AC source, the inductor and capacitor may not have any stored energy, meaning the initial currents and voltages can be zero. However, the inductor and capacitor will begin to store energy as the AC source starts to provide alternating voltage.
- Energy Exchange Between Inductor and Capacitor:**
 - **Inductor:** When current flows through the inductor, it resists changes in current due to its property of inductance. Initially, the inductor opposes the increase in current, causing a voltage drop across it.
 - **Capacitor:** The capacitor, on the other hand, resists changes in voltage. It will initially charge up from the AC source, causing a voltage across its terminals that opposes the source voltage.
- Transient Oscillations:** During the transient period, the RLC circuit exhibits oscillatory behavior where energy is transferred back and forth between the inductor and the capacitor. This oscillation happens because the inductor and capacitor continually exchange energy between their magnetic and electric fields. The circuit's response can be classified based on the damping (resistance) in the system:
 - **Underdamped Response:** If the resistance (R) is small, the circuit will oscillate with decreasing amplitude over time. The current and voltage will oscillate sinusoidally while decaying gradually, eventually reaching a steady state. This is the most common transient behavior in RLC circuits.
 - **Critically Damped Response:** If the resistance is at a critical value, the system will return to steady state without oscillating, but it will do so as quickly as possible.
 - **Overdamped Response:** If the resistance is large, the circuit will return to steady state slowly, without oscillating. The current and voltage will gradually approach their final values over time.

In an underdamped circuit, the current and voltage oscillate at the circuit's natural resonant frequency, given by:

$$\omega_o = \frac{1}{\sqrt{LC}}$$

However, due to the presence of resistance, the actual oscillation frequency (damped natural frequency) is slightly lower than ω_o .

- iv. **Damping Effect of Resistance (R):** The resistor (R) dissipates energy in the form of heat, which causes the oscillations to decay over time. The rate of decay depends on the value of the resistance:

- A higher resistance causes the oscillations to die out faster.
- A lower resistance allows oscillations to persist longer.

- v. **Reaching Steady State:** After the transient period, the circuit reaches steady state, where the response is purely sinusoidal at the same frequency as the AC source. The transient oscillations die out, and the voltages and currents in the circuit are in phase (or have a steady phase relationship) with the AC source, depending on the impedance of the circuit.

In the steady state:

- The voltage and current will have a phase difference that depends on the relative magnitudes of the inductive and capacitive reactances.
- The circuit will behave as if the initial transient oscillations never occurred.

7. Why alternating current (AC) is commonly used for long-distance power transmission?

Ans)

- Ease of Voltage Transformation:** AC can be easily transformed to higher voltages for long-distance transmission using transformers, reducing current and minimizing power losses. Stepping down the voltage at the receiving end is also straightforward.
- Lower Transmission Losses:** Higher voltages reduce current and thus minimize resistive losses (I^2R) in transmission lines, making AC more efficient for long-distance power delivery.
- Cost-Effectiveness:** AC transmission infrastructure, like transformers and switchgear, is well-established, simpler, and more cost-effective than DC for most applications.
- Simplicity of Generation and Distribution:** AC is easily generated by rotating machines, and voltage adjustments for distribution are simpler, making it more practical for large-scale power systems.
- Three-Phase System Advantages:** AC uses a three-phase system that ensures efficient power transfer and better conductor utilization over long distances.

- vi. **Simplicity of Switching and Protection:** AC systems have simpler and more effective switching and protection devices, as current naturally crosses zero in each cycle, simplifying circuit breaking.
- vii. **Integration with the Grid:** AC is widely used in global power grids, enabling easy interconnection and efficient power transmission across vast distances.

8. Describe the purpose and function of a choke coil in AC circuits.

Ans) **Purpose of a Choke Coil in AC Circuits:** A choke coil is primarily used to limit or block higher-frequency alternating current (AC) signals while allowing lower-frequency signals or direct current (DC) to pass. Its purpose is to filter or smooth current flow in AC circuits, helping to remove unwanted high-frequency components and ensuring stable operation in devices like power supplies, radio circuits, and fluorescent lights.

Function of a Choke Coil:

- i. **Inductive Reactance:** The choke coil is essentially an inductor, and inductors oppose changes in current through a property called inductive reactance, which is frequency-dependent. The reactance X_L of a coil increases with frequency according to the formula:

$$X_L = 2\pi fL$$

Where:

- X_L is inductive reactance.
- f is the frequency of the AC signal.
- L is the inductance of the coil.

This means that at higher frequencies, the choke coil offers greater opposition to the current, effectively choking off high-frequency components while allowing lower-frequency or steady (DC) components to pass with little resistance.

- ii. **Current Smoothing:** In power supply circuits, choke coils are used to smooth out fluctuations in current caused by rectification processes. When AC is converted to DC, ripple voltages remain. The choke coil helps to filter out these ripples, providing a smoother, more stable DC output by reducing the AC components from the rectified current.
- iii. **Energy Storage:** The choke coil stores energy in its magnetic field when current passes through it. During periods when the current decreases, the coil releases stored energy, helping to maintain a continuous flow of current. This property is especially useful in circuits where consistent current is critical.
- iv. **Reducing Electromagnetic Interference (EMI):** In power electronics, choke coils help reduce electromagnetic interference (EMI) by blocking high-frequency noise generated by switching devices. This makes them vital in electronic devices, helping to comply with EMI regulations and ensuring the safe and interference-free operation of electrical systems.

- v. **Controlling Current in Fluorescent Lamps:** In fluorescent lamps, choke coils are used in the ballast circuit to limit the current flowing through the lamp. When a fluorescent lamp is powered, the choke limits the current after the initial striking of the arc, protecting the lamp from drawing too much current, which could damage it.

Section(D): Numerical:

1. A resistor (R) of 20 ohms is connected in series with a capacitor (C) of 10 μF in an AC circuit with a frequency of 50 Hz. Calculate the total impedance?

Data:

$$R = 20 \text{ ohms}, C = 10 \mu\text{F}, f = 50 \text{ Hz}, Z = ?$$

Solution:

$$X_C = \frac{1}{2\pi fC}$$

$$X_C = \frac{1}{2\pi(50)(10 \times 10^{-6})}$$

$$X_C = 318.31 \text{ ohms}$$

$$Z = \sqrt{R^2 + X_C^2}$$

$$Z = \sqrt{20^2 + 318.31^2}$$

$$Z = 318.94 \text{ ohms}$$

2. For an inductor with an inductance (L) of 0.5 H and a frequency of 100 Hz, calculate the inductive reactance?

Data:

$$L = 0.5 \text{ H}, f = 100 \text{ Hz}, X_L = ?$$

Solution:

$$X_L = 2\pi fL$$

$$X_L = 2\pi(100)(0.5)$$

$$X_L = 314.16 \text{ ohms}$$

3. In an RL circuit, the resistance (R) is 30 ohms, and the inductance (L) is 0.2 H. Calculate the total impedance at a frequency of 60 Hz?

Data:

$$R = 30 \text{ ohms}, L = 0.2 \text{ H}, f = 60 \text{ Hz}, Z = ?$$

Solution:

$$X_L = 2\pi fL$$

$$X_L = 2\pi(60)(0.2)$$

$$X_L = 75.4 \text{ ohms}$$

$$Z = \sqrt{R^2 + X_L^2}$$

$$Z = \sqrt{30^2 + 75.4^2}$$

$$Z = 81.1 \text{ ohms}$$

4. In an RC circuit, the resistance (R) is 50 ohms, and the capacitance (C) is 20 μ F .Calculate the capacitive reactance?

Data:

$$R = 50 \text{ ohms}, C = 20 \mu\text{F}, X_c = ?$$

Solution:

$$X_C = \frac{1}{2\pi fC}$$

We are assuming the frequency of 50 Hz

$$X_C = \frac{1}{2\pi(50)(20 \times 10^{-6})}$$

$$X_C = 159.15 \text{ ohms}$$

5. An AC circuit has a resistance of 40 ohms, an inductive reactance of 30 ohms, and a capacitive reactance of 20 ohms. Draw the impedance triangle and calculate the total impedance?

Data:

$$R = 40 \text{ ohms}, X_L = 30 \text{ ohms}, X_C = 20 \text{ ohms}, \text{Triangle} = ?, Z = ?$$

Solution:

$$X_T = X_L - X_C$$

$$X_T = 30 - 20$$

$$X_T = 10 \text{ ohms}$$

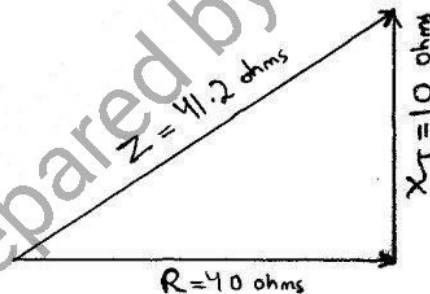
$$Z = \sqrt{40^2 + 10^2}$$

$$Z = 41.2 \text{ ohms}$$

$$\tan \phi = \frac{X_C}{R}$$

$$\tan \phi = \frac{20}{40}$$

$$\phi = 26.56^\circ$$



6. In a series RL circuit, the resistance (R) is 25 ohms, and the inductance (L) is 0.1 H. Calculate the phase angle and impedance at a frequency of 80 Hz?

Data:

$$R = 25 \text{ ohms}, L = 0.1 \text{ H}, \phi = ?, Z = ?, f = 80 \text{ Hz}$$

Solution:

$$X_L = 2\pi fL$$

$$X_L = 2\pi(80)(0.1)$$

$$X_L = 50.26 \text{ ohms}$$

$$Z = \sqrt{R^2 + X_L^2}$$

$$Z = \sqrt{25^2 + 50.26^2}$$

$$Z = 56.13 \text{ ohms}$$

$$\tan \phi = \frac{X_L}{R}$$

$$\tan \phi = \frac{50.26}{25}$$

$$\phi = 63.55^\circ$$

7. In a parallel RC circuit, the resistance (R) is 60 ohms, and the capacitance (C) is 30 μ F. Calculate the total current flowing through the circuit at a frequency of 120 Hz? While voltage is 60 V.

Data:

$$R = 60 \text{ ohms}, C = 30 \mu F, f = 120 \text{ Hz}, V = 60 \text{ V}.$$

Solution:

$$I_R = \frac{V}{R}$$

$$I_R = \frac{60}{60}$$

$$I_R = 1 \text{ A}$$

$$X_C = \frac{1}{2\pi fC}$$

$$X_C = \frac{1}{2\pi(120)(30 \times 10^{-6})}$$

$$X_C = 44.2 \text{ ohms}$$

$$I_C = \frac{V}{X_C}$$

$$I_C = \frac{60}{44.2}$$

$$I_C = 1.36 \text{ A}$$

$$I_T = \sqrt{I_R^2 + I_C^2}$$

$$I_T = \sqrt{(1)^2 + (1.36)^2}$$

$$I_T = 1.69 \text{ A}$$

8. In an RLC circuit, the resistance (R) is 50 ohms, the inductance (L) is 0.1 H, and the capacitance (C) is 50 μ F. Calculate the resonance frequency?

Data:

$$R = 50 \text{ ohms, } L = 0.1 \text{ H, } C = 50 \mu\text{F, } f_r = ?$$

Solution:

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$f_r = \frac{1}{2\pi\sqrt{(0.1)(50 \times 10^{-6})}}$$

$$f_r = 71.2 \text{ Hz}$$

Prepared by: Sir Ahad

UNIT 21: PHYSICS OF SOLIDS

MCQ'S

KEY:

1. a	2. d	3. b	4. a	5. a
6. c	7. a	8. a	9. a	10. b

Section (B): CRQs (Short Answered Questions):

1. Why are the springs made of steel and not of copper?

Ans) Steel is more elastic than copper. Due to this reason that springs are made of steel not copper.

2. The breaking force for a wire is F . What will be the breaking force for two parallel wires of the same size?

Ans) When two wires of the same size are suspended parallel, then the breaking force of each wire being F , the breaking force of two wires in parallel will be $F + F = 2F$.

3. Distinguish between intrinsic and extrinsic semiconductor.

Feature	Intrinsic Semiconductor	Extrinsic Semiconductor
Definition	Pure semiconductor with no impurities.	Semiconductor with intentionally added impurities (dopants).
Conductivity	Low conductivity due to limited free electrons and holes.	Higher conductivity due to the presence of additional charge carriers from dopants.
Carrier Concentration	Carrier concentration depends only on temperature.	Carrier concentration depends on both temperature and the amount of doping.
Types	Only one type, based on pure material (e.g., silicon, germanium).	Two types: n-type (with extra electrons) and p-type (with extra holes).

4. A wire is replaced by another wire of same length and material but of twice the diameter. What will be the effect on the

(a) Increase in its length under a given load?
bear?

(b) Maximum load which it can

Ans)

(a) Young's modulus (Y) is defined as:

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{\Delta L/L}$$

$$\Delta L = \frac{FL}{YA}$$

$$\Delta L = \frac{FL}{Y\left(\frac{\pi d^2}{4}\right)}$$

$$\Delta L = \frac{4FL}{Y\pi d^2}$$

Since diameter is double.

$$\Delta L' = \frac{4FL}{Y\pi(2d)^2}$$

$$\Delta L' = \frac{1}{4} \cdot \frac{4FL}{Y\pi d^2}$$

$$\therefore \Delta L = \frac{4FL}{Y\pi d^2}$$

$$\Delta L' = \frac{1}{4} \cdot \Delta L$$

This means that the increase in length under the same load is reduced to one-fourth.

(b) The maximum load F_{max} that a wire can bear before breaking is related to its cross-sectional area:

$$F_{max} = Y \cdot A \cdot \frac{\Delta L}{L}$$

$$F_{max} = Y \cdot \left(\frac{\pi d^2}{4} \right) \cdot \frac{\Delta L}{L}$$

Since diameter is double.

$$F_{max}' = Y \cdot \left(\frac{\pi (2d)^2}{4} \right) \cdot \frac{\Delta L}{L}$$

$$F_{max}' = 4 \cdot Y \cdot \left(\frac{\pi d^2}{4} \right) \cdot \frac{\Delta L}{L}$$

$$\therefore F_{max} = Y \cdot \left(\frac{\pi d^2}{4} \right) \cdot \frac{\Delta L}{L}$$

$$F_{max}' = 4F_{max}$$

This indicates that the maximum load which the new wire can bear is four times that of the original wire.

5. Sand does not possess any definite shape and volume, still it is solid. Give reason.

Ans) This is due to the fact that there is some space present between the sand particles. So, it does not possess a definite shape. Sand is a type of granular solid and the space between the sand particles is greater in comparison to the solid particles.

6. Specify the importance of stress-strain curve.

Ans) The stress-strain curve provides design engineers with a long list of important parameters needed for application design. A stress-strain graph gives us many mechanical properties such as strength, toughness, elasticity, yield point, strain energy, resilience, and elongation during load. It also helps in fabrication.

7. Why liquids don't possess rigidity?

Ans) Liquids have less intermolecular force of attraction and have high kinetic energy than solids that have high intermolecular force of attraction. Hence, they flow and aren't rigid.

8. Give applications of Curie point.

Ans) Applications of the Curie point include

- **Magnetic Storage Devices:** The Curie point is used in data storage technologies, such as hard drives, to erase magnetic data by heating the material above its Curie temperature.
- **Magnetic Refrigeration:** Materials undergo magnetic phase transitions at their Curie point, used in magnetic refrigeration systems.
- **Temperature Sensors:** Curie point materials are used in temperature sensing devices, like thermostats, where the change in magnetic properties indicates a specific temperature.
- **Permanent Magnets:** Understanding the Curie point helps in designing permanent magnets that retain their magnetism up to desired temperature limits.

9. What are amorphous materials and what are their uses?

Ans) **Amorphous materials:** Amorphous materials are solids that lack a long-range, ordered crystal structure, with atoms arranged randomly, like in glass.

Uses:

- **Glass:** Used For windows, optical lenses, and containers.
- **Amorphous metals:** Used in transformers and magnetic cores due to low energy loss.
- **Semiconductors:** Used in thin-film transistors for displays (e.g., LCD, OLED).
- **Amorphous silicon:** Used in solar panels.

Section (C): BRQs (Long Answered Questions):

1. Explain Force-Extension graph.

Ans) **Force-Extension Graph:** In material science, the force versus extension graph for a material gives the relationship between stress and strain. This is obtained by gradually applying a load to a test component and measuring its deformation according to tensile standards. These curves reveal many of properties of materials, such as the young's modulus, the yield strength, the ultimate tensile strength and so on. For different types of materials as shown in figure below.

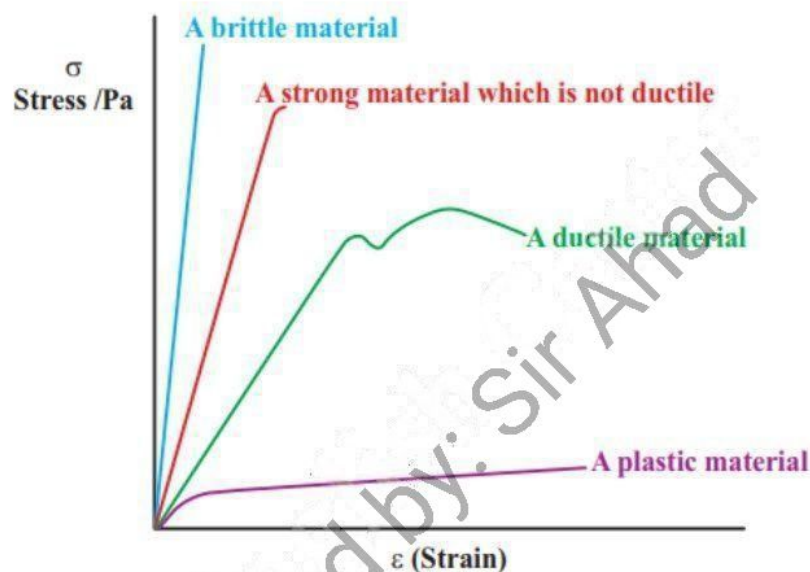


Fig: force extension graph reveals the properties of different materials

The blue line (brittle) represents the behavior of a typical elastic material, which follows Hooke's Law. As the material is stretched, the force applied to it increases linearly with the extension, up to a point called the yield point. The material eventually reaches a maximum point of extension, is called the ultimate extension or fracture point, beyond which it breaks or fractures.

The green line (ductile) represents the behavior of a typical viscoelastic material, which exhibits time-dependent deformation under a constant force. In this case, the material initially undergoes elastic deformation, similar to the elastic material, but then continues to deform slowly over time under a constant force, eventually reaching a maximum extension or creep limit.

The purple line (Plastic) represents the behavior of a typical plastic material, which exhibits permanent deformation when subjected to a force. In this case, the material undergoes plastic deformation immediately, with no linear region or yield point, and continues to deform permanently as the force is increased, until it ultimately fractures or breaks.

The Red line is showing strong material which is not ductile, like steel wires stretch very little and breaks suddenly.

2. Derive relation for Young's Modulus and Shear Modulus.

Ans) **Young's Modulus (modulus of elasticity):** The Hooke's law can be written for Young's Modulus is defined as the ratio of stress to strain.

$$Y = \frac{\text{Stress}}{\text{Strain}}$$

$$Y = \frac{\frac{F}{A}}{\frac{\Delta L}{L}}$$

$$Y = \frac{F \times L}{\Delta L \times A}$$

Shear Modulus: Shear modulus is the measure of the rigidity of the body which is the ratio of shear stress to shear strain.

$$G = \frac{\text{Shear stress}}{\text{Shear strain}}$$

Consider a rigid body as shown in figure below. When acted upon by tangential force to twist it.

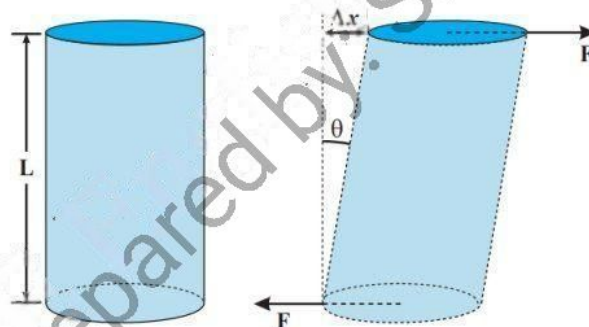


Fig: Shear modulus

The shear stress is F/A and shear strain is $\Delta X/L$. Therefore, the shear modulus becomes

$$G = \frac{\frac{F}{A}}{\frac{\Delta X}{L}}$$

$$G = \frac{F \times L}{\Delta X \times A}$$

3. Distinguish between structure of crystalline, glassy, amorphous, and polymeric solids.

Ans)

Property	Crystalline Solids	Glassy Solids	Amorphous Solids	Polymeric Solids
Atomic Arrangement	Regular, Repeating pattern.	Random, short-range order.	Random, no long-range order.	Long chains or networks of repeating units.
Melting Point	Sharp, Specific temperature.	Gradual, over a range of temperature	Gradual, over a range of temperatures.	Varies, typically lower than crystalline solids.
Rigidity	Very rigid and well-defined structure.	Rigid, but less than crystalline solids.	Not as rigid as crystalline solids.	Can be flexible or rigid.
Transparency	Can be transparent or translucent.	Transparent or opaque depending on composition.	Opaque	Depending on structures, can be transparent or opaque
Examples	Diamond, salt crystals, silicon.	Window glass, certain plastics, amorphous metals.	Rubber, some plastics, glass.	Polyethylene, PVC, nylon.

4. Describe the energy bands in solids.

Ans) **Energy Bands in Solids:** When isolated atoms comes together to form a solid, interactions between neighboring atoms cause the electron energy levels to split and overlap, creating continuous energy bands.

Valence Band: The valence band is the range of energy levels occupied by electrons that are bound to atoms and participate in chemical bonding. It is the highest energy band that contains electrons under normal conditions.

Forbidden Band (Energy Gap): The forbidden band, or energy gap, is the range of energy levels between the valence band and the conduction band where no electron states exist. This gap influences the material's electrical properties; a larger gap typically means the material is less conductive.

Conduction Band: The conduction band is the range of energy levels where electrons are free to move throughout the material, allowing electrical conduction. Electrons in the conduction band are not bound to any particular atom and can carry electric current through the solid.

Example: In the case of a diamond crystal, carbon atoms with the electronic structure $1s^2 2s^2 2p^2$ form two main energy bands as shown in figure below. The valence band and the conduction band are separated by an energy gap known as the "forbidden band." The valence band contains the lower energy states, while the conduction band contains higher energy states. This energy gap prevents electrons in the valence band from easily moving to the conduction band, significantly influencing the electrical properties of the solid, such as its ability to conduct electricity.

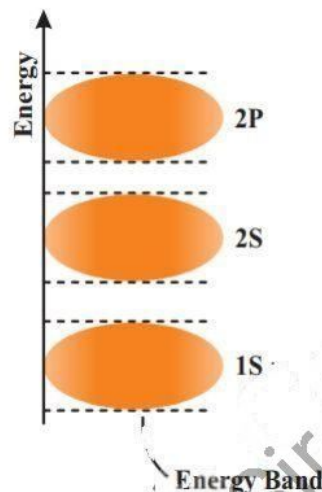


Fig: Energy Bands

5. Describe superconductivity and its applications.

Ans) **Superconductivity:** Superconductivity is a phenomenon where a material exhibits zero electrical resistance and expels magnetic fields (Meissner effect) when cooled below a critical temperature (T_c). This allows current to flow without energy loss.

Key Features of Superconductivity:

- i. **Zero Electrical Resistance:** No resistance to current flow, allowing indefinite circulation of current.
- ii. **Meissner Effect:** Expulsion of magnetic fields when in a superconducting state.
- iii. **Critical Temperature (T_c):** The specific temperature below which a material becomes superconducting.
- iv. **Critical Magnetic Field and Current:** Superconductors lose their properties if exposed to magnetic fields or currents above critical values.

Types of Superconductors:

- i. **Type I Superconductors:** Pure metals, completely lose superconductivity in magnetic fields above the critical level.
- ii. **Type II Superconductors:** Allow partial magnetic flux penetration; used in practical applications.
- iii. **High-Temperature Superconductors (HTS):** Operate at higher temperatures (above 77 K).

Applications of Superconductivity:

Magnetic Resonance Imaging (MRI): MRI scanners provide high resolution picture of the tissues inside the body. Superconducting coils produce a strong magnetic field (up to 60,000 times as strong as the intensity of Earth's magnetic field) that is used to align the protons of hydrogen atoms in the body of the patient, as shown in figure below. Like electrons, protons have a 'spin' property, so they align with a magnetic field, the proton's axis vibrates about the applied magnetic field. Vibrating protons are crashed with a burst of radio waves tuned to push the Protons' spin axes are sideways, perpendicular to the applied magnetic field. When radio waves pass and the protons quickly return to their vibrating pattern, they emit a faint electromagnetic signal whose frequencies depend slightly on the chemical environment in which the proton resides. These signals, detected by sensors, are then analyzed by a computer to reveal varying densities of hydrogen atoms in the body and their interactions with surrounding tissue. The resulting images clearly distinguish between fluid and bone. MRI was formerly called NMRI (Nuclear Magnetic Resonance Imaging) because hydrogen nuclei resonate with the applied fields.

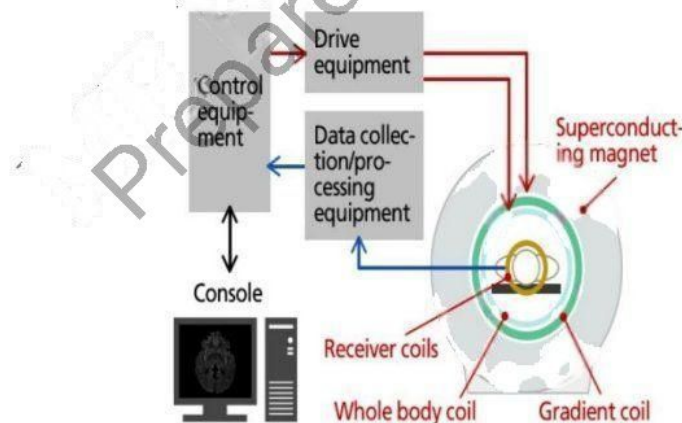


Fig: Working Principle of MRI

Maglev Train Systems: Maglev train systems use powerful electromagnets to float the trains over a guideway, instead of the old steel wheel and track system as shown in figure below. A system called electromagnetic suspension suspends, guides, and propels the trains. A large number of magnets provide controlled tension for lift and propulsion along a track. Maglev (derived from magnetic levitation) is aim of train transportation that uses two sets of magnets: one set to repel and push the train up off the track and another set to move the elevated train ahead, taking advantage of the lack of friction.

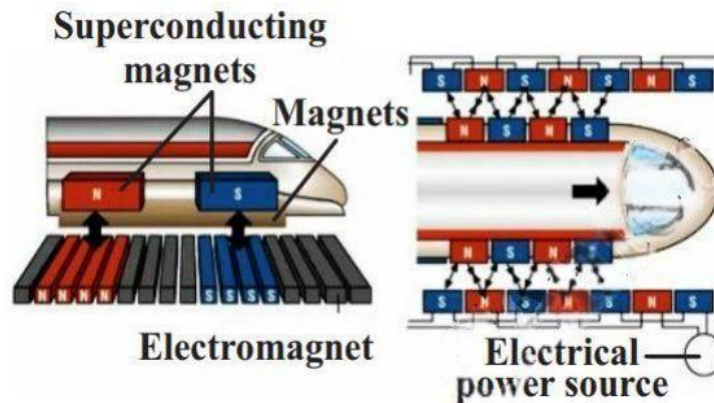


Fig: Magnetic Levitation

6. Discuss the applications of superconductors for MRI, Maglev's, and supercomputers.

Ans) **Super conductor Applications:**

Magnetic Resonance Imaging (MRI): MRI scanners provide high resolution picture of the tissues inside the body. Superconducting coils produce a strong magnetic field (up to 60,000 times as strong as the intensity of Earth's magnetic field) that is used to align the protons of hydrogen atoms in the body of the patient, as shown in figure below. Like electrons, protons have a 'spin' property, so they align with a magnetic field, the proton's axis vibrates about the applied magnetic field. Vibrating protons are crashed with a burst of radio waves tuned to push the Protons' spin axes are sideways, perpendicular to the applied magnetic field. When radio waves pass and the protons quickly return to their vibrating pattern, they emit a faint electromagnetic signal whose frequencies depend slightly on the chemical environment in which the proton resides. These signals, detected by sensors, are then analyzed by a computer to reveal varying densities of hydrogen atoms in the body and their interactions with surrounding tissue. The resulting images clearly distinguish between fluid and bone. MRI was formerly called NMRI (Nuclear Magnetic Resonance Imaging) because hydrogen nuclei resonate with the applied fields.

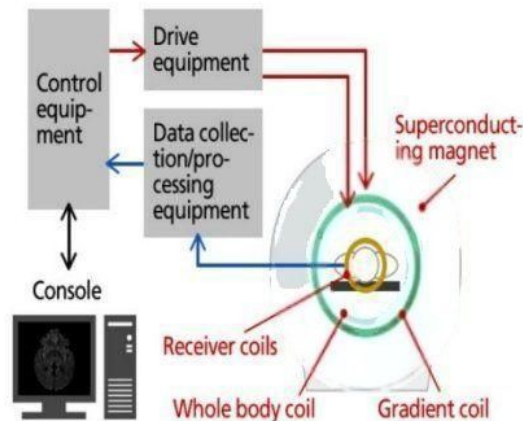


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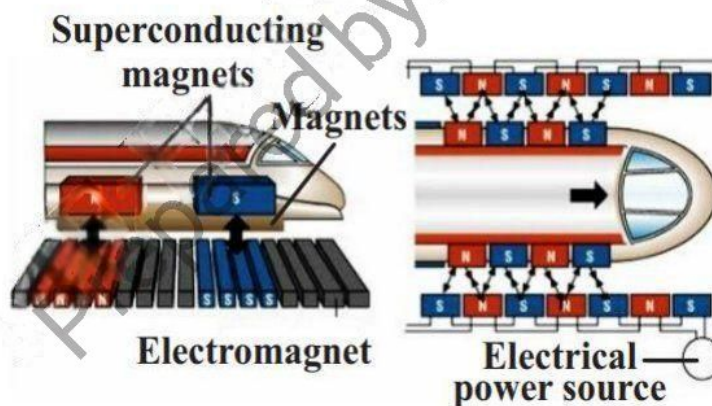


Fig: Magnetic Levitation

Super Conductivity and Super Computers: Super conducting components can improve the performance of super computers in several ways as:

Super conducting qubits: These are the basic building blocks of quantum computers, which are able to perform calculations at speeds that are exponentially faster than classical computers. Superconducting qubits can maintain their quantum states for long periods of time, allowing for complex computations to be performed.

Faster processing speeds: Super conducting components can process information at much faster speeds than traditional electronic components, resulting in faster computing times and the ability to handle larger datasets.

Lower power consumption: Super conducting circuits require less power to operate than traditional electronic circuits, reducing energy costs and making them more environmentally friendly.

Improved reliability: Super conducting components are less prone to errors and can operate for longer periods of time without failure, resulting in more reliable and efficient supercomputers.

However, the use of superconducting components in super computers is still in the experimental stage and faces several challenges.

7. Describe hysteresis loss.

Ans) **Hysteresis Loss:** Hysteresis loss is the energy loss that occurs in a magnetic material when it undergoes cyclic magnetization, i.e., when it is repeatedly magnetized and demagnetized. This loss is due to the internal friction caused by the reorientation of magnetic domains within the material. It is a key factor in determining the efficiency of devices such as transformers, motors, and generators.

When a magnetic material is subjected to an external magnetic field and the field strength is cycled (increased and then decreased), the material's magnetization does not follow the same path. Instead, it forms a loop called the hysteresis loop on a graph of magnetic field strength (H) versus magnetic flux density (B).

The area enclosed by the hysteresis loop represents the energy lost per cycle of magnetization. This energy is dissipated as heat, and this heat is referred to as hysteresis loss.

Cause of Hysteresis Loss: Hysteresis loss is caused by the friction between magnetic domains inside the material as they reorient in response to the changing external magnetic field. When a magnetic field is applied, the domains (regions where the magnetic moments are aligned) tend to align with the field. When the field is reduced or reversed, these domains resist changing their orientation, and energy is required to overcome this resistance. The energy used to reorient the domains is not fully recovered and is lost as heat.

Impact of Hysteresis Loss:

- **Efficiency Reduction:** Hysteresis loss reduces the efficiency of electrical devices such as transformers and motors because the lost energy is dissipated as heat.
- **Heat Generation:** The heat generated by hysteresis loss can cause the temperature of the material to rise, which may require cooling systems to manage.

Minimizing Hysteresis Loss:

- **Use of Soft Magnetic Materials:** Materials with narrow hysteresis loops, such as silicon steel or ferrites, are used in transformers and other AC devices to minimize hysteresis loss.
- **Laminated Cores:** In transformers, laminations in the core can reduce both hysteresis and eddy current losses, improving efficiency.

8. Synthesize hysteresis loop for relationship between magnetic field strength and magnetizing current.

Ans) **Synthesis of hysteresis Loop between Magnetic Field Strength and Magnetizing Current:** The hysteresis loop is a graphical representation, as shown in Figure below that shows the relationship between the magnetic field strength (B) and the magnetization (H) of a material. The loop is particularly useful for understanding how a material responds to changes in an external magnetic field. Here's how you can synthesize the variation of magnetic field strength with magnetizing current from the hysteresis loop.

- Hysteresis Loop:** A hysteresis loop is a closed curve illustrating a material's magnetic behavior under a changing magnetic field, formed by plotting the material's magnetization against the magnetic field strength.
- Variation of Magnetic Field Strength with Magnetizing Current:** The horizontal axis of the hysteresis loop represents the magnetizing current or applied magnetic field strength. The vertical axis represents the resulting magnetization of the material.
- Magnetizing and Demagnetizing:** When the magnetizing current increases, the material gets magnetized, and the magnetic field strength goes up, shown on the right side of the hysteresis loop.
When the magnetizing current decreases, the material may not fully demagnetize, and the magnetic field strength remains at a certain level, shown on the descending left side of the loop.
- Understanding the Loop:** The width of the hysteresis loop indicates energy loss (hysteresis loss) during magnetization and demagnetization cycles. The shape of the loop provides information about the material's magnetic properties, such as coercivity and remanence.

By analyzing the hysteresis loop, the relationship between magnetic field strength and magnetizing current can be observed, aiding in understanding the material's magnetic properties and response to external magnetic field changes.

9. Discuss energy bands and their classification. Explain magnetic properties of soft and hard magnetic materials.

Ans) **Energy Bands in Solids:** When isolated atoms come together to form a solid, interactions between neighboring atoms cause the electron energy levels to split and overlap, creating continuous energy bands.

Valence Band: The valence band is the range of energy levels occupied by electrons that are bound to atoms and participate in chemical bonding. It is the highest energy band that contains electrons under normal conditions.

Forbidden Band (Energy Gap): The forbidden band, or energy gap, is the range of energy levels between the valence band and the conduction band where no electron states exist. This gap influences the material's electrical properties; a larger gap typically means the material is less conductive.

Conduction Band: The conduction band is the range of energy levels where electrons are free to move throughout the material, allowing electrical conduction. Electrons in the conduction band are not bound to any particular atom and can carry electric current through the solid.

Example: In the case of a diamond crystal, carbon atoms with the electronic structure $1s^2 2s^2 2p^2$ form two main energy bands as shown in figure below. The valence band and the conduction band are separated by an energy gap known as the "forbidden band." The valence band contains the lower energy states, while the conduction band contains higher energy states. This energy gap prevents electrons in the valence band from easily moving to the conduction band, significantly influencing the electrical properties of the solid, such as its ability to conduct electricity.

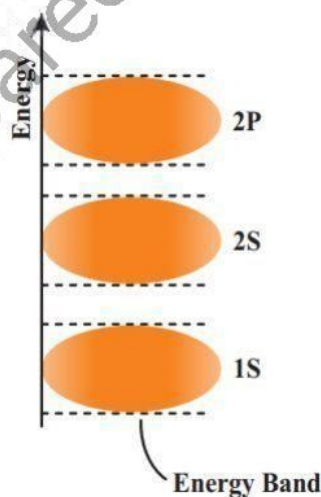


Fig: Energy Bands

Magnetic Properties of Soft Magnetic Materials:

- **Low Coercivity (H_c):** Soft magnetic materials have a low coercive field, meaning they can be easily demagnetized when the external magnetic field is removed.
- **High Permeability (μ):** These materials exhibit high magnetic permeability, allowing magnetic flux to easily pass through them. This results in efficient magnetization with relatively low magnetic fields.
- **Low Retentivity (Residual Magnetism):** They have low residual magnetism (remanence), meaning they do not retain significant magnetization after the external magnetic field is removed.
- **Low Hysteresis Loss:** The area of the hysteresis loop for soft magnetic materials is small, indicating low energy loss during each cycle of magnetization and demagnetization. This makes them ideal for use in alternating current (AC) applications.

Magnetic Properties of Hard Magnetic Materials:

- **High Coercivity (H_c):** Hard magnetic materials have a high coercive field, meaning they resist demagnetization even when subjected to opposing magnetic fields or external factors like heat.
- **High Retentivity (Residual Magnetism):** They have high retentivity, meaning they retain a significant level of magnetization even after the external magnetic field is removed. This makes them suitable for permanent magnets.
- **Low Permeability (μ):** Hard magnetic materials have relatively low permeability, meaning they require stronger external magnetic fields to achieve saturation (complete magnetization).
- **Large Hysteresis Loop:** The hysteresis loop for hard magnetic materials is large, indicating higher energy losses during magnetization cycles. However, this property is desirable in applications where constant magnetization is needed (e.g., permanent magnets).

Section (D): Numerical:

1. The 'lead' in pencils is a graphite composition with a Young's modulus of $1 \times 10^9 \text{ N/m}^2$. Calculate the change in length of the lead in an automatic pencil if you tap it straight into the pencil with a force of 4.0 N. The lead is 0.50 mm in diameter and 60 mm long.

Data:

$$Y = 1 \times 10^9 \frac{\text{N}}{\text{m}^2}, \Delta L = ?, F = 4.0 \text{ N}, d = 0.50 \text{ mm} = 0.50 \times 10^{-3} \text{ m},$$

$$L = 60 \text{ mm} = 60 \times 10^{-3} \text{ m}$$

Solution:

$$Y = \frac{F \times L}{\Delta L \times A}$$

$$1 \times 10^9 = \frac{4.0 \times 60 \times 10^{-3}}{\Delta L \times A}$$

$$\therefore A = \frac{\pi d^2}{4} = \frac{\pi (0.50 \times 10^{-3})^2}{4} = 1.96 \times 10^{-7} \text{ m}^2$$

$$1 \times 10^9 = \frac{4.0 \times 60 \times 10^{-3}}{\Delta L \times 1.96 \times 10^{-7}}$$

$$\Delta L = 1.2 \times 10^{-3} \text{ m} = 1.2 \text{ mm}$$

2. A wire of 2.2 m long and 2.25 mm in diameter, when stretched by a weight of 8.8 kg, its length has been increased by 0.25 mm. Find the stress, strain, and Young's modulus of the material of the wire. Given $g = 9.8 \text{ m/s}^2$.

Data:

$$L = 2.2 \text{ m}, d = 2.25 \text{ mm} = 2.25 \times 10^{-3} \text{ m}, m = 8.8 \text{ kg},$$

$$\Delta L = 0.25 \text{ mm} = 0.25 \times 10^{-3} \text{ m} \quad \sigma = ?, \varepsilon = ?, Y = ?, g = 9.8 \text{ m/s}^2$$

Solution:

$$\sigma = \frac{F}{A}$$

$$\therefore F = W = mg = (8.8)(9.8) = 86.24 \text{ N}$$

$$\text{Also, } A = \frac{\pi d^2}{4} = \frac{\pi (2.25 \times 10^{-3})^2}{4} = 3.97 \times 10^{-6} \text{ m}^2$$

$$\sigma = \frac{86.24}{3.97 \times 10^{-6}} = 2.17 \times 10^7 \approx 2.2 \times 10^7 \text{ N/m}^2$$

$$\varepsilon = \frac{\Delta L}{L}$$

$$\varepsilon = \frac{0.25 \times 10^{-3}}{2.2} = 1.136 \times 10^{-4} \approx 1.14 \times 10^{-4}$$

$$Y = \frac{\sigma}{\varepsilon} = \frac{2.2 \times 10^7}{1.14 \times 10^{-4}} = 1.93 \times 10^{11} \approx 2 \times 10^{11} \text{ N/m}^2$$

3. A farmer making juice fills a glass bottle to the brim and caps it tightly. The juice expands more than the glass when it warms up, in such a way that the volume increases by 0.2% (i.e., $\frac{\Delta V}{V_0} = 2 \times 10^{-3}$) relative to the space available. Calculate the normal force exerted by the juice per square centimeter, if its bulk modulus is $1.8 \times 10^9 \text{ N/m}^2$. Assuming that the bottle does not break.

Data:

$$\frac{\Delta V}{V_0} = 2 \times 10^{-3}, B = 1.8 \times 10^9 \text{ N/m}^2, \Delta P = ?$$

Solution:

$$B = -\frac{\Delta P}{\frac{\Delta V}{V_0}}$$

$$1.8 \times 10^9 = -\frac{\Delta P}{(-2 \times 10^{-3})}$$

$$\Delta P = 3.6 \times 10^6 \text{ N/m}^2$$

$$\Delta P = \frac{3.6 \times 10^6}{10000} \text{ N/cm}^2$$

$$\Delta P = 360 \text{ N/cm}^2$$

4. The elastic limit of copper is $1.5 \times 10^8 \text{ N/m}^2$. It is to be stretched by a load of 10 kg. Find the diameter of the wire if the elastic limit is not to be exceeded.

Data:

$$\sigma_m = 1.5 \times 10^8 \text{ N/m}^2, m = 10 \text{ kg}, d = ?$$

Solution:

$$F = W = mg = (10)(9.8) = 98 \text{ N}$$

$$\sigma_m = \frac{F}{A}$$

$$1.5 \times 10^8 = \frac{98}{A}$$

$$A = 6.53 \times 10^{-7} \text{ m}^2$$

$$\therefore A = \frac{\pi d^2}{4}$$

$$6.53 \times 10^{-7} = \frac{\pi d^2}{4}$$

$$d = 9.12 \times 10^{-4} \text{ m} = 0.912 \text{ mm}$$

5. What would be the greatest length of a steel wire which is fixed at one end, and can it be hanged freely without breaking? The breaking stress of steel is $7.8 \times 10^8 \text{ N/m}^2$ and the density of steel is 7800 kg/m^3 .

Data:

$$\sigma_b = 7.8 \times 10^8 \text{ N/m}^2, \rho = 7800 \frac{\text{kg}}{\text{m}^3}, L = ?$$

Solution:

$$\sigma_b = \frac{F}{A}$$

$$\therefore F = W = mg \text{ \& } m = \rho AL$$

$$\sigma_b = \frac{\rho ALg}{A}$$

$$7.8 \times 10^8 = (7800)L(9.8)$$

$$L = 1.02 \times 10^4 \text{ m}$$

6. A mild steel wire of radius 0.55 mm and length 3.5 m is stretched by a force of 52 N. Calculate:

(a) Longitudinal stress,

(b) Longitudinal strain, and

(c) Elongation produced in the wire if Young's modulus is $2.1 \times 10^{11} \text{ N/m}^2$.

Data:

$$r = 0.55 \text{ mm} = 0.55 \times 10^{-3} \text{ m}, L = 3.5 \text{ m}, F = 52 \text{ N}, \sigma = ?, \varepsilon = ?, \Delta L = ?,$$

$$Y = 2.1 \times 10^{11} \text{ N/m}^2.$$

Solution:

$$\sigma = \frac{F}{A}$$

$$\therefore A = \pi r^2 = \pi(0.55 \times 10^{-3})^2 = 9.5 \times 10^{-7} \text{ m}^2$$

$$\sigma = \frac{52}{9.5 \times 10^{-7}} = 5.47 \times 10^7 \text{ N/m}^2$$

$$Y = \frac{\sigma}{\varepsilon}$$

$$2.1 \times 10^{11} = \frac{5.47 \times 10^7}{\varepsilon}$$

$$\varepsilon = 2.6 \times 10^{-4}$$

$$\varepsilon = \frac{\Delta L}{L}$$

$$2.6 \times 10^{-4} = \frac{\Delta L}{3.5}$$

$$\Delta L = 9.1 \times 10^{-4} \text{ m} = 0.91 \text{ mm}$$

7. Calculate the change in volume of a lead block of volume 1.3 m^3 subjected to a pressure of 12 atm . Also, calculate the compressibility of lead. Given the bulk modulus as $B = 80 \times 10^9 \text{ N / m}^2$.

Data:

$$V_o = 1.3 \text{ m}^3, \Delta P = 12 \text{ atm} = 12 \times 1.03 \times 10^5 = 1.24 \times 10^6 \text{ Pa, compressibility} = ?,$$

$$B = 80 \times 10^9 \text{ N / m}^2$$

Solution:

$$B = -\frac{\Delta P}{\frac{\Delta V}{V_o}}$$

$$80 \times 10^9 = -\frac{1.24 \times 10^6}{\frac{\Delta V}{1.3}}$$

$$\Delta V = -2.015 \times 10^{-5} \text{ m}^3$$

After omitting the -ve sign

$$\Delta V = 2.015 \times 10^{-5} \text{ m}^3$$

$$\text{compressibility} = \frac{1}{B}$$

$$\text{compressibility} = \frac{1}{80 \times 10^9}$$

$$\text{compressibility} = 1.25 \times 10^{-11} \text{ m}^2/\text{N}$$

8. The thickness of a metal plate is 0.35 inches. It's drilled to have a hole of radius 0.08 inches on the plate. If the shear strength is $4 \times 10^4 \text{ lbs/in}^2$, determine the force needed to make that hole.

Data:

$$t = 0.35 \text{ in}, r = 0.08 \text{ in}, \text{shear strength} = 4 \times 10^4 \text{ lbs/in}^2, F = ?$$

Solution:

$$4 \times 10^4 = \frac{F}{A}$$

$$A = 2\pi r t = 2\pi(0.08)(0.35) = 0.176 \text{ in}^2$$

$$4 \times 10^4 = \frac{F}{0.176}$$

$$F = 7 \times 10^3 \text{ lbs}$$

UNIT 22: SOLID STATE ELECTRONICS

KEY

1. a	2. b	3. b	4. d	5. c
6. d	7. b	8. b	9. d	10. d

Section (B): ORQs (Short Answered Questions):

1. Describe a p-n junction (diode) and how holes and electrons are produced in semiconductor?

Ans) **p-n Junction (Diode):** A p-n junction is a semiconductor device that allows current to flow in one direction. It is formed by joining p-type and n-type semiconductors.

Structure:

- **P-type Semiconductor:** Doped with trivalent elements (e.g., boron), creating holes as majority carriers.
- **N-type Semiconductor:** Doped with pentavalent elements (e.g., phosphorus), producing electrons as majority carriers.

Depletion Region: When the p-type and n-type materials are joined, electrons and holes recombine at the junction, forming a depletion region that acts as a barrier to current flow.

Operation:

- **Forward Bias:** Reduces the depletion region, allowing current to flow.
- **Reverse Bias:** Widens the depletion region, preventing current flow.

Production of Holes and Electrons:

- **Holes:** Created in p-type material through doping with trivalent elements.
- **Electrons:** Produced in n-type material through doping with pentavalent elements.

2. Define and distinguish between p-n-p & n-p-n transistors?

Ans)

Feature	p-n-p Transistor	n-p-n Transistor
Definition	A p-n-p transistor is a type of bipolar junction transistor (BJT) where a thin n-type material is sandwiched between two p-type materials.	An n-p-n transistor is a type of BJT where a thin p-type material is sandwiched between two n-type materials.
Structure	p-type (Emitter) , n-type (Base) , p-type (Collector)	n-type (Emitter) , p-type (Base) , n-type (Collector)
Current Flow Direction	From Emitter to Collector	From Collector to Emitter
Majority Charge Carriers	Holes in the emitter and collector regions	Electrons in the emitter and collector regions

3. Explain common-base and common collector configurations?

Ans) **Common Collector (CC) Circuit:** In CC configuration, the input signal is applied to the base, and the output signal is taken from the emitter, with the collector terminal serving as the common connection.

Figure below depicts this setup. Here, when the base-emitter junction is forward biased, a small base current I_B , causes a much larger collector current I_C to flow. The emitter current I_E , is approximately equal to I_C .

Since $I_E = I_B + I_C$ and I_B is significantly smaller than I_C .

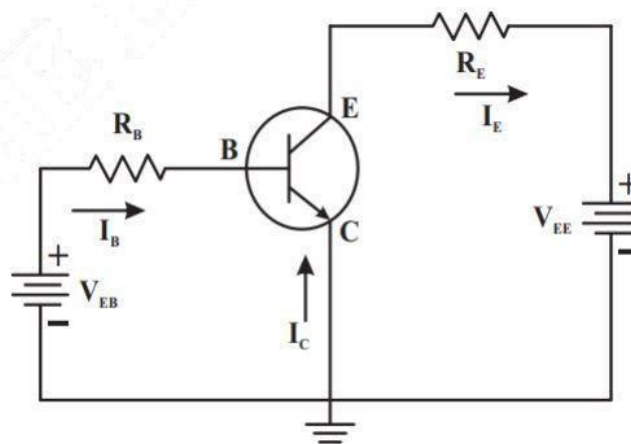


Fig: Common collector circuits

Common Base (CB) Circuit: In CB configuration, the input and output signals share the base terminal of the transistor.

Figure below demonstrates this configuration. Here, the input signal is applied to the emitter, and the output signal is taken from the collector. The base-emitter junction is forward biased to allow transistor operation.

CB amplifiers provide low input impedance and approximately unity voltage gain. The current gain in CB configuration is expressed as $\frac{I_{out}}{I_{in}}$ or the formula $\frac{I_C}{I_E}$.

However, as the base current is extremely small compared to the collector current, the emitter current is therefore approximately equal to the collector current. Thus $I_E \approx I_C$.

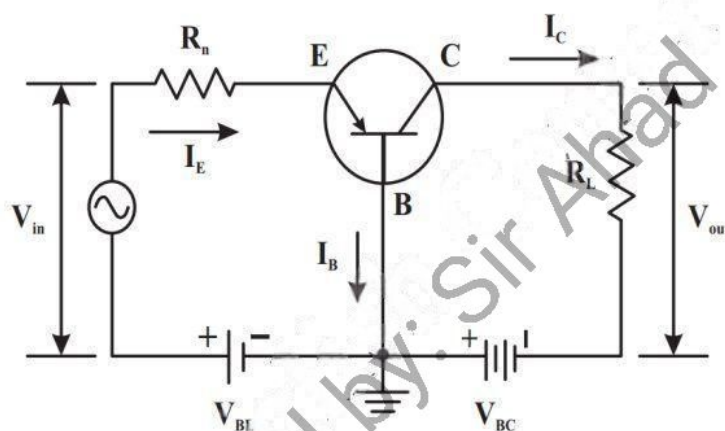


Fig: Common Base (CB) Circuit

4. Describe the operations of transistors.

Ans) **Operation of transistors:** Transistors are categorized as PNP and NPN. For simplicity the NPN transistor is taken as shown in figure (a).

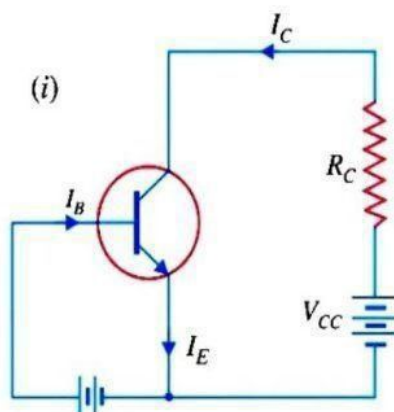


Fig (a): NPN transistor

Its action has two requirements:

- The base-emitter junction is forward biased so that a current I_B is generated. Once this junction is conducting, $V_{BE} \approx 0.6\text{ V}$.
- The base-collector junction is reverse biased. Transistor action then translates into relationship

$$I_C = \beta I_B$$

Where β is the current gain, which is typically 100. For completeness, the emitter current is then given by

$$I_E = I_B + I_C = (1 + \beta)I_B$$

Figure (b) graphs a set of characteristics curves of the transistor.

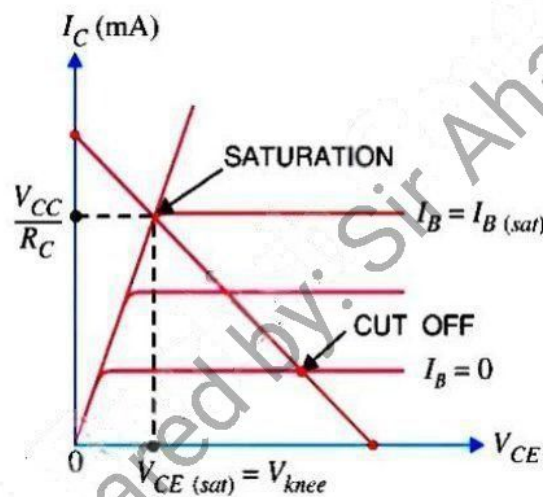


Fig (b): Transistors characteristics curves

Those graphs show the collector current I_C as function of the collector-emitter voltage V_{CE} , for different values of the base current I_B . When the transistor-action conditions are not met, the transistor is off and $I_C = 0$. This situation is called cutoff.

Now assume that V_{BE} is forward biased and I_B is above some minimum value. As the collector-base junction becomes reverse biased, which we represent here by increasing V_{CE} , transistor action begins to unfold:

In a transistor's characteristic curve, I_C (collector current) initially rises sharply until it reaches a certain point. Beyond this point, I_C continues to increase, but at a much slower rate. This might seem counterintuitive, but the rapid rise is called "saturation," indicating full current flow. The flat region that follows is when the transistor operates normally, where further increases in base current do not significantly increase I_C .

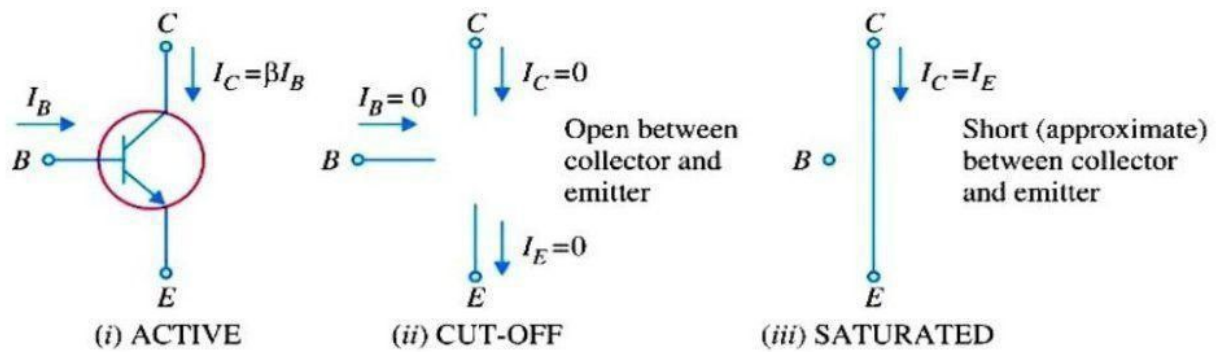


Fig: Transistors active-cut off and saturated

5. Explain the use of transistors as a switch and an amplifier (common-emitter).

Ans) **The transistors as a switch:** A transistor can be used as a switch in electronic circuits, where it functions to either allow or block the flow of current. There are two main configurations for using a transistor as a switch: the NPN (negative-positive-negative) and PNP (positive-negative-positive) configurations.

NPN Transistor as a Switch: The transistor Q_1 in figure below shows how to control output current with input.

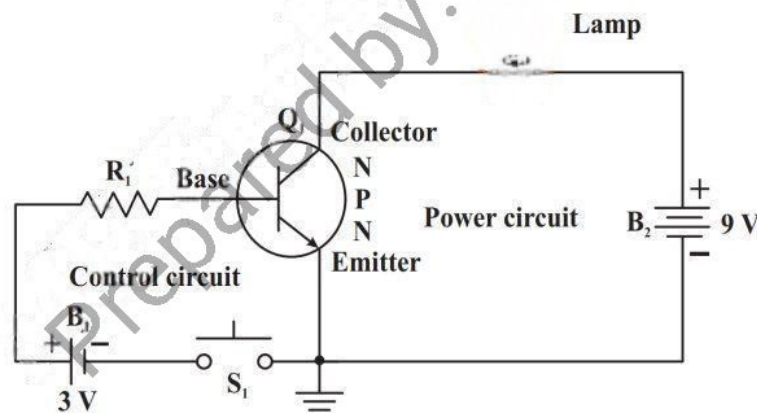


Fig: Transistors as a switch

Here are key points to note.

- Off State:** Normally, Q_1 allows no output current unless we apply forward voltage to its base-emitter circuit.
- Forward Voltage:** The amount of output current is controlled by the forward voltage that controls base current. In figure above:
 - The input control circuit determines base current.
 - The output current is collector current for the power circuit.

- Q_1 is an NPN transistor, needing a positive V_{BE} for forward voltage.
- The emitter is common to both input control and power circuits.
- Common-emitter (CE) circuit is the most common transistor arrangement.

The base-emitter junction of Q_1 in figure above can be forward biased by battery B_1 . Switch S_1 must be closed to apply forward voltage. Battery B_2 provides reverse voltage to the collector of Q_1 . Reverse polarity means the collector is more positive than the base.

When switch S_1 is open:

- No current flows in the base-emitter or control circuit because no forward voltage is applied.
- Resistance from emitter to collector of the transistor is very high.
- No current flows in the power circuit, and the lamp does not glow.

When switch S_1 is closed:

- A small current flows in the control circuit.
- R_1 limits current in the base circuit.
- Resistance from emitter to collector is low.
- A large current flows in the power circuit, and the lamp glows.

Opening switch S_1 in the control circuit causes the resistance from emitter to collector to increase again, almost to infinity. The lamp in the power circuit because it does not glow.

Transistor as an Amplifier:

Figure (a) shows the circuit diagram of a transistor amplifier. The input and output voltages are shown in figure (a), without any input signal. The dc biasing level is set by the feedback resistor R_2 . The NPN transistor is biased so that the collector-to-emitter voltage V_C is half of the supply voltage. For the supply voltage of 10 V, therefore, the collector voltage is set at one-half the total, or +5 V. The 0.7 V at the base is partially turning on the transistor. The transistor acts as an amplifier when in this partially turned-on condition. The level of the forward bias that determines the operating level of the transistor.

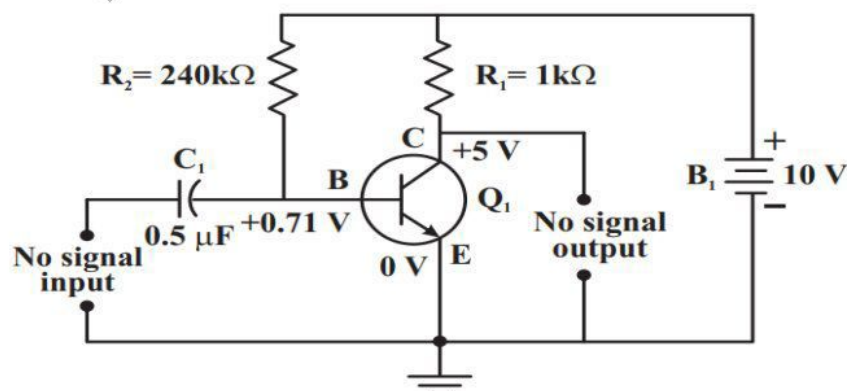


Fig (a): Transistors as an Amplifier

An input signal has been added to the amplifier in figure (b), Input is coupled to the base by C_1 . Amplified output is taken from the collector. The input signal is $0.02 V_{pp}$ as measured on an oscilloscope. The measured output signal voltage is $3 V_{pp}$. The ac gain of the amplifier, therefore is calculated as

$$A_v = V_{out}/V = 3/0.02 = 150$$

The output signal ($3 V_{pp}$) is 150 times greater than the input signal of $0.02 V_{pp}$. This amplifier stage is said to have a voltage gain of 150.

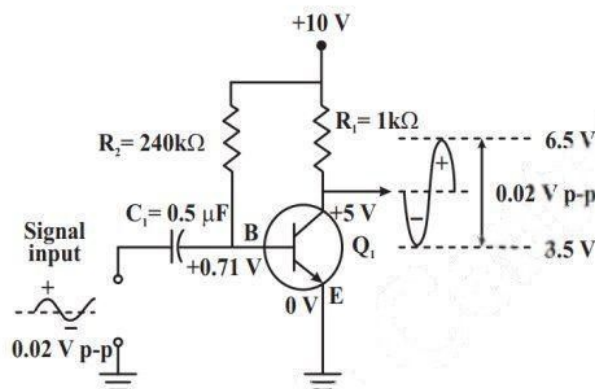


Fig (b): Transistors as an Amplifier

6. How would you understand the effects of negative feedback on the gain of an operational amplifier?

Ans) Negative feedback in an operational amplifier (op-amp) reduces the overall gain but increases stability, bandwidth, and linearity. By feeding a portion of the output signal back to the input in opposition to the original input signal, the op-amp's gain becomes more controlled and predictable, resulting in a more accurate and stable output. The trade-off is that the gain is lowered compared to the open-loop gain (gain without feedback).

7. Draw and briefly describe the circuit diagrams for both the inverting and the non-inverting amplifier for single signal input?

Ans) The circuit diagrams for both the inverting and the non-inverting amplifier for single signal input:

Inverting operational amplifier: In inverting operational amplifiers, the op amp forces the negative terminal to equal the positive terminal, which is commonly ground. Therefore, the input current is determined by the V_{IN}/R_1 ratio as shown in figure below: Inverting Operational Amplifier.

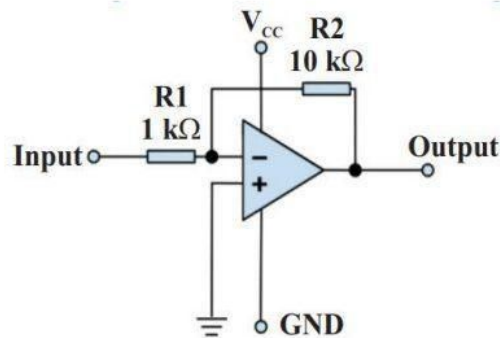


Fig: Circuit diagrams for inverting Amp

In this configuration, the same current flows through R_2 to the output. Ideally, current does not flow into the operational amplifier's negative terminal due to its high Z_{IN} . The current flowing from the negative terminal through R_2 creates an inverted voltage polarity with respect to V_{IN} . This is why these op amps are labeled with an inverting configuration. Note that the op amp's output can only swing between its positive and negative supplies, so creating a negative output voltage requires an op amp with a negative supply rail. V_{OUT} can be calculated with equation

$$V_{out} = -\frac{R_2}{R_1} \times V_{in}$$

Non-inverting operational amplifier: In a non-inverting amplifier circuit, the input signal from the source is connected to the non-inverting (+) terminal as shown in figure below: Non-Inverting Operational Amplifier.

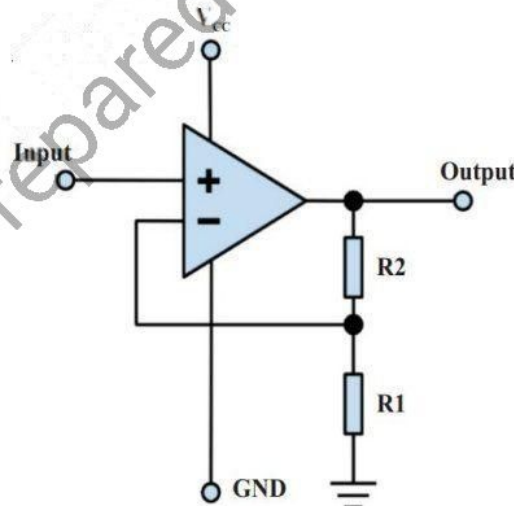


Fig: Circuits for non-inverting amplifier

The operational amplifier forces the inverting (-) terminal voltage to equal the input voltage, which creates a current flow through the feedback resistors. The output voltage is always in phase with the input voltage, which is why this topology is known as non-inverting. Note that with a

non-inverting amplifier, the voltage gain is always greater than 1, which is not always the case with the inverting configurations. V_{out} can be calculated with Equation:

$$V_{out} = \frac{1 + R_2}{R_1} \times V_{in}$$

8. Derive an expression for the gain of inverting amplifiers by using virtual earth approximation.

Ans) **Gain of an Inverting Amplifier:** If the current in the input resistor R_{in} is I_{in} and the current in the feedback resistor R_f is I_f , then point P is at 0 V:

$$I_{in} = \frac{V_{in}}{R_{in}} \text{ and } I_f = \frac{V_{out}}{R_f}$$

The input resistance of the op-amp is very high and so virtually no current enters or leaves the inverting input (-) of the op-amp. This means that I_{in} and I_f must be equal in size.

If V_{in} is a positive potential, then the current in two resistors flows from left to right. V_{out} will be negative because the current flows from P, which is at 0 V, to the output connection, which must have a lower voltage than 0 V. Thus

$$I_f = -I_{in} \text{ and } \frac{V_{out}}{R_f} = -\frac{V_{in}}{R_{in}}$$

The gain of the inverting amplifier is thus given by

$$G = \frac{V_{out}}{V_{in}} = -\frac{R_f}{R_{in}}$$

The negative sign shows that when the input voltage is positive the output voltage is negative and when the input is positive the output is negative. If the input voltage is alternating then there will be a phase difference of 180° or π rad between the input and the output voltages.

9. Describe the properties of an ideal operational amplifier.

Ans) **Properties of an Ideal Operational Amplifier (Op-Amp):** The ideal op-amp has the following properties:

Characteristics	Ideal Op-Amp
Infinite open- loop voltage gain	An ideal op amp is a device often used as an amplifier. When you input voltage into it, the op amp outputs a amplified voltage. In an ideal scenario, the op amp would provide extremely high gain, essentially infinite, amplifying the signal countless times for maximum gain as needed.
Infinite input resistance (Impedance)	An ideal op amp has super high input impedance, which means it won't load the circuit. If input impedance is low, the op amp draws more current; if it's high, less current is drawn. We aim for high input impedance to avoid disturbing the original circuit by minimizing current pulled from it, ideally with infinite input impedance.
Zero output resistance (Impedance)	In an ideal op amp, the output impedance is zero. This means when the op amp generates a signal, we want it to have zero resistance, ensuring that the maximum voltage goes to the output load. In a circuit, voltage gets divided based on the impedance. Higher impedance means more voltage drop. For the voltage to drop across the output load, the load's impedance should be greater than the op amp's output. That's why, ideally, we aim for zero output impedance in the op amp.
Gain independent of frequency	In an ideal op amp, the produced gain remains constant regardless of the input signal's frequency. This means the amplification stays reliable and consistent across all frequencies.

Section (C): ERQs (Long Answered Questions):

1. Define Intrinsic (pure) and doped semiconductors. And How the N-type and P-type semiconductors are produced.

Ans) **Intrinsic Semiconductors:** Intrinsic semiconductors are materials that are chemically pure, meaning they do not contain any significant amount of impurity atoms.

Doped Semiconductors: Doped semiconductors are the materials that are created by introducing impurity atoms into the intrinsic semiconductor material.

N-type semiconductors production: A small amount of a group-V element (penta-valent) is doped into the semiconductor's crystal lattice, as illustrated in figure below. Common donor impurities include phosphorus (P) or arsenic (As).

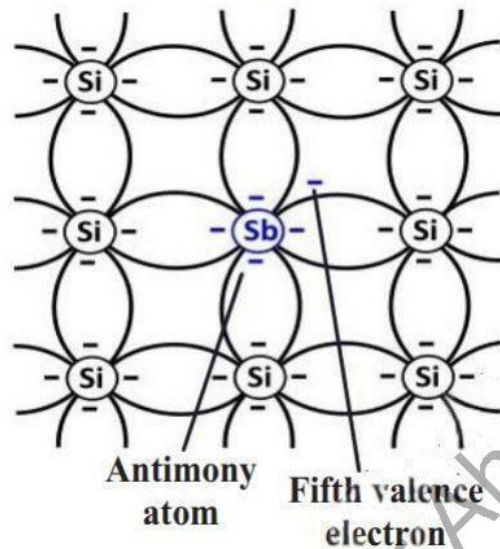


Fig: N-type

These impurity atoms have one more valence electron than the semiconductor material, creating extra electrons that are free to move and conduct electricity.

P-type semiconductors production: If element of group-III (tri-valent) is added into the crystal lattice of the semiconductor as shown in figure below. Common acceptor impurities include boron (B) or gallium (Ga). These impurity atoms have one fewer valence electron than the semiconductor material, creating "holes" in the crystal lattice where electrons can move.

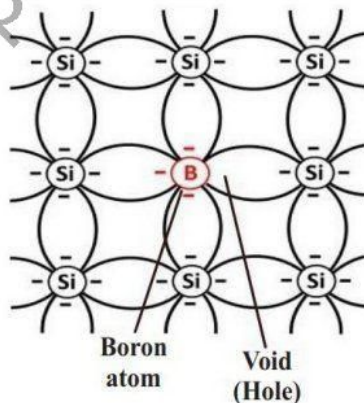


Fig: P-type

2. Describe a P-N junction (diode), discuss its forward and reverse biasing.

Ans) **P-N Junction Diode:** A diode is a semiconductor device that is formed through P-N junction and used in allowing the flow of electric current in one direction and blocking in the opposite. The symbol of the P-N junction diode is shown in figure below.

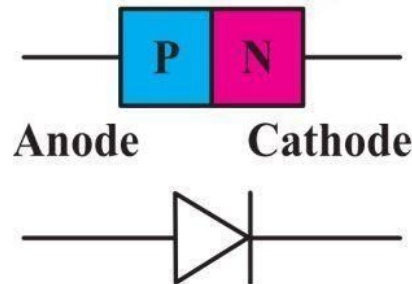


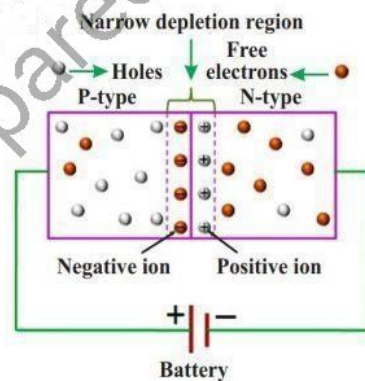
Fig: Diode

Properties of Diode: Below are some of the common properties of a diode:

1. Diode has the ability to rectify electric current.
2. It can create a potential barrier and make use of its capacitance properties.
3. Diode creates various nonlinear current-voltage characteristics.

Diode Biasing:

Forward Bias: When the P-type is connected to the positive terminal of the battery and the N-type is connected to the negative terminal is called Forward bias as shown in figure.



Forward Bias

In this condition, the applied electric field and the built-in electric field at the P-N junction are in opposing directions. Adding both the electric fields gives a resultant electric field, which is found to be smaller than the built-in electric field. So the depletion region becomes thinner and less resistant.

When the applied voltage is high, the resistance of the depletion region becomes insignificant. At 0.3V to 0.6 V, the resistance of the depletion region in silicon becomes absolutely insignificant, allowing current to flow freely through it.

Reverse Bias: When the P-type is connected to the negative terminal of the battery and the N-type is connected to the positive side is called Reverse Bias as shown in figure below. In this condition, the applied electric field and the built-in electric field are both in the same direction. The resultant electric field and the built-in electric field are also in the same direction, resulting in a more resistive, thicker depletion region. If applied voltage increased, it results in a thicker and more resistant depletion region.

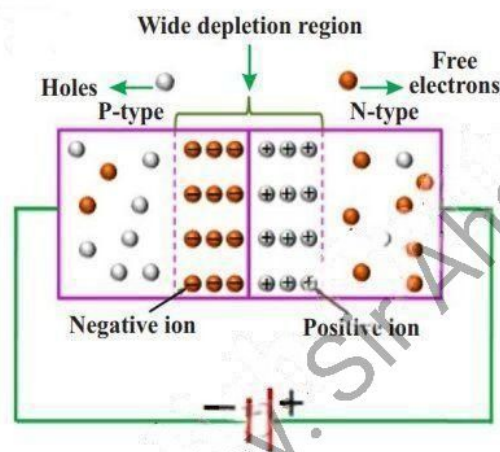


Fig: Reverse Bias

3. Describe the I-V characteristic curves of P-N junction.

Ans) **I-V Characteristics of p-n Junction:** The relationship between the voltage across the junction and current through the circuit is known as the (V-I) characteristics of a P-N junction or semiconductor diode.

The V-I characteristics of the P-N junction can be explained in three cases;

- Zero bias or unbiased
- Forward bias
- Reverse bias

At zero bias, no electric current flows through the diode because there's no external voltage applied to enable the movement of electrons or holes. In forward bias as shown in figure (a) when the diode voltage (V_d) reaches 0.7 V for silicon and 0.3 V for germanium, current starts flowing. The current increases gradually at first, creating a non-linear curve until the diode surpasses the potential barrier, after which it operates normally and the curve steepens linearly with increasing external voltage.

In reverse bias, only a small leakage current flows, represented to the left of the origin in the graph as shown in figure (c). This current remains low until the diode breaks down, at which point it can be destroyed unless a high series resistance limits the current.

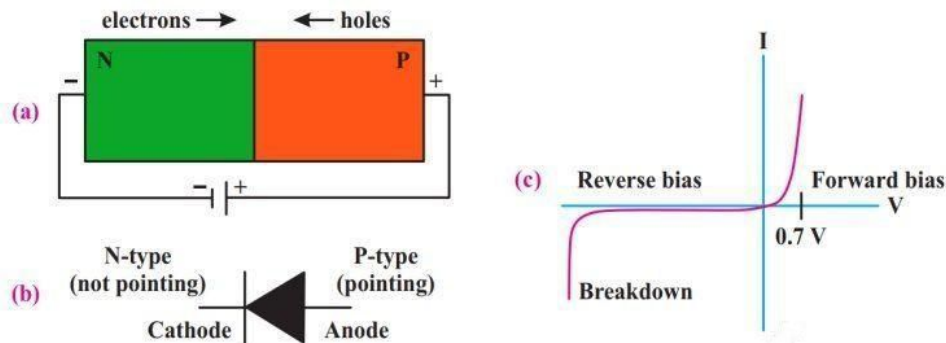


Fig: I-V Characteristics of p-n Junction (a,b,c)

4. Define rectification and describe the use of diodes for half and full wave rectifications.

Ans) **Rectification:** Rectification is the process of converting an alternating current (AC) waveform into a direct current (DC) waveform, i.e., creating a new waveform that has only a single polarity.

Types of Rectification: Rectification is classified into two types according to the output characteristics which are:

- i. half-wave rectification and
- ii. full-wave rectification

Half-wave Rectification: Since a diode allows AC current to flow only in one direction, it can serve as a rectifier. As shown in figure (a), the AC source applies a voltage across the diode alternately positive and negative. When the positive cycle of AC voltage passes through the diode, the diode is forward biased and acts as a closed circuit; there is current through the resistor R_L .

During the negative half cycle, the diode is connected with the negative supply which reverse biases the diode; the diode behaves like an open circuit and does not produce the output across the load.

Hence a graph of the voltage V_{ab} across R_L as a function of time looks like the output voltage shown in figure (c). This is called half wave rectification which seems unidirectional from its output signal.

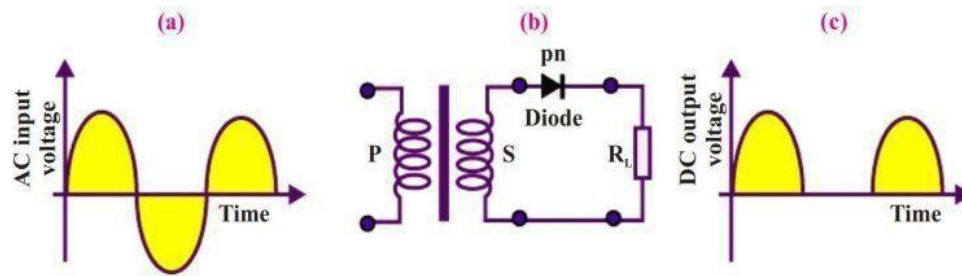


Fig: Half Wave Rectifications

Full-Wave Rectification: Full-wave rectification uses two diodes (as shown in figure b). During the positive half cycle of input AC signal, this makes the diode 1 forward biased (acts as closed switch) and diode 2 reverse biased (acts as open switch). Therefore, current flows through the load resistor R_L .

During the negative half cycle of input AC signal, this makes the diode 2 forward biased and the diode 1 reverse biased. Therefore, the current will flow through diode 2 and through load resistor R_L and lower half of the secondary winding.

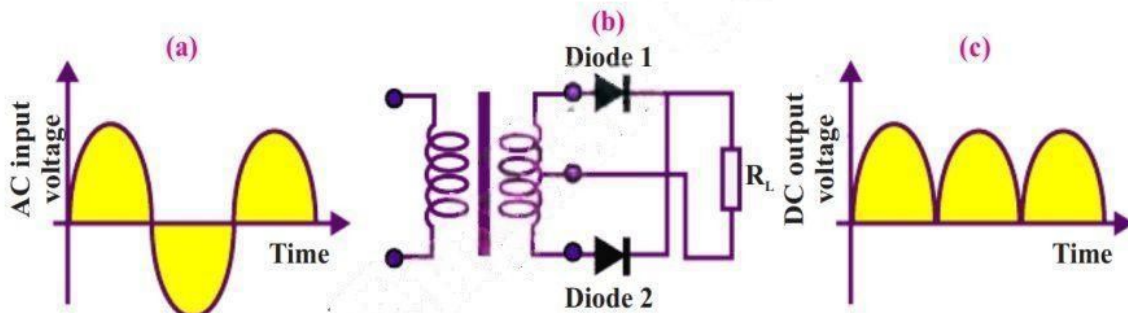


Fig: Full-Wave Rectifications

5. Describe the function and use of LED, Photodiode and Photo voltaic cell.

Ans)

1. Light Emitting Diode (LED):

Function: A light-emitting diode (LED) is a special type of junction in which current flows, when it is activated in a forward direction as shown in figure below.

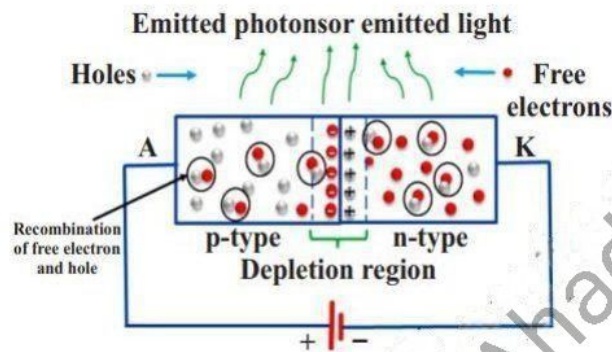


Fig: Light emitting diode

Silicon diodes don't work well for this, but compounds like gallium and arsenic create efficient LEDs. Gallium arsenide (GaAs), with a crystal structure similar to silicon, is often used.

Use: LED bulbs replace traditional lighting in various applications such as flashlights, streetlights, traffic signals, car brake lights, billboards, and LCD screens. LEDs, also known as solid-state lighting, last longer (50000 hours vs 2000 hours for regular bulbs), are more efficient, and durable.

2. Photodiode:

Function: A photodiode is a type of light detector that converts light into current or voltage. It includes optical filters, built-in lenses, and surface areas.

Photodiodes are often used in reverse bias, where a voltage encourages the flow of photocurrent, maximizing sensitivity. These detectors are usually made of semiconductor materials like silicon, which can absorb photons of light.

When light particles (photons) hit the semiconductor material, they transfer energy to electrons, creating electron-hole pairs. This process generates an electric current proportional to the incident light's intensity. In simple terms, brighter light results in a higher current from the photodiode.

Use: Photodiodes find widespread use in various applications, including:

- **Optical Communication:** Photodiodes are employed in optical communication systems, such as fiber optics, to detect and convert transmitted light signals into electrical signals.
- **Light Sensors:** They are used in electronic devices like cameras, light meters, and automatic lighting systems to sense ambient light levels.

- **Barcode Readers:** Photodiodes are often used in barcode scanners to detect the reflected light from the barcode.
- **Medical Devices:** In some medical instruments, photodiodes are used for tasks like measuring oxygen levels in blood.

3. Photo Voltaic Cells:

Function: Solar cell or photovoltaic cell is heavily doped P-N junction diodes used to convert sunlight into electric energy.

If the energy of incident photon is greater than the band gap energy, it creates electron-hole pairs, as shown in figure below. That absorbed photon excites an electron from the valence band and produce a current, when connected to an external circuit, work as source of emf and power. The basic element of a photovoltaic cell is a semiconductor material, usually made of silicon. Silicon is chosen for its widespread availability. Other semiconductor materials, such as cadmium telluride or copper indium gallium selenide, are also used in different types of solar cells.

A typical silicon P-N junction may produce about 0.6 V. Many are connected in series to produce higher voltage. Such series strings are connected in parallel within a photovoltaic panel i.e., solar panel. A good photovoltaic panel can have an output of perhaps 50 W/m², averaged over day and night, sunny and cloudy.

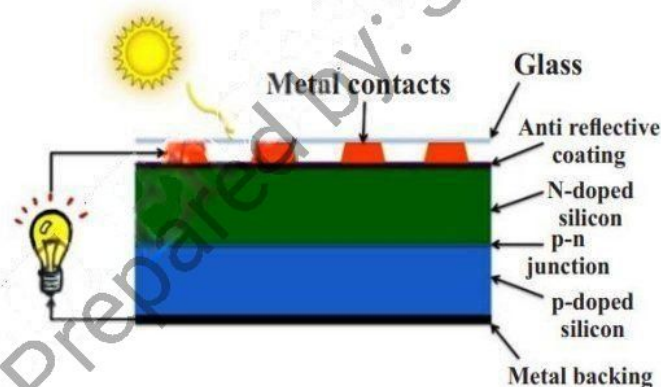


Fig: Photo Voltaic Cells

Uses:

- **Camera:** Light meters, Automatic Shutter Control, Photographic Flash Control.
- **Medical:** CAT Scanners, X-Ray Detection, Pulse Oximeters, Blood Particle Analysis.
- **Automotive:** Headlight Dimmer, Twilight Detectors.
- **Communication:** Fiber Optic Link, Optical Remote Control.
- **Industry:** Bar Code Scanners, Light Pen, Encoders, Surveying Instruments, Copiers Density of Toner.

6. Explain the use of transistors as a switch and an amplifier (common-emitter).

Ans) **The transistors as a switch:** A transistor can be used as a switch in electronic circuits, where it functions to either allow or block the flow of current. There are two main configurations for using a transistor as a switch: the NPN (negative-positive-negative) and PNP (positive-negative-positive) configurations.

NPN Transistor as a Switch: The transistor Q_1 in figure below shows how to control output current with input.

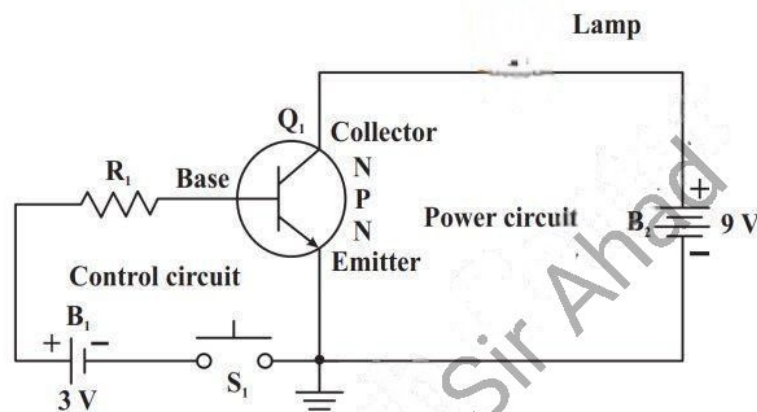


Fig: Transistors as a switch

Here are key points to note.

- Off State:** Normally, Q_1 allows no output current unless we apply forward voltage to its base-emitter circuit.
- Forward Voltage:** The amount of output current is controlled by the forward voltage that controls base current. In figure above:
 - The input control circuit determines base current.
 - The output current is collector current for the power circuit.
 - Q_1 is an NPN transistor, needing a positive V_{BE} for forward voltage.
 - The emitter is common to both input control and power circuits.
 - Common-emitter (CE) circuit is the most common transistor arrangement.

The base-emitter junction of Q_1 in figure above can be forward biased by battery B_1 . Switch S_1 must be closed to apply forward voltage. Battery B_2 provides reverse voltage to the collector of Q_1 . Reverse polarity means the collector is more positive than the base.

When switch S_1 is open:

- No current flows in the base-emitter or control circuit because no forward voltage is applied.
- Resistance from emitter to collector of the transistor is very high.

- No current flows in the power circuit, and the lamp doesn't light up.

When switch S_1 is closed:

- A small current flows in the control circuit.
- R_1 limits current in the base circuit
- Resistance from emitter to collector of Q_1 decreases.
- A large current flows in the power circuit, lighting up the lamp.

Opening switch S_1 in the control circuit turns off the lamp in the power circuit because resistance from emitter to collector of Q_1 increases again, almost to infinity.

Transistor as an Amplifier: The dc operating voltages are shown in figure (a), without any input signal. The dc biasing levels are set by the feedback resistor R_2 . The NPN transistor is biased so that the collector-to-emitter voltage V_C is half of the supply voltage. For the supply voltage of 10 v, therefore, the collector voltage is set at one-half the total, or +5 V. The 0.7 V at the base is partially turning on the transistor. The transistor acts as an amplifier when in this partially turned-on condition. It is the amount of dc forward bias that determines the operating level of the transistor.

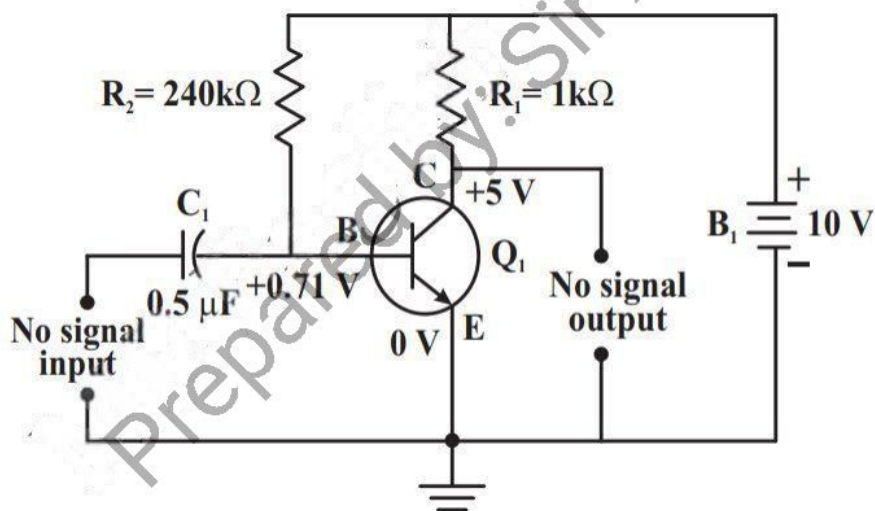
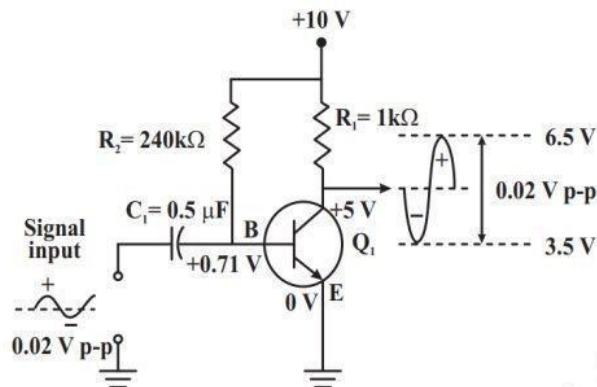


Fig (a): Transistors as an Amplifier

An input signal has been added to the amplifier in figure (b), Input is coupled to the base by C_1 . Amplified output is taken from the collector. The input signal is $0.02 V_{pp}$ as measured on an oscilloscope. The measured output signal voltage is $3 V_{pp}$. The ac gain of the amplifier, therefore is calculated as

$$A_v = V_{out}/V = 3/0.02 = 150$$

The output signal ($3 V_{pp}$) is 150 times greater than the input signal of $0.02 V_{pp}$. This amplifier stage is said to have a voltage gain of 150.



7. Describe the properties of an ideal operational amplifier.

Ans) **Properties of an Ideal Operational Amplifier (Op-Amp):** The ideal op-amp has the following properties:

Characteristics	Ideal Op-Amp
Infinite open-loop voltage gain	An ideal op amp is a device often used as an amplifier. When you input voltage into it, the op amp outputs a amplified voltage. In an ideal scenario, the op amp would provide extremely high gain, essentially infinite, amplifying the signal countless times for maximum gain as needed.
Infinite input resistance (Impedance)	An ideal op amp has super high input impedance, which means it won't load the circuit. If input impedance is low, the op amp draws more current; if it's high, less current is drawn. We aim for high input impedance to avoid disturbing the original circuit by minimizing current pulled from it, ideally with infinite input impedance.
Zero output resistance (Impedance)	In an ideal op amp, the output impedance is zero. This means when the op amp generates a signal, we want it to have zero resistance, ensuring that the maximum voltage goes to the output load. In a circuit, voltage gets divided based on the impedance. Higher impedance means more voltage drop. For the voltage to drop across the output load, the load's impedance should be greater than the op amp's output. That's why, ideally, we aim for zero output impedance in the op amp.
Gain independent of frequency	In an ideal op amp, the produced gain remains constant regardless of the input signal's frequency. This means the amplification stays reliable and consistent across all frequencies.
Infinite bandwidth	The bandwidth of an op-amp is the range of frequencies it amplifies equally. An ideal op-amp amplifies all frequencies, so it should ideally have an infinite bandwidth.

Infinite slow rate	An ideal op-amp should change the output instantaneously as the input is changed. The slew rate of the op-amp is the factor that affects this time delay. An infinite slew rate means there is no time delay.
Zero noise contribution	Any signal includes a small amount of noise. The ideal op- amp does not produce any noise itself, although it will amplify any noise that is present in its input.

Section (D): Numerical:

1. A Ge diode has a voltage drop of 0.4 V when 12 mA flow through it. If the same 470 Ohm is used, what battery voltage is needed?

Data:

$$V_{diode} = 0.4 \text{ V}, I = 12 \text{ mA} = 12 \times 10^{-3} \text{ A}, R = 470 \Omega, V_{battery} = ?$$

Solution:

$$V_{resistor} = IR$$

$$V_{resistor} = (12 \times 10^{-3})(470)$$

$$V_{resistor} = 5.64 \text{ V}$$

$$V_{battery} = V_{diode} + V_{resistor}$$

$$V_{battery} = 0.4 + 5.64$$

$$V_{battery} = 6.04 \text{ V}$$

2. A semiconductor diode laser has a peak emission wavelength of 1.55 μm . Find its band gap in eV.

Data:

$$\lambda = 1.55 \mu\text{m} = 1.55 \times 10^{-6} \text{ m}, E_g = ?$$

Solution:

$$E_g = \frac{hc}{\lambda}$$

$$\therefore h = \text{Planck's constant} = 6.626 \times 10^{-34} \text{ Js}, c = \text{Speed of light} = 3 \times 10^8 \text{ m/s}$$

$$E_g = \frac{(6.626 \times 10^{-34})(3 \times 10^8)}{1.55 \times 10^{-6}}$$

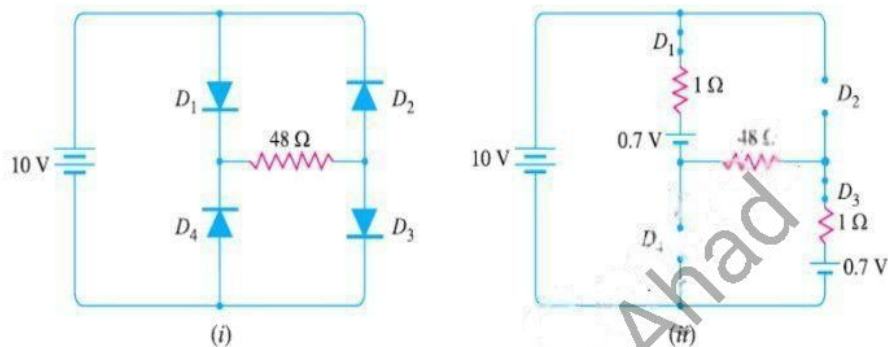
$$E_g = 1.282 \times 10^{-19} \text{ J}$$

To convert joules to electron volts, we use the conversion factor: $1\text{eV} = 1.602 \times 10^{-19} \text{ J}$

$$E_g = \frac{1.282 \times 10^{-19}}{1.602 \times 10^{-19}} \text{eV}$$

$$E_g = 0.8 \text{ eV}$$

3. Calculate the current through 48Ω resistor in the circuit shown in Fig (i). Assume the diodes to be of silicon and forward resistance of each diode is 1Ω .



Data:

Data is given in above figures, while we have to find "I"

Solution:

Fig (ii) shows we must take resistances and barrier voltages of only two diodes.

$$\text{Net circuit voltage} = 10 - 0.7 - 0.7 = 8.6 \text{ V}$$

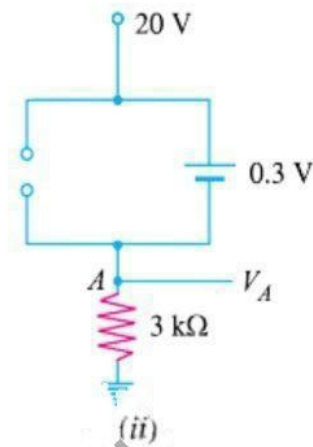
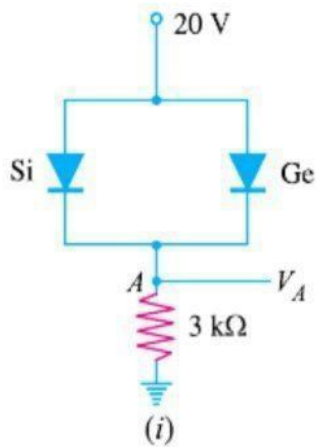
$$\text{Total circuit resistance} = 1 + 48 + 1 = 50 \Omega$$

$$I = \frac{\text{Net circuit voltage}}{\text{Total circuit resistance}}$$

$$I = \frac{8.6}{50}$$

$$I = 0.172 \text{ A} = 172 \text{ mA}$$

4. Find the voltage V_A in the circuit shown in Fig.(i). Use simplified model.



Data:

$$V = 20\text{ V}, V_{Ge} = 0.3\text{ V}, V_A = ?, R = 3\text{ k}\Omega$$

Solution:

We know that, Cut in voltage of Ge diode is 0.3 V and of silicon is 0.7 V .

The diode which has lower cut-in voltage will turned on first. So, here Ge diode will conduct, and the V_A is given by

$$V_A = 20 - 0.3 = 19.7\text{ V}$$

5. In a common base connection, $I_E = 1\text{ mA}$, $I_C = 0.95\text{ mA}$. Calculate the value of I_B .

Data:

$$I_E = 1\text{ mA}, I_C = 0.95\text{ mA}, I_B = ?$$

Solution:

$$I_B = I_E - I_C$$

$$I_B = 1 - 0.95$$

$$I_B = 0.05\text{ mA}$$

6. Find the value of β if (i) $\alpha = 0.9$ (ii) $\alpha = 0.98$ (iii) $\alpha = 0.99$.

Data:

$$\beta = ?$$

$$(i) \quad \alpha = 0.9$$

$$(ii) \quad \alpha = 0.98$$

$$(iii) \quad \alpha = 0.99$$

Solution:

(i)

$$\beta = \frac{\alpha}{1-\alpha}$$

$$\beta = \frac{0.9}{1-0.9}$$

$$\beta = 9$$

(ii)

$$\beta = \frac{\alpha}{1-\alpha}$$

$$\beta = \frac{0.98}{1-0.98}$$

$$\beta = 49$$

(iii)

$$\beta = \frac{\alpha}{1-\alpha}$$

$$\beta = \frac{0.99}{1-0.99}$$

$$\beta = 99$$

7. Calculate I_E in a transistor for which $\beta = 50$ and $I_B = 20\mu A$.

Data:

$$\beta = 50, I_B = 20\mu A = 0.02 \text{ mA}, I_E = ?$$

Solution:

$$\beta = \frac{I_C}{I_B}$$

$$50 = \frac{I_C}{0.02}$$

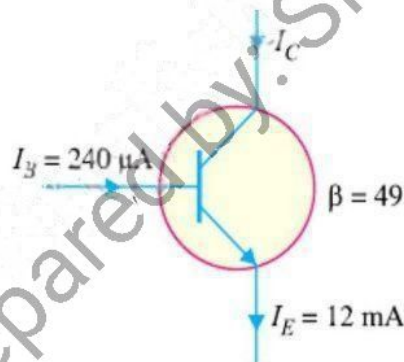
$$I_C = 1 \text{ mA}$$

$$I_E = I_B + I_C$$

$$I_E = 0.02 + 1$$

$$I_E = 1.02 \text{ mA}$$

8. Find the α rating of the transistor shown in Fig. Hence determine the value of I_C using both α and β rating of the transistor.



Data:

$$I_B = 240 \mu A = 240 \times 10^{-3}, \beta = 49, I_E = 12 \text{ mA}, \alpha = ?, I_C = ? \text{ (using both } \alpha \text{ and } \beta \text{ rating)}$$

Solution:

$$\alpha = \frac{\beta}{1 + \beta}$$

$$\alpha = \frac{49}{1 + 49}$$

$$\alpha = 0.98$$

$$I_C = \alpha I_E$$

$$I_C = (0.98)(12)$$

$$\mathbf{I_C = 11.76\ mA}$$

$$I_C = \beta I_B$$

$$I_C = (49)(240 \times 10^{-3})$$

$$\mathbf{I_C = 11.76\ mA}$$

UNIT 23: DIGITAL ELECTRONICS

MCQ'S

KEY

1. b	2. a	3. c	4. b	5. b
6. b	7. b	8. b	9. b	10. d

Section (B): GRQs (Short Answered Questions):

1. Explain the significance of signal levels in digital electronics and how they are represented in terms of voltage.

Ans) **Significance of Signal Levels:**

- Data Representation:** Signal levels (high and low) represent binary data (0s and 1s) used for digital processing and communication.
- Logic Operations:** Logic gates perform operations based on these voltage levels, determining circuit behavior.
- Noise Margin:** Adequate levels provide tolerance to noise, ensuring accurate signal interpretation.
- Power Consumption:** Voltage levels affect power usage, with different levels influencing energy efficiency.
- Interfacing:** Proper voltage levels are crucial for interfacing between different components or systems.

Digital signal levels: In digital electronics, signal levels are represented in various ways. Here are common digital signal levels:

- Low Level (0):**
 - This represents the binary digit 0.
 - In terms of voltage, it is associated with a lower voltage level.
 - Often referred to as "low" or "logic 0."
- High Level (1):**
 - This represents the binary digit 1.
 - In terms of voltage, it is associated with a higher voltage level.
 - Often referred to as "high" or "logic 1."
- Threshold Level:**
 - The threshold level is the voltage level that separates low and high states.
 - Signals below this threshold are interpreted as 0, and signals above it are interpreted as 1.
 - The threshold level helps define the noise margin and ensure reliable signal interpretation.

iv. **Logic Levels:**

- Depending on the technology and the system, different logic levels might be used.

v. **Swing or Voltage Range:**

- The difference in voltage levels between low and high states is often referred to as the swing or voltage range. For example, voltage varies from 5V to +5V will have a swing or voltage range of 10V.
- A larger voltage swing can enhance noise immunity and signal reliability.

2. Describe the binary representation used in digital electronics. How is this representation related to the 'high' and 'low' states?

Ans) In digital electronics, the 'high' and 'low' states are often symbolically represented as '1' and '0' respectively. This binary notation helps describe the states of open and closed circuits.

High State ('1'): When a digital signal is in a 'high' state, it is typically associated with a closed circuit or a voltage level close to the maximum specified value. The symbolic representation for this state is '1'. It signifies the 'on' or 'active' state in many digital systems.

Low State ('0'): Conversely, the 'low' state is linked to an open circuit or a voltage level close to the minimum specified value. It is symbolically represented as '0'. This state signifies the 'off' or 'inactive' state in digital systems.

High and Low represent the signal levels in the circuit. Digital waveforms consist of voltage levels that are changing back and forth between the HIGH and LOW states. These can also be described as the one that is composed of series of pulses, or a pulse train as shown in Figure below.

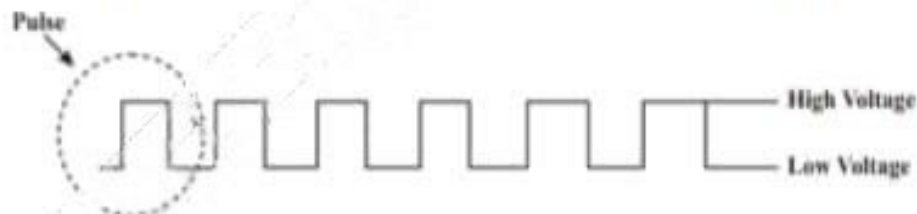


Fig: Pulse showing High and Low states

3. Choose any three logic gate symbols and explain their operations.

Ans) **AND Gate:** The AND gate produces a 'high' output only when all its inputs are 'high'. In other words, the output of an AND gate attains state 1 if and only if all the inputs are in state 1.

OR Gate: An OR gate generates a 'high' output if any of its inputs are 'high'. In other words, the output of an OR gate attains state 1 if one or more inputs attain state 1.

NOT Gate: It is used to perform the inversion operation in digital circuits. For example, the output of a NOT gate attains state 1 if and only if the input does not attain state 1.

4. Create a combination of logic gates that mimics the behavior of an XOR gate.

Ans) As we know that, $A \text{ XOR } B = \bar{A}B + A\bar{B}$

If we use two inverters, one inverter to create \bar{A} and the second to create \bar{B} and input \bar{A} and B to the first AND gate and input A and \bar{B} to the second AND gate. Then we take the outputs of the two AND gates and input them to an OR gate. The output of the OR gate is $\bar{A}B + A\bar{B}$ which is A XOR B.

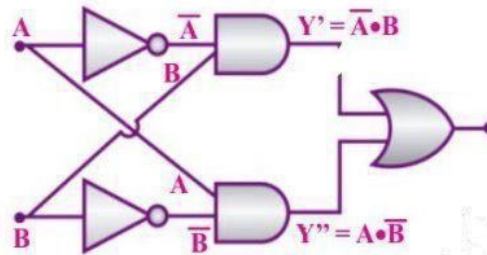


Fig: XOR gate and its equivalent diagram

Section (C): ERQs (Long Answered Questions)

1. Design a simple circuit using a 2-input AND gate, a switch, a lamp, and a battery. Explain how the circuit operates and the conditions under which the lamp will be illuminated.

Ans) **Circuit Layout:**

- **Input 1:** Connected to the switch. When the switch is closed, Input 1 receives a 1. When the switch is open, Input 1 receives a 0.
- **Input 2:** This is a constant signal that can either be 1 (if a condition is met, such as power being available) or 0 (if a condition like a safety feature is not satisfied).
- **AND Gate Output:** The output of the AND gate is connected to the lamp.
- The battery provides power to the circuit and the lamp.

Circuit Operation:

- **The AND gate takes two inputs:**
 - i. **Input 1 (Switch):** This input depends on whether the switch is open or closed.
 - ii. **Input 2 (Constant Input):** This could represent a condition like power being available or some other required system condition (e.g., safety checks).
- The AND gate outputs a 1 (high) only if both inputs are 1. When the output of the AND gate is 1, the current flows from the battery to power the lamp, illuminating it.

Conditions for the Lamp to be Illuminated:

- **Condition 1:** The switch must be closed, which means Input 1 is 1.
- **Condition 2:** Input 2 must also be 1, which could represent a constant logic signal (like the system being powered on or a safety condition being satisfied).

If either Input 1 (switch open) or Input 2 (some condition not met) is 0, the AND gate output will be 0, and the lamp will remain off.

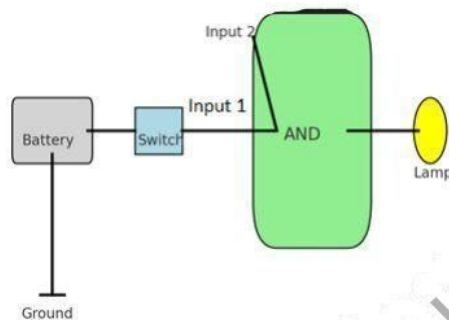


Fig: Circuit Diagram

Truth Table for the Circuit:

Switch (Input 1)	Condition (Input 2)	AND Gate Output	Lamp Status
0	0	0	OFF
0	1	0	OFF
1	0	0	OFF
1	1	1	ON

2. Construct a truth table for a 3-input OR gate. Explain how the truth table represents the relationship between input combinations and the resulting output.

Ans) **Truth Table:**

Three Input ($Y=A+B+C$)			
A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Truth Table Representation: A truth table is a mathematical table used in logic and computer science to represent the relationship between input combinations and their corresponding outputs in a logical operation or system, such as a digital circuit. It lists all possible combinations of inputs and shows the resulting output for each combination.

Here's how it works:

- i. **Input Combinations:** Each row of a truth table represents a different combination of input values (e.g., 0 and 1 for binary). For n inputs, the truth table will have 2^n rows, as there are 2^n possible combinations of binary values (0 and 1) for the inputs.
- ii. **Output Corresponding to Inputs:** The output column(s) of the truth table show how the system or logical operation responds to each specific combination of input values. The output is determined by the type of logical function or operation being represented (e.g., AND, OR, NOT, NAND, etc.).

Example: OR Gate Truth Table

Input A	Input B	Output (A OR B)
0	0	0
0	1	1
1	0	1
1	1	1

In this case:

- The output is 1 if either or both inputs (A or B) are 1.
- The output is 0 only when both inputs are 0, as this is the behavior of an OR gate, which produces 1 if at least one of the inputs is 1.

UNIT 24: RELATIVITY

MCQ'S

KEY

1. b	2. c	3. a	4. c	5. b
6. a	7. d	8. a	9. b	10. b

Section (B): CRQs (Short Answered Questions):

1. Show that for values of $v \ll c$, Lorentz transformation reduces to the Galilean transformation.

Ans) Lorentz transformation is

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Since $v \ll c$, therefore $\frac{v^2}{c^2}$ is negligible

$$x' = \frac{x - vt}{\sqrt{1}}$$

$$x' = x - vt$$

This is the Galilean Transformation.

2. If a particle could move with the velocity of light, how much K.E. would it possess?

Ans) If a particle could move at the velocity of light, its kinetic energy (K.E.) would be infinite. This is because, according to special relativity, as an object with mass approaches the speed of light, its relativistic mass increases without bound, and thus the energy required to continue accelerating it also becomes infinite.

However, only massless particles (like photons) can travel at the speed of light, and they have zero rest mass but still possess energy due to their motion.

3. Explain the difference between Special and General Relativity in simple terms.

Ans) **Special Relativity:**

- i. **Scope:** Deals with objects moving at constant speeds, particularly those close to the speed of light.
- ii. **Key Concepts:**
 - **Relative Motion:** Observers moving relative to each other can perceive time and space differently.
 - **Time Dilation:** Time moves slower for objects moving at high speeds compared to stationary observers.
 - **Length Contraction:** Objects moving at high speeds appear shorter in the direction of motion from the perspective of a stationary observer.
- iii. **Inertial Frames:** It applies to inertial frames of reference (where objects are not accelerating).
- iv. **No Gravity:** It does not account for gravitational effects.

General Relativity:

- i. **Scope:** Expands on Special Relativity to include acceleration and gravity.
- ii. **Key Concepts:**
 - **Curved Spacetime:** Mass and energy can warp the fabric of spacetime, causing the path of objects (and light) to curve.
 - **Gravity as Geometry:** Gravity is not a force in the traditional sense but rather the effect of this curvature. Objects follow curved paths in spacetime caused by massive bodies.
 - **Time Dilation Near Massive Objects:** Time runs slower in stronger gravitational fields compared to weaker fields.
- iii. **Non-Inertial Frames:** It applies to non-inertial frames of reference (where objects can be accelerating).
- iv. **Incorporates Gravity:** It fundamentally changes how we understand gravity compared to Newtonian physics.

4. Differentiate between Inertial Frames of Reference and Non- Inertial Frames of Reference.

Ans)

Criteria	Inertial frame of Reference	Non-Inertial frame of Reference
Definition	A frame of reference with a constant	A frame of reference with an accelerating motion.
Motion of objects	Objects in uniform motion appear to follow straight lines or constant velocities.	Objects may appear to accelerate or experience fictitious forces.
Newton's first law	Newton's first law (Law of inertia) is valid in this frame.	Newton's first law is not valid due to accelerating motion.
Appearance of forces	Real forces are observed and can be directly measured.	Fictitious forces (e.g., centrifugal force, coriolis force) may appear due to the acceleration of the frame
Equations of motion	Newton's laws of motion hold true in this frame.	Additional terms or transformations may be required to account of the frame.

5. Why can't any object move at the speed of light?

Ans) No object with mass can move at the speed of light because, as an object approaches this speed, its relativistic mass increases, requiring an infinite amount of energy to reach light speed. At light speed, this would imply infinite mass and energy, which is physically impossible. Only massless particles, like photons, can travel at the speed of light.

6. What is the limitation in the Galilean Transformation Equation, and how did Lorentz solve it?

Ans) **Limitation of the Galilean Transformation equations:** The limitation of the Galilean Transformation equations is that they assume time and space are absolute, leading to the conclusion that the speed of light can vary depending on the observer's motion. This contradicts the findings of experiments (like the Michelson-Morley experiment) that showed the speed of light is constant for all observers, regardless of their relative motion.

Lorentz's Solution: Hendrik Lorentz addressed this limitation by developing the Lorentz Transformation, which incorporates the following key principles:

- i. **Constancy of the Speed of Light:** The Lorentz Transformation assumes that the speed of light in a vacuum is the same for all observers, regardless of their relative motion.
- ii. **Time Dilation:** It introduces the concept that time is not absolute; it can vary for observers in different inertial frames. Moving clocks tick slower compared to stationary clocks (time dilation).
- iii. **Length Contraction:** It posits that objects in motion are measured to be shorter in the direction of motion from the perspective of a stationary observer.

Lorentz Transformation Equations: The Lorentz Transformation equations mathematically relate the coordinates and time of events in different inertial frames, accounting for the effects of relative motion and ensuring the speed of light remains constant.

7. Calculate the value of γ (Lorentz factor) if the object is moving at the speed of light.

Ans) The Lorentz factor γ is calculated using the formula:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Where:

- v is the velocity of the object,
- c is the speed of light.

If the object is moving at the speed of light, $v = c$:

$$\gamma = \frac{1}{\sqrt{1 - \frac{c^2}{c^2}}}$$

$$\gamma = \frac{1}{\sqrt{1 - 1}}$$

$$\gamma = \frac{1}{\sqrt{0}}$$

$$\gamma = \infty$$

Section (C): ERQs (Long Answered Questions):

1. State and explain the basic postulates of Einstein's special theory of relativity. Discuss length-contraction, mass variation and time-dilation.

Ans) **The Postulates of Special Relativity:** The special theory of relativity is based on two essential assumptions, commonly known as postulates.

Postulate I (Principle of Relativity):

The laws of Physics have the same form in all inertial frames of reference.

Consider a vehicle moving at a constant speed. Inside the vehicle, a passenger throws a ball straight up into the air. If we ignore any effects from the air, the passenger inside the moving vehicle sees the ball move up and then back down in a straight line, just as it would appear if someone standing still on the Earth threw a ball upwards. This means that the laws of physics that govern the motion of the ball, including gravity and equations for constant acceleration, work the same way whether the vehicle is moving or at rest.

Both observers, the one inside the vehicle and the one on the ground, see the ball go up and come back down. However, they see the path of the ball differently. The person on the ground sees the ball's path as a curved shape (a parabola), while the person in the vehicle sees it as a simple up-and-down motion. As shown in figure below.

Additionally, the person on the ground thinks that the ball has a horizontal component of velocity, which is the same as the vehicle's velocity. For the outside observer, the distance traveled along the parabolic path is longer than the path straight down but the time for the fall is the same. For the outside observer, the average velocity is greater.

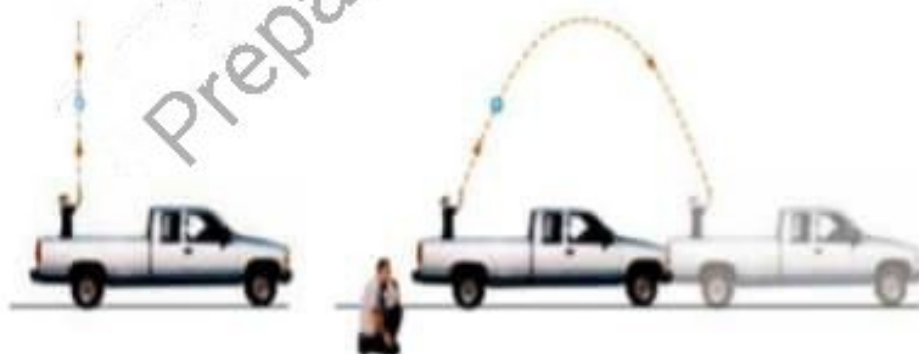


Fig: Principle of Relativity

Postulate II (Constancy of the speed of light):

The speed of light in vacuum has the same value, $c = 3 \times 10^8 \text{ m/s}$ in all inertial reference frames, regardless of the velocity of the observer or the velocity of the source emitting the light.

Consider again the same example of throwing a ball in moving vehicle. As shown in figure below, But this time, instead of throwing a ball, we shine a flashlight. If we were to do the same thing with light as we did with the ball, common sense might suggest that the speed of light would increase if the vehicle is moving in the same direction as the light according to Galilean Transformation Equation: $v = c + u$ (if we replace ball's speed v' with speed of light c) In fact, such an increase in the speed of light has never been found. In fact, in experiments carried out to test for the effect of the movement of the source on the speed of light (Michelson Morley), the results indicate that the speed of light is completely unaffected by the motion of the source. It appears that the speed of light in a vacuum is constant regardless of relative motion.

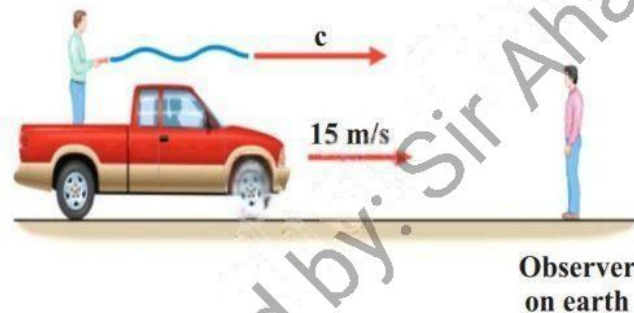


Fig: Two different frames of reference but speed of light is constant for both frames of references

Length Contraction: Let L_0 be the length of a rod when the rod is stationary as shown in fig (a). If there is relative motion at speed v between an observer at rest and the rod along the length of the rod, then observer will calculate a relativistic length L given by

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

Where the length L_0 is called proper length, L is relativistic length and c is speed of light ($3 \times 10^8 \text{ m/s}$). The relative motion causes a length contraction as shown in fig (b).

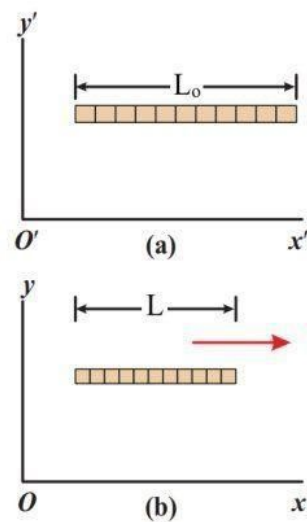


Fig: Proper length and relative length of a rod moving with velocity v

A greater speed v results in a greater contraction. Space contracts in only one direction, the direction of motion as shown in figure below.

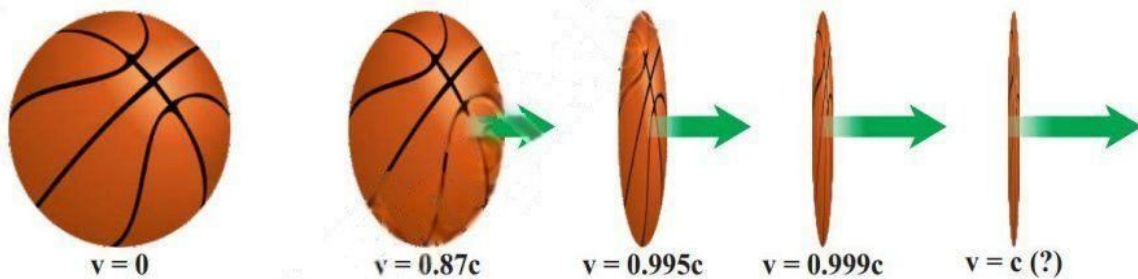


Fig: Relative speed increases the contraction in only in one direction the direction of motion. It doesn't affect other directions.

Mass Variation: Let m_0 be the rest mass of an object. If the object is moving at speed v then its relativistic mass m will be given by

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Where m is relativistic mass and c is speed of light. The relative motion causes a mass variation.

Time Dilation: The time interval, between two events occurring at a given point in the moving frame S' appears to be longer to the observer in the stationary frame S . This effect is called time dilation.

Let Δt_0 be the proper time measured by a clock that is at rest. The relative time t measured in another frame of reference is given by

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Where Δt is relativistic time and c is speed of light ($3 \times 10^8 \text{ m/s}$). Here, the value of $\sqrt{1 - \frac{v^2}{c^2}}$ should not be equal to zero; this will only occur when the speed of an object is less than the speed of light c . If the speed of an object v equals c in the above equation, then Δt becomes infinite. This implies that time will seemingly stop, which is impossible. Therefore, no material object can travel at the speed of light. As shown in figure below.

2. Explain the concept of mass-energy equivalence. Derive Einstein's mass-energy relation and demonstrate that 1 atomic mass unit (u) is equivalent to 931 MeV.

Ans) **Concept of Mass-Energy Equivalence:** The concept of mass-energy equivalence is one of the most profound results of Einstein's theory of special relativity. It states that mass and energy are interchangeable; mass can be converted into energy and vice versa. This relationship is summarized by the famous equation:

$$E = mc^2$$

Where

- E is the total energy of an object,
- m is the mass of the object,
- c is the speed of light in a vacuum.

This equation implies that even a small amount of mass corresponds to an enormous amount of energy due to the factor of c^2 , where $c = 3 \times 10^8 \text{ m/s}$. It also shows that mass is just another form of energy.

Physical Interpretation:

- **Rest energy:** The energy associated with the rest mass m_0 of an object when it is not moving is given by $E = m_0 c^2$.
- **Kinetic energy:** As an object moves and its velocity increases, it gains kinetic energy, which adds to the total energy.

- **Energy transformation:** In nuclear reactions, for instance, a small amount of mass is converted into energy, explaining why nuclear processes release such vast amounts of energy (as seen in nuclear bombs and stars).

Einstein's Mass Energy Relationship: According to Einstein's special theory of relativity, mass and energy are interchangeable. At rest, an object's mass m_0 and the equivalent energy E_0 are related by:

$$E_0 = m_0 c^2$$

If the object is moving, it has additional energy in the form of kinetic energy $K.E$. The total energy E is the sum of its mass energy and its kinetic energy:

$$E = E_0 + K.E = m_0 c^2 + \frac{1}{2} m_0 v^2$$

$$E = m_0 c^2 \left(1 + \frac{v^2}{2c^2} \right)$$

$$\therefore 1 + \frac{v^2}{2c^2} = \gamma = \text{Lorentz factor}$$

$$E = \gamma m_0 c^2$$

Energy equivalent of 1 a.m.u./u: According to Einstein mass-energy relation, the energy equivalent to mass m given by: $E = mc^2$ where c is velocity of light.

Let mass = $1 \text{ a.m.u} = 1.680665 \times 10^{-27} \text{ kg}$, $c = 3 \times 10^8 \text{ m/s}$. Then energy equivalent to 1 a.m.u is given as:

$$1 \text{ a.m.u} = 1.680665 \times 10^{-27} \times (3 \times 10^8)^2 = 1.4925 \times 10^{-10} \text{ Joules}$$

$$1 \text{ a.m.u} = \frac{1.4925 \times 10^{-10}}{1.6 \times 10^{-19}} = 931 \text{ MeV}$$

3. Discuss the important conclusions derived from General Theory of relativity. What are the experimental observations in favour of these conclusions?

Ans) **General relativity has a number of consequences:**

- Einstein's general theory of relativity states that time is a fourth dimension adding to the three dimensions of space. Einstein called this four-dimensional geometry as space-time.
- The general theory of relativity also predicted light coming from a strong gravitational field would have its wavelength shifted toward longer wavelengths, called a red-shift.
- The theory also predicted that when gravity becomes great enough, it would produce objects called black holes. Black holes are objects whose gravity is so massive that light cannot escape from the surface at all. Since no light can escape, such objects would appear black.
- Light is bent as it passes through curved space-time. This can cause distant objects to appear distorted or magnified. Gravitational lensing has been used to study distant galaxies and other objects.
- The expansion of the universe: General relativity predicts that the universe is expanding.

Experimental observations in favour of these conclusions:

- The expansion of the universe has been confirmed by astronomical observations.
- Dr. Katie Bouman captures the first image of a black hole. She and her team of NASA scientists developed a new technology to make the discovery possible. In 2019, astronomers observed the first direct evidence of existence of a black hole.

4. How does the principle of relativity differ from the classical Galilean view?

Ans) The Principle of Relativity in Einstein's Special Theory of Relativity fundamentally differs from the classical Galilean Relativity in how it treats the concepts of space, time, and motion. Let's explore these differences in detail:

Nature of Space and Time:

- **Galilean Relativity (Classical View):**
 - Space and time are treated as absolute and independent of each other. Time flows uniformly for all observers, and space is considered a fixed, unchanging stage where objects move.
 - The equations of motion (e.g., Newton's laws) apply in any inertial frame, and transformations between these frames (Galilean transformations) assume time is the same for all observers

- **Einstein's Special Relativity:**

- Space and time are relative and interconnected, forming a single entity called spacetime. The flow of time and the measurement of distances depend on the observer's relative motion.
- Time dilates (slows down) and lengths contract (shrink) for objects moving close to the speed of light, as observed from a different inertial frame. These effects are governed by the Lorentz transformations, not the Galilean transformations.
- This leads to the idea that time is not absolute; it can be different for different observers moving relative to each other.

Transformation between Reference Frames:

- **Galilean Transformations (Classical View):**

- The transformations assume that time is the same for all observers, regardless of their motion. The velocity of one object relative to another is simply the sum or difference of their velocities.

Galilean Transformation Equations:

$$x' = x - vt \text{ and } t' = t$$

where x and t are the position and time in one reference frame, and x' and t' are the position and time in another frame moving with velocity v relative to the first.

- **Lorentz Transformations (Relativity View):**

- The Lorentz transformations replace the Galilean transformations when dealing with speeds close to the speed of light. These transformations account for the fact that the speed of light c is constant for all observers, regardless of their motion.

Lorentz Transformation Equations:

$$x' = \gamma(x - vt) \text{ and } t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

where γ is the Lorentz factor:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

This shows that space and time are mixed together and depend on the relative velocity between observers. The addition of velocities is also different, where velocities do not add linearly as in Galilean relativity.

Constancy of the Speed of Light:

- **Galilean Relativity (Classical View):**

- There is no universal speed limit, and the speed of light can vary depending on the relative motion of the observer and the source of light. For example, if a light source is moving toward you, classical mechanics would suggest you see the light approaching faster than if the source were stationary.

- **Einstein's Relativity:**

- The speed of light is constant for all observers, regardless of their motion relative to the light source. This is a central postulate of Einstein's special relativity and directly contradicts the Galilean view. It means that no matter how fast an observer is moving, they will always measure the speed of light as $c = 3 \times 10^8$.

Relativity of Simultaneity:

- **Galilean Relativity (Classical View):**

- Events that occur at the same time are considered simultaneous for all observers, regardless of their motion. Time is absolute, so if two events are simultaneous in one reference frame, they are simultaneous in all others.

- **Einstein's Relativity:**

- The concept of simultaneity is relative. Whether two events occur at the same time depends on the observer's frame of reference. For example, two events that appear simultaneous to a stationary observer may not be simultaneous to an observer in motion relative to the events. This is a consequence of time dilation and the finite speed of light.

Time Dilation and Length Contraction:

- **Galilean Relativity (Classical View):**

- Time and space are independent and absolute, meaning clocks tick at the same rate, and lengths do not change regardless of motion.

- **Einstein's Relativity:**

- Time dilates (slows down) and length contracts (shortens) for objects moving at speeds close to the speed of light relative to an observer. These effects are measurable and have been confirmed in experiments with high-speed particles and atomic clocks on fast-moving airplanes.

Energy and Mass:

- **Galilean Relativity (Classical View):**

- Mass is considered constant, regardless of the speed of an object. Energy and mass are treated as separate entities, and energy is purely kinetic at high speeds.

- **Einstein's Relativity:**

- Mass and energy are equivalent as expressed by $E = mc^2$. An object's relativistic mass increases with speed, meaning it takes more and more energy to accelerate it as its velocity approaches the speed of light. No object with mass can reach or exceed the speed of light.

5. Explain the concept of spacetime curvature in general relativity and how it is used to visualize the effects of gravity.

Ans) **Gravity as Space Time Continuum/Curvature:** According to General Theory of Relativity, when you put mass in space-time, it bends the geometry of space-time. According to this theory, gravity is not just a force between masses, as described by Newtonian physics, but rather a manifestation of the curvature of space-time caused by mass and energy.

Here's a breakdown of what this concept entails:

Space-time Continuum:

- Space and time are unified into a single, four-dimensional entity known as space-time. In the absence of gravity or significant mass, space-time is flat.
- However, the presence of mass and energy warps or curves the fabric of space-time. This curvature is what we perceive as the force of gravity.

Curvature of Space-time:

- Massive objects, such as stars, planets, and galaxies, curve the space-time around them. The greater the mass or energy density, the greater the curvature of space-time.

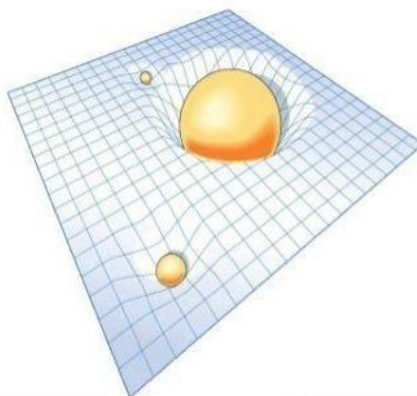
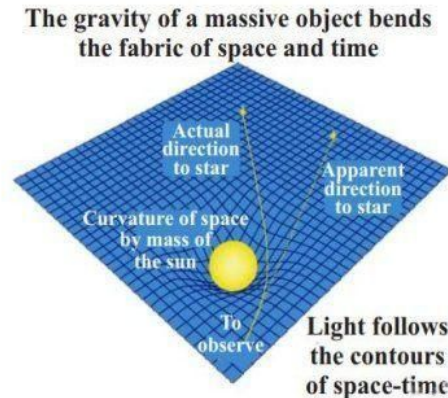


Fig: Heavier object bends the geometry of spacetime more than lighter object

- Objects move along paths dictated by the curvature of space-time, which we perceive as being influenced by gravity. For example, Earth orbits the Sun because the Sun's mass curves the space-time around it, causing Earth to follow a curved path.



Physical Interpretation:

- Gravity is thus interpreted as the result of objects moving through the curved space-time created by mass and energy. Massive objects "warp" the space-time around them, causing other objects to move in response to this curvature.
- This concept provides a unified explanation for both the gravitational force and the motion of objects in space, incorporating both space and time into a single framework.

6. Derive the basic equations of the Galilean transformation and explain how they relate the positions and velocities of objects in different inertial frames.

Ans) **Relation of Positions in Different Frames:** Let S and S' be two inertial frames as shown in figure below. Let S be at rest and S' moves with uniform velocity v along the positive X direction. We assume that $v \ll c$. Let the origins of the two frames coincide at $t = 0$. Suppose some event occurs at the point P. The observer O in frame S determines the position of the event by the coordinates x, y, z . The observer O' in frame S' determines the position of the event by the coordinate's x', y', z' . Let the time elapsed at the same rate in both frames, i.e., $t=t'$. There is no relative motion between S and S' along the axes of Y and Z. Measurements in the X direction made in S frame will be greater than those made in S' frame by the amount vt , which is the distance x' has moved in the X direction.

Therefore,

$$x' = x - vt$$

$$y' = y$$

$$z' = z$$

$$t' = t$$

The set of above equations is known as the Galilean transformation equations.

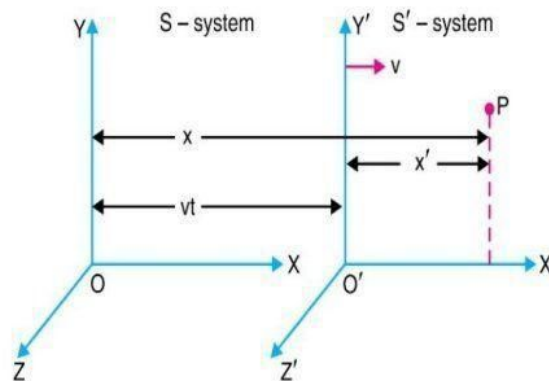


Fig: Inertial frames of references: moving and stationary

Relation of Velocities in Different Frames: The velocity of an object is the rate of change of its position with respect to time. If the object's position in frame S is given by $x(t)$, then its velocity in frame S , denoted as v_x , is:

$$v_x = \frac{dx}{dt}$$

In frame S' , the position of the object is $x' = x - vt$. Taking the derivative of this expression with respect to time gives the velocity in frame S' , denoted as $v_{x'}$:

$$v_{x'} = \frac{dx'}{dt} = \frac{d}{dt} (x - vt) = \frac{dx}{dt} - v = v_x - v$$

This shows that the velocity of the object in frame S' is simply the velocity of the object in frame S minus the relative velocity v between the two frames.

7. Discuss the Lorentz transformation equations in special relativity and how they describe the relationship between space and time coordinates in different inertial frames. Give examples to illustrate their application.

Ans) **Lorentz Transformation Equations in Special Relativity:** The Lorentz transformation equations are central to Einstein's theory of special relativity, describing how space and time coordinates of an event transform between two inertial reference frames moving at a constant velocity relative to each other. These transformations replace the Galilean transformations of classical mechanics when dealing with objects moving at speeds close to the speed of light, ensuring that the laws of physics, including the constancy of the speed of light, hold true for all observers.

The two postulates of special relativity lead to the Lorentz transformation, which accounts for relativistic effects like **time dilation** and **length contraction**.

The Lorentz Transformation Equations: Consider two inertial reference frames as shown in figure below.

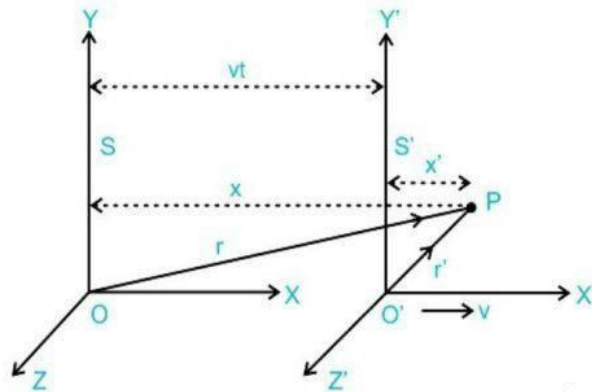


Fig: The Lorentz Transformation

- S : A stationary frame (relative to an observer).
- S' : A frame moving with a constant velocity v along the x – axis relative to S .

Let:

- (x, y, z, t) : The position and time of an event in frame S .
- (x', y', z', t') : The position and time of the same event in frame S' .

The Lorentz transformation equations relate these coordinates in the two frames:

- Transformation for the x -coordinate:

$$x' = \gamma(x - vt)$$

- Transformation for the time-coordinate:

$$t' = \gamma \left(t - \frac{vx}{c^2} \right)$$

- Transformation for the y - and z -coordinates: Since there is no relative motion along the y and z -axes, these coordinates remain the same:

$$y' = y \text{ and } z' = z$$

Here, γ is the Lorentz factor, defined as:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Where v is the relative velocity between the frames, and c is the speed of light.

Example 1 (Time Dilation): Let's consider a spaceship traveling at a speed $v = 0.8c$ (where c is the speed of light) relative to Earth. Suppose astronauts on the spaceship observe a clock ticking at regular intervals of 1 second (in their rest frame).

Using the Lorentz transformation, the time interval Δt observed on Earth (stationary frame) would be longer due to time dilation.

The Lorentz factor γ is:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$\gamma = \frac{1}{\sqrt{1 - \frac{(0.8c)^2}{c^2}}} = 1.67$$

Thus, the time interval Δt on Earth is:

$$\Delta t = \gamma \Delta t_0 = 1.67 \times 1 = 1.67 \text{ s}$$

So, for every 1 second on the spaceship, 1.67 seconds elapse on Earth. This is time dilation, where the moving clock (spaceship clock) appears to tick slower to the stationary observer on Earth.

Example 2 (Length Contraction): Now, suppose the same spaceship has a proper length (its length in its own rest frame) of $L_0 = 100 \text{ m}$. From the perspective of an observer on Earth, the spaceship is moving at $0.8c$, so its length appears contracted.

Using the length contraction formula:

$$L = \frac{L_0}{\gamma} = \frac{100}{1.67} = 59.88 \text{ m}$$

Therefore, to the Earth observer, the length of the spaceship is only about 59.88 meters due to relativistic length contraction.

Implications of Lorentz Transformations:

- i. **Speed of Light:** The Lorentz transformations preserve the constancy of the speed of light in all inertial frames. This means that if an observer in S measures the speed of light as c , an observer in S' will also measure the speed of light as c , regardless of the relative motion between S and S' .
- ii. **Relativity of Motion:** The Lorentz transformations show that space and time are not independent; they are linked in such a way that an object's position and time in one frame depend on the relative motion between frames. This leads to phenomena like time dilation and length contraction, which have been experimentally verified (e.g., in particle accelerators and with GPS satellites).

- iii. **Transformation of Velocities:** The velocity of an object in one frame can be transformed into another frame using a velocity transformation derived from the Lorentz transformations. For velocities close to the speed of light, these transformations show that no object can exceed the speed of light, which differs from the classical Galilean velocity addition formula.

Section (D): Numerical:

1. A rod 1 meter long is moving along its length with a velocity $0.6c$. Calculate its length as it appears to an observer (a) on the earth (b) moving with the rod itself.

Data:

$$L_0 = 1, v = 0.6c, (a)L = ?, (b)L = ?, v = 0$$

Solution:

(a)

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$L = (1) \sqrt{1 - \frac{(0.6c)^2}{c^2}}$$

$$L = (1) \sqrt{1 - \frac{0.36c^2}{c^2}}$$

$$L = 0.8 \text{ m}$$

(b)

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

Since observer is moving with rod, therefore $v = 0$.

$$L = (1) \sqrt{1 - \frac{0^2}{c^2}}$$

$$L = 1 \text{ m}$$

2. How fast would a rocket have to go relative to an observer for its length to be contracted to 99% of its length at rest?

Data:

$$v = ?, L = 0.99 L_0$$

Solution:

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$0.99 L_0 = L_0 \sqrt{1 - \frac{v^2}{(3 \times 10^8)^2}}$$

$$v = 42.32 \times 10^6 \text{ m/s}$$

3. A particle with a proper lifetime of $1 \mu\text{s}$ moves through the laboratory at $2.7 \times 10^8 \text{ m/s}$.

(a) What is its lifetime, as measured by observers in the laboratory? (b) What will be the distance traversed by it before disintegrating?

Data:

$$\Delta t_0 = 1 \times 10^{-6} \text{ s}, v = 2.7 \times 10^8 \text{ m/s}, (a) t = ?, (b) S = ?$$

Solution:

(a)

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Delta t = \frac{1 \times 10^{-6}}{\sqrt{1 - \frac{(2.7 \times 10^8)^2}{(3 \times 10^8)^2}}}$$

$$\Delta t = 2.3 \times 10^{-6} \text{ s}$$

(b)

$$v = \frac{S}{\Delta t}$$

$$2.7 \times 10^8 = \frac{S}{2.3 \times 10^{-6}}$$

$$S = 621 \text{ m}$$

4. At what speed is a particle moving if the mass is equal to three times its rest mass?

Data:

$$v = ?, m = 3m_0$$

Solution:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$3m_0 = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$3 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\sqrt{1 - \frac{v^2}{c^2}} = \frac{1}{3}$$

$$1 - \frac{v^2}{c^2} = \frac{1}{9}$$

$$v^2 = \left(1 - \frac{1}{9}\right)c^2$$

$$v = \frac{2\sqrt{2}}{3}c$$

5. If 4 kg of a substance is fully converted into energy, how much energy is produced?

Data:

$$m = 4 \text{ kg}, E = ?$$

Solution:

$$E = mc^2$$

$$E = (4)(3 \times 10^8)^2$$

$$E = 3.6 \times 10^{17} \text{ J}$$

6. Calculate the rest energy of an electron in joules and in electron volts.

Data:

$$E_e(\text{in } J) = ?, E_e(\text{in } eV) = ?$$

Solution:

$$E_e(\text{in } J) = mc^2$$

$$E_e(\text{in } J) = (9.11 \times 10^{-31})(3 \times 10^8)^2$$

$$E_e(\text{in } J) = 8.2 \times 10^{-14} J$$

$$E_e(\text{in } eV) = \frac{8.2 \times 10^{-14}}{1.6 \times 10^{-19}}$$

$$E_e(\text{in } eV) = 0.512 \text{ MeV}$$

7. Calculate the K.E. of an electron moving with a velocity of 0.98 times the velocity of light in the laboratory system.

Data:

$$K.E = ?, v = 0.98 c$$

Solution:

$$K.E = \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right) m_0 c^2$$

$$K.E = \left(\frac{1}{\sqrt{1 - \frac{(0.98c)^2}{c^2}}} - 1 \right) (9.11 \times 10^{-31})(3 \times 10^8)^2$$

$$K.E = 3.3 \times 10^{-13} J$$

8. At what velocity does the K.E of a particle equal its rest energy?

Data:

$$K.E = E_0, v = ?$$

Solution:

$$E_0 = m_0 c^2$$

$$K.E = \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right) m_0 c^2$$

$$\therefore K.E = E_0$$

$$\left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right) m_0 c^2 = m_0 c^2$$

$$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 = 1$$

$$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 2$$

$$\sqrt{1 - \frac{v^2}{c^2}} = \frac{1}{2}$$

$$\frac{v^2}{c^2} = 1 - \frac{1}{4}$$

$$v^2 = \left(1 - \frac{1}{4} \right) c^2$$

$$v = \sqrt{1 - \frac{1}{4}} c$$

$$v = \frac{\sqrt{3}}{2} c$$

9. A particle of rest mass m_0 moves with speed $c/\sqrt{2}$. Calculate its mass, momentum, total energy and kinetic energy.

Data:

$$v_0 = c/\sqrt{2}, m = ?, p = ?, E = ?, K.E = ?$$

Solution:

$$m = \frac{m_0}{\sqrt{1 - \frac{(c/\sqrt{2})^2}{c^2}}}$$

$$m = \sqrt{2}m_0$$

$$p = mv$$

$$p = (\sqrt{2}m_0)(c/\sqrt{2})$$

$$p = m_0c$$

$$E = mc^2$$

$$E = \sqrt{2}m_0c^2$$

$$K.E = \left(\frac{1}{\sqrt{1 - \frac{(c/\sqrt{2})^2}{c^2}}} - 1 \right) m_0c^2$$

$$K.E = 0.41m_0c^2$$

10. The nearest star to Earth is Proximal Centauri, 4.3 light-years away. A spaceship with a constant speed of $0.8c$ relative to the Earth travels toward the star.

(a) How much time would elapse on a clock as measured by travelers on the spacecraft?

(b) How long does the trip take according to Earth observers?

Data:

$$S = 4.3 \text{ light-years}, v = 0.8c, (a)t_{\text{spacecraft}} = ?, (b)t_{\text{Earth}} = ?$$

Solution:

(b)

$$\Delta t_{\text{Earth}} = \frac{S}{v}$$

$$\Delta t_{\text{Earth}} = \frac{4.3c}{0.8c}$$

$$\Delta t_{\text{Earth}} = 5.375 \text{ years} \approx 5.38 \text{ years}$$

(a)

$$\Delta t_{\text{spacecraft}} = \Delta t_{\text{Earth}} \sqrt{1 - \frac{v^2}{c^2}}$$

$$\Delta t_{\text{spacecraft}} = (5.38) \sqrt{1 - \frac{(0.8c)^2}{c^2}}$$

$$\Delta t_{\text{spacecraft}} = 3.22 \text{ years}$$

UNIT 25: QUANTUM PHYSICS

MCQ'S

KEY

1. d	2. d	3. b	4. c	5. d
6. d	7. b	8. a	9. b	10. c

Section (B): CRQs (Short Answered Questions):

1. Differentiate between wave and particle.

Ans)

Aspect	Wave	Particle
Nature	A wave is a continuous oscillation that spreads through space and time.	A particle is a discrete, localized object with a definite position.
Behavior	Exhibits interference and diffraction.	Follows classical trajectories; does not exhibit interference or diffraction.
Energy Distribution	Energy is spread over a region of space.	Energy is concentrated in a specific point or region.
Motion	Described by wave equations (e.g., wave propagation, amplitude, frequency).	Described by Newton's laws (e.g., momentum, velocity, position).

2. Is it possible for the de Broglie wavelength of a particle?

Ans) Yes, it is possible for a particle to have a de Broglie wavelength, which is given by:

$$\lambda = \frac{h}{p}$$

Where λ is the wavelength, h is Planck's constant, and p is the particle's momentum. This concept applies to all particles, indicating their wave-like nature in quantum mechanics.

3. Estimated Broglie wavelength of a cricket ball on the pitch?

Ans) According to the de-Braglie equation

$$\lambda = \frac{h}{p}$$

$$\therefore p = mv$$

$$\lambda = \frac{h}{mv}$$

Assuming a typical mass of a cricket ball is $m \approx 0.16 \text{ kg}$ and its velocity is around $v \approx 30 \text{ m/s}$ (a fast delivery in cricket), then

$$\lambda = \frac{6.63 \times 10^{-34}}{(0.16)(30)}$$

$$\lambda = 1.38 \times 10^{-34} \text{ m}$$

4. Differentiate between the continuous and discrete emission of radiation?

Ans)

Aspect	Continuous Emission	Discrete Emission
Definition	Radiation emitted over a continuous range of wavelengths or frequencies.	Radiation emitted at specific, distinct wavelengths or frequencies.
Spectrum	Produces a smooth, uninterrupted spectrum.	Produces a line spectrum with distinct lines at specific wavelengths.
Source	Typically occurs in hot, dense objects like solids, liquids, or high-pressure gases (e.g., blackbody radiation).	Typically occurs in low-density gases or atomic/molecular transitions (e.g., atomic spectra).
Energy Levels	Emitted energy is not confined to specific energy levels; transitions between many energy states.	Emission occurs when electrons transition between quantized energy levels in atoms or molecules.

5. What is threshold frequency?

Ans) The threshold frequency is the minimum frequency of incident light required to eject electrons from a material in the photoelectric effect.

In other words, it is the lowest frequency at which photons have enough energy to overcome the work function of the material and release photoelectrons. If the frequency of the incident light is below this threshold, no electrons will be emitted regardless of the intensity of the light.

Mathematically, the threshold frequency f_0 is related to the work function ϕ of the material by:

$$\phi = hf_0$$

6. How has the photoelectric effect been applied in real-world technologies or devices, and what are its practical implications?

Ans) The photoelectric effect has several important real-world applications and practical implications in various technologies and devices. Here are some key applications:

- i. **Photodetectors and Photodiodes:** Photodetectors and photodiodes are used in a range of applications including digital cameras, light sensors, and optical communication systems. They convert light into electrical signals, enabling devices to detect and process optical information accurately.
- ii. **Solar Cells (Photovoltaic Cells):** Solar cells use the photoelectric effect to convert sunlight into electrical energy. They are a major technology for renewable energy, contributing to sustainable power generation and reducing reliance on fossil fuels.
- iii. **Astronomical Instruments:** Instruments like telescopes and spectrometers use photoelectric detectors to capture and analyze light from distant celestial objects. They provide valuable data for astronomical research and space exploration.
- iv. **X-ray and Gamma-ray Detectors:** X-ray and gamma-ray detectors in medical imaging and radiation monitoring use photoelectric effects to detect high-energy photons. They enhance diagnostic capabilities and ensure safety in environments with radiation exposure.

7. What are the advantages of an electron microscope over an optical microscope?

Ans) Electron microscopes offer several advantages over optical microscopes, primarily due to their ability to achieve much higher resolution and magnification. Here are some key advantages:

- i. **Higher Resolution:** Electron microscopes can achieve resolution down to the atomic level (around 0.1 nanometers), while optical microscopes are limited to resolutions of about 200 nanometers due to the wavelength of visible light.
- ii. **Greater Magnification:** Electron microscopes can magnify specimens up to several million times, compared to the few thousand times achievable with optical microscopes.
- iii. **Detailed Imaging of Internal Structures:** Transmission electron microscopes (TEM) can provide detailed images of the internal structure of specimens, including organelles, cellular membranes, and crystal lattices, while optical microscopes can't.

- iv. **Enhanced Contrast:** Electron microscopes use electron beams and can achieve contrast through different imaging techniques, such as phase contrast and dark-field imaging, which can enhance the visibility of fine details.

8. Is it possible to create only an electron through matter and photon interaction?

Ans) No, it is not possible to create only an electron through matter and photon interaction. An electron is always created with its antiparticle, a positron, due to conservation laws.

9. Give construction of electron microscope?

Ans) **Electron microscope:** The construction of an electron microscope involves several key components and systems. Here's a concise overview of the main components:

- i. **Electron Source:**
 - **Component:** Electron gun (often a field emission or thermionic emission source).
 - **Function:** Generates a beam of electrons.
- ii. **Electromagnetic Lenses:**
 - **Component:** Magnetic lenses.
 - **Function:** Focuses and directs the electron beam, similar to how optical lenses work with light.
- iii. **Sample Holder:**
 - **Component:** Specimen stage or holder.
 - **Function:** Holds and positions the sample within the electron beam.
- iv. **Vacuum System:**
 - **Component:** High-vacuum chamber.
 - **Function:** Maintains a vacuum environment to prevent electron scattering by air molecules.
- v. **Detectors:**
 - **Component:** Backscattered electron detector and secondary electron detector.
 - **Function:** Captures and converts the electron image into a visible format for analysis.

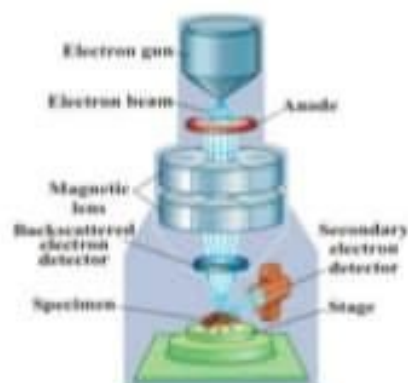


Fig: Electron Microscope

10. Elaborate the particle nature of electromagnetic radiation.

Ans) **Particle Nature of Electromagnetic Radiation:** Electromagnetic radiation can be described in terms of a stream of mass-less particles, called photons. Each photon contains a certain amount of energy. The different types of radiation are defined by the amount of energy found in the photons.

The particle nature of light states that light consists of particles called 'Photons' (Particles do not interfere) That is, when the space is occupied by some particle, other particles cannot occupy the same space. Experiments from the last century, such as the photoelectric effect and atomic line spectra, can only be explained if EM radiation is assumed to behave as particles.

Section (C): ERQs (Long Answered Questions):

1. Describe how energy is distributed over the wavelength range for different temperatures.

Ans) The distribution of energy over different wavelengths at various temperatures is described by blackbody radiation. A blackbody is an idealized object that absorbs all incident radiation and re-emits energy in a characteristic spectrum that depends solely on its temperature. The distribution of this emitted energy across wavelengths at different temperatures can be understood through Planck's law, Wien's displacement law, and the Stefan-Boltzmann law.

- i. **Planck's Law:** Planck's law gives the spectral radiance of a blackbody, which describes the amount of energy emitted per unit area, per unit wavelength, per unit time. The law is written as:

$$I(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

Where:

- $I(\lambda, T)$ is the spectral radiance at wavelength λ and temperature T .
- h is Planck's constant
- c is the speed of light
- k_B is Boltzmann's constant

Key Features of Planck's Law:

- At higher temperatures, the emitted energy increases and shifts to shorter wavelengths (towards the visible or even ultraviolet spectrum).
- At lower temperatures, the energy is mostly emitted at longer wavelengths (infrared or beyond), with less total radiation.

- ii. **Wien's Displacement Law:** Wien's displacement law describes the relationship between the temperature of a blackbody and the wavelength at which it emits the maximum intensity of radiation. It shows that as the temperature increases, the peak of the emitted spectrum shifts to shorter wavelengths.

$$\lambda_{max}T = b$$

where:

- λ_{max} is the wavelength at which the blackbody emits the maximum radiation
- T is the temperature of the blackbody
- b is Wien's constant

Implication:

- At higher temperatures, λ_{max} becomes smaller, meaning the peak of the emitted spectrum shifts toward shorter wavelengths (from infrared to visible to ultraviolet).
- At lower temperatures, λ_{max} moves to longer wavelengths, mostly in the infrared region.

For example:

- The Sun, with a surface temperature around 5778 K, emits most of its radiation in the visible spectrum, with λ_{max} around 500 nm (green light).
- A cooler object, like a person at room temperature (~300 K), emits primarily in the infrared, with λ_{max} around 9.7 μm (far infrared).

- iii. **Stefan-Boltzmann Law:** The Stefan-Boltzmann law provides the total energy radiated per unit surface area of a blackbody across all wavelengths. It states that the total radiative energy is proportional to the fourth power of the blackbody's temperature:

$$P = \sigma T^4$$

Where:

- P is the total power radiated per unit area
- σ is the Stefan-Boltzmann constant
- T is the absolute temperature in kelvins (K).

Implication:

- As the temperature of an object increases, the total energy radiated increases dramatically (since it depends on T^4).
- A small increase in temperature leads to a significant rise in the energy radiated.

Energy Distribution Over Wavelength Range at Different Temperatures:

i. **Low Temperatures (e.g., room temperature):**

- Most radiation is emitted in the infrared part of the spectrum.
- Little to no visible light is emitted.
- Peak wavelength is at long wavelengths, around 10 μm .

ii. Moderate Temperatures (e.g., incandescent light bulb, ~3000 K):

- Radiation shifts toward shorter wavelengths.
- A significant portion of the radiation is emitted in the visible spectrum, hence the light appears yellowish-white.
- Peak wavelength is around 1000 nm (near-infrared to visible).

iii. High Temperatures (e.g., the Sun, ~6000 K):

- The peak of the radiation moves further into the visible spectrum, with a lot of energy emitted in the blue and ultraviolet regions.
- The radiation intensity increases significantly across all wavelengths.
- Peak wavelength is around 500 nm (visible green light).

iv. Extremely High Temperatures (e.g., stars hotter than the Sun, ~10,000 K):

- Peak emission shifts into the ultraviolet range.
- The amount of visible light emitted increases, with stars appearing bluish.
- Peak wavelength is around 300 nm.

2. Explain the particle model of light in terms of photons with particular energy and frequency.

Ans) **Particle Model of Photons with Energy and Frequency:** In Compton Effect experiment, there is increase in wavelength of photons, due to their scattering by an electron. The impact results is one of the fundamentals of quantum mechanics, which represents wave and particle properties of light.

Arthur Holly Compton successfully demonstrated that X-rays can be treated as discrete bundles, or quanta, of electromagnetic energy. This concept was later termed "photon" by American physicist Gilbert Lewis.

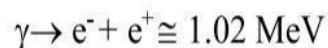
Photons exhibit both particle-like properties, such as energy and momentum, and wave-like properties, such as frequency and diffraction. The energy of a photon is dependent on its frequency, with lower energy photons having lower frequencies and longer wavelengths, as described by the equation $E = \frac{h\nu}{\lambda}$.

In Compton's experiment, photons collide with free or loosely bound electrons in matter. During these collisions, photons transfer part of their energy and momentum to the electrons, causing the electrons to recoil. This interaction results in the emission of new photons with lower energy and longer wavelengths, a phenomenon known as the Compton shift. The shift in frequency of the scattered photons depends on the energy transferred to the electrons and is independent of the initial wavelength of the incident photons.

3. Describe conservation laws in pair production and annihilation of matter.

Ans) **Conservation laws of Pair-Production:** For pair production to occur, a photon must have a minimum energy of 1.022 MeV. This requirement arises because the rest mass of both an electron and a positron is 0.511 MeV each. Therefore, the combined energy needed to create an electron-positron pair is $0.511 \text{ MeV} \times 2$ (1.022 MeV); If the incident photon possesses energy exceeding 1.022 MeV, the surplus energy is distributed as kinetic energy between the electron and the positron.

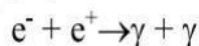
Therefore, the pair production reaction is given as under:



Facts of pair production process are:

- i. The pair production process obeys law of conservation of energy, momentum and electric charge respectively.
- ii. During collision, the antiparticle of an electron i.e., positron has the same physical properties as electron, except its charge, as both have opposite charge to each other. The sum of charges happens to be zero which is equal to photon before interaction. Therefore, electric charge is conserved.
- iii. The law of conservation of energy is
 - (a) $hf = (K.E.)_{e^-} + (K.E.)_{e^+}$
 - (b) $hf = 2m_0c^2 + (K.E.)_{e^-} + (K.E.)_{e^+}$

Conservation laws of annihilation of matter:



Each photon has an energy equal to the rest mass of electrons 0.51 MeV. The two photons are produced moving in opposite direction in order to conserve momentum and energy which forbids the creation of a single gamma-ray, as shown in figure below. The charge is also conserved as net charge before and after is zero.

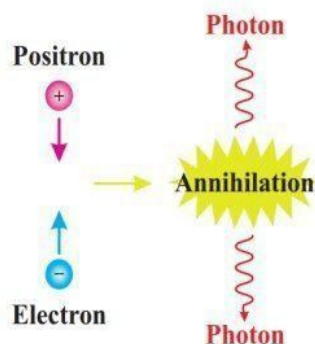


Fig: Annihilation of Matter

4. Describe Compton's effect qualitatively.

Ans) **Compton Effect Qualitatively:** Compton's Effect, as illustrated in figure below, can be qualitatively explained as follows:

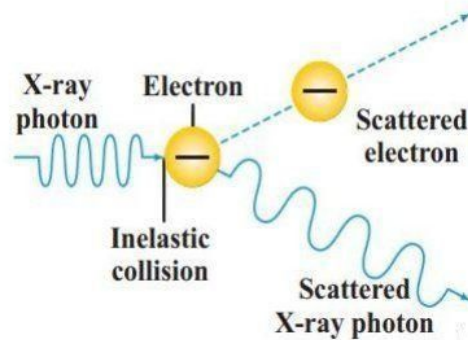


Fig: Compton Effect

Initial Interaction: A photon, which is a packet of electromagnetic energy, interacts with an electron in a material, typically a target like a metal or graphite.

Scattering Process: During the interaction, the photon transfers some of its energy and momentum to the electron. This transfer causes the photon to change its direction and wavelength (or equivalently, its frequency).

Change in Photon Energy: The scattered photon emerges with less energy (longer wavelength) than the initial incident photon. The amount of energy lost by the photon is directly related to the energy and momentum gained by the electron.

Quantum Nature: Compton's Effect cannot be explained using classical wave theory alone. Classical wave theory predicts that light should scatter uniformly without a change in wavelength. However, Compton's observations demonstrated that the scattered light has a shifted wavelength, indicating a particle-like interaction.

Experimental Confirmation: Compton conducted experiments where X-rays were targeted at graphite and the scattered X-rays were observed. By measuring the scattering angle and change in wavelength, he confirmed that the results were consistent with the predictions of quantum theory.

Wavelength Shift: The wavelength shift observed in Compton scattering is directly proportional to the Compton wavelength, which depends on the mass of the electron and the Planck constant. This relationship provides crucial evidence for the particle nature of photons. The Compton shift can be evolved as under:

$$\lambda_2 - \lambda_1 = \frac{hc}{E_0} (1 - \cos\theta) = \frac{h}{m_0c} (1 - \cos\theta)$$
$$\Delta\lambda = \frac{h}{m_0c} (1 - \cos\theta)$$

This equation describes the phenomenon known as Compton Effect. $\Delta\lambda$ gives the change in photon wavelength due to scattering with a free electron and it is called Compton shift.

It's clear that the Compton Shift is independent of the wavelength of the incident photon and depends on scattering angle.

The term $\lambda_c = \frac{h}{m_0 c} = 2.426 \times 10^{-12} \text{ m}$, is called Compton Wavelength of the scattering particle i.e., electron.

5. Explain how the very short wavelength of electrons, and the ability to use electrons and magnetic fields to focus them, allows electron microscope to achieve very high resolution.

Ans) Electron microscopes achieve very high resolution due to the very short wavelength of electrons and the ability to focus these electrons using magnetic fields. Here's how these factors contribute:

i. Wavelength of Electrons:

- The resolving power of a microscope is limited by the wavelength of the radiation used to illuminate the sample, as described by the Abbe diffraction limit. According to this principle, smaller wavelengths allow for greater resolution.
- In light microscopes, visible light is used, which has a wavelength range of 400-700 nanometers. This limits the resolution to about 200 nanometers.
- In electron microscopes, electrons are used instead of light. When electrons are accelerated to high velocities, their wavelength (calculated using the de Broglie equation) becomes extremely small—on the order of fractions of a nanometer. For example, an electron accelerated at 100,000 volts has a wavelength around 0.0037 nanometers, which is thousands of times smaller than visible light.
- This short wavelength allows electron microscopes to resolve much smaller structures, down to the atomic level (less than 1 nanometer), which is far beyond the capability of optical microscopes.

ii. Focusing of Electrons:

- Electrons are charged particles, and unlike light (which is focused by glass lenses), they can be controlled using magnetic or electrostatic lenses.
- In an electron microscope, strong electromagnetic lenses are used to focus and direct the electron beam. These lenses generate magnetic fields that bend the electron paths, similar to how glass lenses bend light in an optical microscope.
- The ability to precisely manipulate and focus the electron beam is crucial in achieving high magnification and resolving extremely fine details of a sample.

iii. Combination of Wavelength and Focusing:

- The combination of a very short wavelength and highly controllable magnetic fields allows electron microscopes, particularly transmission electron microscopes (TEM), to achieve resolutions as fine as 0.1 nanometers (1 Å), revealing atomic structures that are impossible to see with conventional light microscopes.

6. Describe the impact of de Broglie proposal that any kind of particle has both wave and particle properties.

Ans) The de Broglie hypothesis, proposed by French physicist Louis de Broglie in 1924, had a profound impact on the development of quantum mechanics and modern physics. His groundbreaking idea was that any particle, not just light, has both wave and particle properties. This concept, often referred to as wave-particle duality, transformed our understanding of the nature of matter and energy. Here's how it impacted various fields:

i. Fundamental Shift in Physics:

- Before de Broglie's proposal, wave-particle duality was mainly associated with light, following the results of the photoelectric effect (which showed light behaves as particles called photons) and interference experiments (which showed light behaves as a wave). Matter was still considered purely as particles.
- De Broglie suggested that if light (waves) could behave like particles, particles such as electrons could also exhibit wave-like behavior. His equation $\lambda = \frac{h}{p}$ (where λ is wavelength, h is Planck's constant, and p is the momentum of the particle) described the wavelength of any particle in terms of its momentum.

ii. Development of Quantum Mechanics:

- De Broglie's hypothesis led directly to the development of wave mechanics in quantum physics. Soon after, Erwin Schrödinger developed the Schrödinger equation, which describes particles as wave functions that spread out in space and time. This equation became central to quantum mechanics.
- The wave-particle duality idea became fundamental in describing the behavior of subatomic particles like electrons, protons, and neutrons, influencing the design of new models for atomic and molecular structures.

iii. Experimental Verification:

- In 1927, Davisson and Germer performed an experiment where they observed electron diffraction, confirming that electrons (normally considered particles) exhibited wave-like behavior when they scattered off a crystal. This provided direct experimental support for de Broglie's hypothesis.
- Electron microscopy is based on this principle, where electrons are used as waves to image very small structures with extremely high resolution, as their wavelength can be made very small.

iv. Heisenberg's Uncertainty Principle:

- De Broglie's wave-particle duality led to the development of Werner Heisenberg's Uncertainty Principle, which states that one cannot precisely measure both the position and momentum of a particle at the same time. The particle's wave-like nature means it has an inherent uncertainty in these measurements.
- This principle is fundamental to quantum mechanics and reflects the limitations of our classical understanding of particle behavior.

v. Quantum Tunneling:

- De Broglie's wave nature of particles also explained phenomena like quantum tunneling, where particles can pass through energy barriers that would be insurmountable under classical physics. This is possible because particles, as waves, have a probability of being found on the other side of a barrier.

vi. Impact on Chemistry and Material Science:

- In chemistry, wave-particle duality allowed scientists to understand the behavior of electrons in atoms and molecules more deeply. The wave nature of electrons explains the structure of atomic orbitals and their quantized energy levels, which are critical for understanding chemical bonds and reactions.
- Semiconductor technology, which powers modern electronics, is also based on quantum principles derived from wave-particle duality, especially in the understanding of electron behavior in materials.

7. Describe the confirmation of de Broglie proposal by Davisson and Germer experiment in which the diffraction of electrons by the surface layers of a crystal lattice was observed.

Ans) The Davisson and Germer experiment (1927) provided crucial experimental confirmation of de Broglie's hypothesis that particles, such as electrons, exhibit wave-like behavior. This experiment demonstrated the diffraction of electrons by the surface layers of a crystal lattice, which is a hallmark of wave phenomena. Here's a detailed description of the experiment and its significance:

i. Background:

- Louis de Broglie proposed in 1924 that all matter, including particles like electrons, could exhibit wave-like properties, with a wavelength λ given by the equation:

$$\lambda = \frac{h}{p}$$

where h is Planck's constant and p is the momentum of the particle.

- This was a bold extension of wave-particle duality, which had been observed in light. De Broglie's theory suggested that electrons, previously regarded as point-like particles, could also behave as waves under certain conditions.

ii. Experimental Setup:

- Clinton Davisson and Lester Germer were conducting experiments at Bell Labs to study the behavior of electrons scattered off a nickel crystal surface.
- They accelerated a beam of low-energy electrons (using a vacuum tube) and directed it at the surface of a nickel target. The electrons were scattered off the surface, and the intensity of the scattered electrons was measured as a function of the scattering angle.
- Initially, the experiment was designed to study how electrons reflect off surfaces, but during the course of their work, they observed something remarkable.

iii. Observation of Electron Diffraction:

- The key result occurred when the electron beam was directed at a nickel crystal at the correct energy. The nickel crystal acted like a diffraction grating for the electrons.
- The scattered electrons formed a pattern of intensity peaks at specific angles, which closely resembled the diffraction patterns observed with waves (such as X-rays) interacting with crystals.
- This diffraction pattern is strong evidence of wave-like behavior because diffraction occurs when a wave encounters an obstacle or slit that is comparable in size to its wavelength.

iv. Confirmation of de Broglie's Hypothesis:

- By analyzing the diffraction pattern, Davisson and Germer calculated the wavelength of the electrons based on the angles of constructive interference using Bragg's law:

$$n\lambda = 2d \sin\theta$$

Where:

- λ is the wavelength of the electrons
- d is the spacing between atomic planes in the crystal
- θ is the angle of diffraction
- n is an integer representing the diffraction order
- The calculated wavelength of the electrons matched the wavelength predicted by de Broglie's equation $\lambda = \frac{h}{p}$, confirming that electrons behave like waves with a wavelength inversely proportional to their momentum.

v. Significance of the Experiment:

- The Davisson-Germer experiment was the first direct experimental confirmation of de Broglie's theory of matter waves. It proved that particles such as electrons, which were previously thought to behave purely as particles, could also exhibit wave-like behavior.
- This finding was pivotal in the development of quantum mechanics, as it provided experimental validation for the concept of wave-particle duality, not just for light, but for all matter.
- It also supported the growing understanding that at the atomic and subatomic scales, particles do not follow classical mechanics but instead behave according to the rules of quantum mechanics, where the wave-like nature of particles plays a key role.

Section (D): Numerical:

1. The Sun's surface temperature is 5700 K.

(i) How much power is radiated by the Sun?

(ii) Given that the distance to Earth is about 200 Sun radii, what is the maximum power possible from one square kilometer solar energy installation?

(iii) What is the wavelength of maximum intensity of solar radiation?

Data:

$$T = 5700 \text{ K}, (i) P = ?, (ii) P_{max} = ?, (iii) \lambda_{max} = ?$$

Solution:

(i)

$$P = \sigma T^4$$

$$\therefore \sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$$

$$P = (5.67 \times 10^{-8})(5700)^4$$

$$P = (5.67 \times 10^{-8})(5700)^4$$

$$P = 5.98 \times 10^7 \text{ W/m}^2$$

(ii)

$$P_{max} = PA$$

The sun has a radius of $6.96 \times 10^8 \text{ m}$

The area of the sun is $A = 4\pi R^2$

$$A = 4\pi(6.96 \times 10^8)^2$$

$$A = 6.087 \times 10^{18} \text{ m}^2$$

$$P_{max} = (5.98 \times 10^7)(6.087 \times 10^{18})$$

$$P_{max} = 3.6 \times 10^{26} \text{ Watts}$$

(iii)

$$\lambda_{max} = \frac{b}{T}$$

$$\lambda_{max} = \frac{2.897 \times 10^{-3}}{5700} \rightarrow \lambda_{max} = 5.1 \times 10^{-7} \text{ m}$$

2. The temperature of your skin is approximately 32°C . What is the wavelength at which the peak occurs in the radiation emitted from your skin?

Data:

$$T = 32^{\circ}\text{C} = 32 + 273 = 305\text{ K}, \lambda = ?$$

Solution:

$$\lambda = \frac{b}{T}$$

$$\lambda = \frac{b}{T}$$

$$b = 2.897 \times 10^{-3} \text{ mK}$$

$$\lambda = \frac{2.897 \times 10^{-3}}{305}$$

$$\lambda = 9.5 \times 10^{-6} \text{ m}$$

3. An FM radio transmitter has a power output of 100 kW and operates at a frequency of 94 MHz. How many photons per second does the transmitter emit?

Data:

$$P = 100 \text{ kW} = 100 \times 10^3 \text{ W}, \nu = 94 \text{ MHz} = 94 \times 10^6 \text{ Hz}, N = ?$$

Solution:

$$E = h\nu$$

$$E = (6.63 \times 10^{-34})(94 \times 10^6)$$

$$E = 6.23 \times 10^{-26} \text{ J}$$

$$N = \frac{P}{E}$$

$$N = \frac{100 \times 10^3}{6.23 \times 10^{-26}}$$

$$N = 1.605 \times 10^{30} \text{ s}^{-1}$$

4. A light source of wavelength illuminates a metal and ejects photoelectrons with a maximum kinetic energy of 1.0 eV. A second light source with half the wavelength of the first ejects photoelectrons with a maximum kinetic energy of 4.0 eV. Determine the work function of the metal.

Data:

$$K.E_1 = 1.0 \text{ eV}, \lambda_2 = \frac{\lambda_1}{2}, K.E_2 = 4.0 \text{ eV}, \phi = ?$$

Solution:

- When light of wavelength λ_1 is incident on a metal then,

$$E_1 = \frac{hc}{\lambda_1} = K.E_1 + \phi$$

$$\frac{hc}{\lambda_1} = 1.0 + \phi$$

- When light of wavelength $\lambda_2 = \frac{\lambda_1}{2}$ is incident on a metal then

$$E_2 = \frac{hc}{\lambda_2} = \frac{hc}{\frac{\lambda_1}{2}} = K.E_2 + \phi$$

$$\frac{2hc}{\lambda_1} = 4 + \phi$$

$$2\left(\frac{hc}{\lambda_1}\right) = 4 + \phi$$

Putting $\frac{hc}{\lambda_1} = 1.0 + \phi$ in above equation, we get

$$2(1.0 + \phi) = 4 + \phi$$

$$\phi = 2 \text{ eV}$$

5. A 430 nm violet light is an incident on a calcium photo electrode with a work function of 2.71 eV. Find the energy of the incident photons and the maximum kinetic energy of ejected electrons.

Data:

$$\lambda = 430 \text{ nm} = 430 \times 10^{-9} \text{ m}, \phi = 2.71 \text{ eV}, E = ?, K.E = ?$$

Solution:

$$E = \frac{hc}{\lambda}$$

$$E = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{430 \times 10^{-9}}$$

$$E = 4.625 \times 10^{-19} \text{ J}$$

$$E = \frac{4.625 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV}$$

$$E = 2.89 \text{ eV}$$

$$E = K.E + \phi$$

$$2.89 = 2.71 + \phi$$

$$\phi = 0.18 \text{ eV}$$

6. Cut-off frequency for the photoelectric effect in some materials is $8 \times 10^{13} \text{ Hz}$. When the incident light has a frequency of $1.2 \times 10^{14} \text{ Hz}$, the stopping potential is measured as - 0.16 V. Estimate a value of Planck's constant from these data and determine the percentage error of your estimation.

Data:

$$\nu_0 = 8 \times 10^{13} \text{ Hz}, \nu = 1.2 \times 10^{14} \text{ Hz}, V_0 = -0.16 \text{ V}, h = ?, \%error = ?$$

Solution:

$$h\nu - h\nu_0 = eV_0$$

$$h(\nu - \nu_0) = eV_0$$

$$h(1.2 \times 10^{14} - 8 \times 10^{13}) = (1.6 \times 10^{-19})(0.16)$$

$$h = \frac{(1.6 \times 10^{-19})(0.16)}{1.2 \times 10^{14} - 8 \times 10^{13}}$$

$$h = 6.4 \times 10^{-34} \text{ Js}$$

$$\%error = \frac{|V_{true} - V_{approx.}|}{V_{true}} \times 100\%$$

$$\%error = \frac{|6.63 \times 10^{-34} - 6.4 \times 10^{-34}|}{6.63 \times 10^{-34}} \times 100\%$$

$$\%error = 3.47 \%$$

7. The work function of some metals is listed below. The number of metals which will show photoelectric effect when light of 300 nm wavelength falls on the metal is

Metal	Li	Na	K	Mg	Cu	Ag	Fe	Pt	W
ϕ in eV	2.4	2.3	2.2	3.7	4.8	4.3	4.7	6.3	4.75

Data:

$$\lambda = 300 \text{ nm} = 300 \times 10^{-9} \text{ m},$$

Number of metals which will show photoelectric effect = ?

Solution:

$$E = \frac{hc}{\lambda}$$

$$E = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{300 \times 10^{-9}}$$

$$E = 6.63 \times 10^{-19} \text{ J}$$

$$E = \frac{6.63 \times 10^{-19}}{1.6 \times 10^{-19}} = 4.14 \text{ eV}$$

Those metals which have work function equals to this energy or less, they will show photoelectric effect. So four metals namely Li, Na, K and Mg will show photoelectric effect.

8. X-rays with an energy of 300 keV undergo Compton scattering with a target. If the scattered X-rays are detected at 30° relative to the incident X-rays, determine the Compton shift at this angle, the energy of the scattered X-rays, and the energy of the recoiling electron.

Data:

$$E_0 = 300 \text{ keV} = 300000 \text{ eV} = 300000 \times 1.6 \times 10^{-19} = 4.8 \times 10^{-14} \text{ J}, \theta = 30^\circ,$$

$$\Delta\lambda = ?, E' = ?, E_e = ?$$

Solution:

$$\Delta\lambda = \lambda_2 - \lambda_1 = \frac{h}{m_0 c} (1 - \cos\theta)$$

$$\Delta\lambda = \lambda_2 - \lambda_1 = \frac{6.63 \times 10^{-34}}{(9.11 \times 10^{-31})(3 \times 10^8)} (1 - \cos 30^\circ)$$

$$\Delta\lambda = 3.25 \times 10^{-13} \text{ m} = 0.325 \text{ pm}$$

$$E_0 = \frac{hc}{\lambda_0}$$

$$4.8 \times 10^{-14} = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{\lambda_0}$$

$$\lambda_0 = 4.14 \times 10^{-12} \text{ m}$$

$$\lambda' = \lambda_0 + \Delta\lambda$$

$$\lambda' = 4.14 \times 10^{-12} + 0.325 \times 10^{-12}$$

$$\lambda' = 4.46 \times 10^{-12} \text{ m}$$

$$E' = \frac{hc}{\lambda'}$$

$$E' = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{4.46 \times 10^{-12}}$$

$$E' = 4.46 \times 10^{-14} \text{ J} = \frac{4.46 \times 10^{-14}}{1.6 \times 10^{-19}} = 278 \text{ keV}$$

$$E_e = E_0 - E'$$

$$E_e = 300 - 278$$

$$E_e = 22 \text{ keV}$$

9. A photon with a wavelength of $6.0 \times 10^{-12} \text{ m}$ collides with an electron. After the collision the photon wavelength is found to have been changed by exactly one (Compton Wavelength is $2.43 \times 10^{-12} \text{ m}$).

- (i) What is the photon's wavelength after collision?
- (ii) Through what angle has been deflected in this collision?
- (iii) What is the angle for the electron after the collision?
- (iv) What is the electron's kinetic energy, in eV, after collision?

Data:

$$\lambda_1 = 6.0 \times 10^{-12} \text{ m}, \Delta\lambda = 2.43 \times 10^{-12} \text{ m}, (i) \lambda_2 = ?$$

Solution:

(i)

$$\Delta\lambda = \lambda_2 - \lambda_1$$

$$2.43 \times 10^{-12} = \lambda_2 - 6.0 \times 10^{-12}$$

$$\lambda_2 = 8.4 \times 10^{-12} m$$

(ii)

$$\Delta\lambda = \lambda_2 - \lambda_1 = \frac{h}{m_0 c} (1 - \cos\theta)$$

$$2.43 \times 10^{-12} = \frac{6.63 \times 10^{-34}}{(9.11 \times 10^{-31})(3 \times 10^8)} (1 - \cos\theta)$$

$$\theta = 90^\circ$$

(iii)

$$\tan \phi = \frac{\lambda_2}{\lambda_1}$$

$$\tan \phi = \frac{8.4 \times 10^{-12}}{6.0 \times 10^{-12}}$$

$$\phi = 54.4^\circ$$

(iv)

$$\Delta E_e = E_{final} - E_{initial}$$

$$\Delta E_e = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2}$$

$$\Delta E_e = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2}$$

$$\Delta E_e = hc \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right)$$

$$\Delta E_e = (6.63 \times 10^{-34})(3 \times 10^8) \left(\frac{1}{6.0 \times 10^{-12}} - \frac{1}{8.4 \times 10^{-12}} \right)$$

$$\Delta E_e = 9.47 \times 10^{-15} J$$

$$\Delta E_e = \frac{9.47 \times 10^{-15}}{1.6 \times 10^{-19}} = 5.9 \times 10^4 eV$$

10. Find the de Broglie wavelength of an electron in the ground state of hydrogen.

Data:

$$\lambda = ?, n = 1$$

Solution:

The de-Broglie wavelength (λ) is given by the formula:

$$\lambda = \frac{h}{p} \dots\dots(i)$$

According to Bohr's model of the hydrogen atom, the angular momentum (L) of the electron in a hydrogen atom is quantized and given by:

$$L = n \frac{h}{2\pi} \dots\dots(ii)$$

The angular momentum can also be expressed in terms of the radius (r) and linear momentum (p):

$$L = mvr \dots\dots(iii)$$

Comparing equations (ii) and (iii) we get

$$mvr = n \frac{h}{2\pi}$$

$$v = \frac{nh}{2\pi rm}$$

$$\therefore p = mv$$

$$p = m \left(\frac{nh}{2\pi rm} \right) = \frac{nh}{2\pi r}$$

Putting value of p in equation (i), we get

$$\lambda = \frac{h}{\frac{nh}{2\pi r}} = \frac{2\pi r}{n}$$

The radius of ground state ground state is

$$\therefore r = 5.29 \times 10^{-11} m$$

$$\lambda = \frac{h}{\frac{nh}{2\pi r}} = \frac{2\pi(5.29 \times 10^{-11})}{1}$$

$$\lambda = 3.324 \times 10^{-10} m = 3.324 \text{ \AA}$$

11. Determine the minimum uncertainties in the positions of the following objects if their speeds are known with a precision of 1.0×10^{-3} m/s: (a) an electron and (b) a bowling ball of mass 6.0 kg.

Data:

$$\Delta v = 1.0 \times 10^{-3} \frac{m}{s}, (a) \Delta x = ?, (b) \Delta x = ?, m_b = 6.0 \text{ kg},$$

Solution:

(a)

$$\Delta p = m_e \Delta v = (9.11 \times 10^{-31})(1.0 \times 10^{-3}) = 9.11 \times 10^{-34} \text{ N}\cdot\text{s}$$

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

$$\Delta x \cdot (9.11 \times 10^{-34}) = \frac{(6.63 \times 10^{-34})}{4\pi}$$

$$\Delta x = 5.8 \text{ cm}$$

(b)

$$\Delta p = m_b \Delta v = (6)(1.0 \times 10^{-3}) = 6 \times 10^{-3} \text{ N}\cdot\text{s}$$

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

$$\Delta x \cdot (6 \times 10^{-3}) = \frac{(6.63 \times 10^{-34})}{4\pi}$$

$$\Delta x = 8.8 \times 10^{-33} \text{ m}$$

UNIT 26: ATOMIC PHYSICS

MCQ'S

KEY

1. b	2. a	3. a	4. b	5. a
6. a	7. a	8. a	9. d	10. b

Section (B): CRQs (Short Answered Questions):

1. Why do different elements have different spectra?

Ans) Different elements have different spectra because each element has a unique electronic structure. The arrangement of electrons in distinct energy levels (or orbitals) around the nucleus varies between elements. When electrons absorb or emit energy, they transition between these energy levels, and the energy differences correspond to specific wavelengths of light. These transitions produce unique sets of spectral lines for each element, known as its atomic spectrum, which serves as a fingerprint to identify the element.

2. In the Bohr model, how many times larger is the radius of the fifth Bohr orbit compared to that of the first Bohr orbit?

Ans) In the Bohr model, the radius of an electron's orbit is proportional to the square of the principal quantum number n . The formula for the radius of the n -th orbit is:

$$r_n = n^2 \cdot r_1$$

Where

- r_1 is the radius of the first Bohr orbit.

For the fifth orbit ($n = 5$) compared to the first orbit ($n = 1$):

$$r_5 = 5^2 \cdot r_1 = 25 \cdot r_1$$

Thus, the radius of the fifth Bohr orbit is 25 times larger than that of the first Bohr orbit.

3. What is the difference between X-Rays and Gamma Rays?

Ans)

Property	X-Rays	Gamma Rays
Source	Produced by electronic transitions in atoms, typically due to the deceleration of high-energy electrons or inner-shell electron transitions.	Produced by nuclear reactions, including radioactive decay, nuclear fission, or fusion.
Energy Range	Lower energy compared to gamma rays (typically 0.1 to 100 keV, though can extend higher).	Higher energy (typically greater than 100 keV and extending into MeV range).
Wavelength	Longer wavelength (around 10^{-11} to 10^{-8} meters).	Shorter wavelength (less than 10^{-11} meters).
Penetration Power	High penetration, but generally less than gamma rays.	Extremely high penetration power due to higher energy.

4. State the properties of X-Rays, which makes it possible to detect cracks in bones.

Ans) The properties of X-rays that make it possible to detect cracks in bones are:

- Penetration Ability:** X-rays can pass through soft tissues like skin and muscle, but are absorbed more by denser materials such as bone, making bones visible in X-ray images.
- Differential Absorption:** Bones, being denser than surrounding tissues, absorb more X-rays, creating a contrast that makes cracks or fractures in the bones visible.
- Short Wavelength:** The short wavelength of X-rays allows them to resolve fine details, including small fractures or cracks in bones.
- Ionizing Radiation:** X-rays interact with matter by ionizing atoms, which contributes to creating a clear image of internal structures based on how much radiation is absorbed by different tissues.

5. What is the energy of a photon that, when absorbed by a hydrogen atom, could cause an electronic transition from (a) the $n=3$ state to the $n=5$ state and (b) the $n=5$ state to the $n=8$ state?

Ans)

(a) Transition from $n = 3$ to $n = 5$:

$$E_n = -\frac{13.6}{n^2}$$

$$E_3 = -\frac{13.6}{3^2} = -1.511 \text{ eV}$$

$$E_5 = -\frac{13.6}{5^2} = -0.544 \text{ eV}$$

$$\Delta E = E_{final} - E_{initial}$$

$$\Delta E = -0.544 + 1.511$$

$$\Delta E = 0.967 \text{ eV}$$

(b) Transition from $n = 5$ to $n = 8$:

$$E_n = -\frac{13.6}{n^2}$$

$$E_5 = -\frac{13.6}{5^2} = -0.544 \text{ eV}$$

$$E_8 = -\frac{13.6}{8^2} = -0.2125 \text{ eV}$$

$$\Delta E = E_{final} - E_{initial}$$

$$\Delta E = -0.2125 + 0.544$$

$$\Delta E = 0.3315 \text{ eV}$$

6. What are the (a) wavelength range and (b) frequency range of the Lyman series and the Balmer series?

Ans)

(a) Wavelength Range:

- i. **Lyman Series:** In the Lyman series, electrons transition to the $n = 1$ energy level from higher levels $n > 1$

$$\frac{1}{\lambda} = R_H \left(1 - \frac{1}{n^2} \right)$$

Where:

- $R_H = \text{Rydberg constant} = 1.097 \times 10^7 \text{ m}^{-1}$
- n is the principal quantum number of the initial state.
- **Shortest wavelength:** Transition from $n = \infty$ to $n = 1$

$$\lambda_{min} = \frac{1}{R_H} = \frac{1}{1.097 \times 10^7} = 91.2 \text{ nm}$$

- **Longest wavelength:** Transition from $n = 2$ to $n = 1$

$$\lambda_{max} = \frac{1}{R_H \left(1 - \frac{1}{2^2} \right)} = \frac{1}{(1.097 \times 10^7) \left(1 - \frac{1}{2^2} \right)} = 121.6 \text{ nm}$$

Result: Therefore, the wavelength range for the Lyman series is approximately 91.2 nm to 121.6 nm.

- ii. **Balmer Series:** In the Balmer series, electrons transition to the $n = 2$ energy level from higher levels $n > 2$.

$$\frac{1}{\lambda} = R_H \left(\frac{1}{2^2} - \frac{1}{n^2} \right)$$

- **Shortest wavelength:** Transition from $n = \infty$ to $n = 2$

$$\lambda_{min} = \frac{1}{R_H \left(\frac{1}{2^2} \right)} = \frac{1}{(1.097 \times 10^7) \left(\frac{1}{2^2} \right)} = 364.6 \text{ nm}$$

- **Longest wavelength:** Transition from $n = 3$ to $n = 2$

$$\lambda_{max} = \frac{1}{R_H \left(\frac{1}{2^2} - \frac{1}{3^2} \right)} = \frac{1}{(1.097 \times 10^7) \left(\frac{1}{2^2} - \frac{1}{3^2} \right)} = 656.3 \text{ nm}$$

Result: Therefore, the wavelength range for the Balmer series is approximately 364.6 nm to 656.3 nm.

7. Distinguish between spontaneous and stimulated emission.

Ans)

Aspect	Spontaneous Emission	Stimulated Emission
Definition	The process by which an excited electron randomly decays to a lower energy level, emitting a photon without external influence.	The process where an excited electron is induced to decay to a lower energy level by an incoming photon, emitting an identical photon.
Photon Emission	Emission occurs randomly and independently.	Emission occurs when triggered by an incoming photon.
Photon Properties	The emitted photon has random phase, direction, and polarization.	The emitted photon is coherent with the incoming photon, sharing the same phase, direction, and polarization.
Coherence	Photons are incoherent, leading to uncorrelated light.	Photons are coherent, producing correlated and amplifiable light.

8. Explain why population inversion is necessary in a laser?

Ans)

Dominance of Stimulated Emission: Population inversion ensures that the majority of atoms or molecules are in an excited state, making stimulated emission more likely than absorption.

Amplification of Light: With more excited atoms, each incoming photon can stimulate the emission of additional photons, resulting in coherent and amplified light.

Laser Action: Without population inversion, the process of light amplification would not occur, as absorption would dominate over stimulated emission, preventing the laser from functioning.

9. In an optically pumped laser, the light that causes optical pumping is always shorter in wavelength than the laser beam. Explain

Ans) In an optically pumped laser, the light used for optical pumping has a shorter wavelength (and therefore higher energy) than the emitted laser beam because the pumping light must provide enough energy to excite electrons from a lower energy level to a higher energy state.

This energy difference between the ground state and the excited state must be greater than or equal to the energy of the emitted laser light. After reaching the excited state, the electrons undergo stimulated emission, releasing photons with a longer wavelength (lower energy) corresponding to the energy difference between two specific energy levels in the laser medium.

Thus, the pumping light has higher energy to achieve population inversion, while the laser beam corresponds to the lower energy transition between specific laser states.

10. A hydrogen atom is in its first excited state ($n=2$). Calculate (a) the kinetic energy of the electron, (b) the potential energy of the system, and (c) the total energy of the system.

Ans)

(a) $K = -E_n = \frac{13.6}{n^2}$

$$K = \frac{13.6}{2^2}$$

$$K = 3.4 \text{ eV}$$

(b) $U = 2E_n = 2 \left(-\frac{13.6}{n^2} \right)$

$$U = 2 \left(-\frac{13.6}{2^2} \right)$$

$$U = -6.8 \text{ eV}$$

(c)

$$\text{Total Energy} = K + U$$

$$\text{Total Energy} = 3.4 + (-6.8)$$

$$\text{Total Energy} = -3.4 \text{ eV}$$

Section (C): ERQs (Long Answered Questions):

1. What are the postulates of Bohr's Model of hydrogen atom? Discuss the importance of this model to explain various series of line spectra in hydrogen atom. Do any of the assumptions of the Bohr's theory of hydrogen atom contradict with the classical Physics? Derive the expression for total energy of electron in nth Bohr orbit and show that $E_n \propto 1/n^2$.

Ans) **Bohr's Model and its Postulates:** In 1913, Niels Bohr introduced atomic model in order to give quantitative determination of frequency emitted during de-excitation of an electron in Hydrogen atom.

The following are the postulates of Bohr

Postulates I: The electron in a hydrogen atom orbits the nucleus has

$$F_c = \frac{m \times v^2}{r} = F_e = k \times \frac{q_1 \times q_2}{r^2}$$

Postulates II: The magnitude of the angular momentum L of the electron in its orbit is equal to the integral multiple of $\frac{h}{2\pi}$ i.e.,

$$L = n \frac{h}{2\pi}, n = 1, 2, 3, 4, \dots$$

Where h is called Plank's constant and n is positive integer (quantum number).

Postulates III: Only certain orbits are stable in which electrons are revolving and these orbits are called stationary states. The atom emits radiation (photon) when the electron makes a transition from a higher energy state (E_n) to the lower energy state (E_p).

$$h\nu = E_n - E_p$$

where ν is the frequency of emitted photon.

Importance to explain various series of line spectra: Its importance lies in how it successfully explains the discrete spectral lines observed in hydrogen's emission spectrum, something classical physics struggled to account for. Here's a breakdown of its significance:

- **Quantization of Energy Levels:** Bohr's model postulates that electrons in a hydrogen atom can only occupy certain allowed orbits or energy levels around the nucleus. Each of these orbits corresponds to a specific quantized energy level, meaning that the electron can only exist in these fixed energy states and cannot exist in between them. This quantization directly explains why only specific wavelengths (or spectral lines) are observed, rather than a continuous spectrum.
- **Explanation of Line Spectra:** The model explains that when an electron transitions between these quantized energy levels, it absorbs or emits energy in the form of photons. The energy difference between the initial and final states corresponds to the energy (and

- therefore wavelength) of the emitted or absorbed light. This concept explains why hydrogen's emission spectrum consists of distinct lines rather than a broad spectrum.
- **The Rydberg Formula and Spectral Series:** Bohr's model also provided a theoretical foundation for the empirical Rydberg formula, which describes the wavelengths of the spectral lines of hydrogen. The model predicts the wavelengths of light in different spectral series, including:
 - **Lyman Series:** Electron transitions from higher energy levels to the ground state ($n=1$), resulting in ultraviolet radiation.
 - **Balmer Series:** Transitions to the $n=2$ energy level, responsible for visible light.
 - **Paschen, Brackett, Pfund Series:** Transitions to levels ($n=3, 4, 5$, etc.), producing infrared radiation.
- **Success in Explaining Hydrogen Spectrum:** Bohr's model was particularly successful in explaining the hydrogen atom's spectrum, especially the visible Balmer series. It could accurately calculate the wavelengths of these lines, matching experimental data. This success validated the concept of quantized energy levels and laid the foundation for further developments in quantum theory.

Contradiction between Bohr's Theory and Classical Physics:

- Quantization of Angular Momentum:**
 - Bohr's Assumption: Electrons occupy specific orbits with quantized angular momentum ($L=n\hbar$).
 - Classical Physics: Angular momentum is continuous, not quantized.
- Stationary Orbits:**
 - Bohr's Assumption: Electrons in quantized orbits do not radiate energy.
 - Classical Physics: Accelerating charges (like orbiting electrons) must radiate energy, causing them to spiral into the nucleus.
- Energy Quantization:**
 - Bohr's Assumption: Electrons have specific, discrete energy levels.
 - Classical Physics: Energy can take any continuous value, leading to a continuous spectrum.
- Stable Orbits Despite Acceleration:**
 - Bohr's Assumption: Electrons can remain in stable orbits indefinitely.
 - Classical Physics: Accelerating electrons should continuously lose energy and collapse into the nucleus.
- Energy Emission Only During Transitions:**
 - Bohr's Assumption: Energy is emitted/absorbed only during transitions between energy levels.
 - Classical Physics: Electrons should emit radiation continuously while orbiting.

Derivation of Total Energy of Electron: The allowed energy levels and quantitative values of the emission wavelengths of the hydrogen atom can be calculated from the postulate (III) suggests the qualitative existence of a characteristic discrete emission spectrum and corresponding absorption spectrum for hydrogen. The electron has kinetic energy $(K = \frac{1}{2}mV^2)$ and electric potential energy $(U = -\frac{kq_1q_2}{r^2})$. The total energy will be

$$E = K + U$$

$$E = \frac{1}{2}mV^2 - \frac{ke^2}{r}$$

Putting the value of $mV^2 = \frac{ke^2}{r}$ from postulates I of Bohr's atomic model.

$$E = \frac{ke^2}{2r} - \frac{ke^2}{r}$$

$$E = -\frac{ke^2}{2r}$$

$$\therefore r = \frac{n^2\epsilon_0 h^2}{\pi m e^2} \text{ \& } k = \frac{1}{4\pi\epsilon_0}$$

$$E = -\frac{\left(\frac{1}{4\pi\epsilon_0}\right)e^2}{2\left(\frac{n^2\epsilon_0 h^2}{\pi m e^2}\right)}$$

$$E = -\frac{me^4}{8\epsilon_0^2 h^2} \left(\frac{1}{n^2}\right)$$

Putting numerical values of following in above equation

- $e = 1.6 \times 10^{-19} \text{ C}$
- $h = 6.63 \times 10^{-34} \text{ Js}$
- $m = 9.1 \times 10^{-31} \text{ kg}$
- $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$

$$E_n = \frac{-13.6}{n^2} \text{ eV}$$

Since -13.6 is a constant so this equation shows that orbital energy of hydrogen is inversely proportional to square of principle quantum number.

$$E_n \propto \frac{1}{n^2}$$

2. How X-rays are produced? State the purpose of cooling fins in the X-ray tube.

Ans) **Production of X-rays:** X-rays as electromagnetic waves, but of much shorter wavelength: about 0.1nm to 10nm. They are produced when fast electrons, or cathode rays, strike a target, such as the walls or anode of a low-pressure discharge tube. In a modern X-ray tube there is no gas, or as little as high-vacuum technique can achieve, the pressure is about 10^{-5} mm Hg. The electrons are provided by thermionic emission from a white-hot tungsten filament.

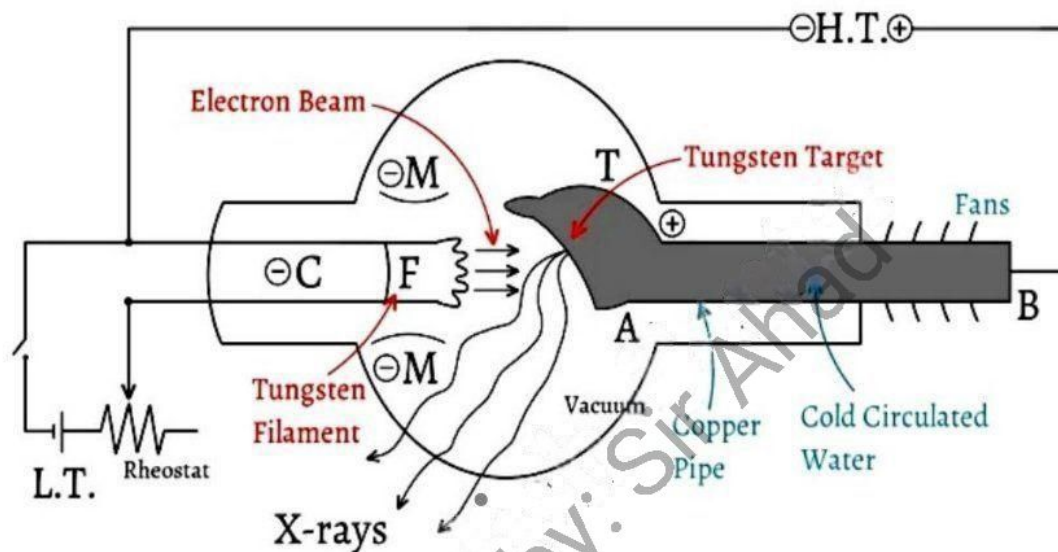


Fig: Production of X-Rays

In figure above is the filament and T is the target, or anode. Because there is so little gas, the electrons on their way to the anode do not lose any noticeable amount of their energy in ionizing atoms. From the a.c supply, transformers provide about 10 volts (L.T) for heating the filament, and about 100 kV (HT) for accelerating the electrons. On the half cycles when the target is positive, the electrons bombard it, and generate X-rays. On the half-cycles when the target is negative, nothing happens at all there is too little gas in the tube for it to break down. Thus the tube acts, in effect, as its own rectifier providing pulses of direct current between target and filament.

Purpose of Cooling Fins in the X-ray Tube: The heat generated at the target by the electronic bombardment is so enormous that the target must be cooled artificially.

3. Explain why X-rays are appropriate in study of crystalline structure material? Write some main properties of X-rays.

Ans) **Reasons why X-rays appropriate in study of crystalline structure:**

- Wavelength Compatibility:** The wavelength of X-rays is on the order of 1 to 10 angstroms, which is comparable to the distance between atoms in a crystal. This allows X-rays to effectively probe the atomic-scale structure of the material.

- ii. **Diffraction Phenomenon:** When X-rays hit a crystal, they are diffracted by the periodic atomic planes within the crystal. The angles and intensities of the diffracted X-rays provide information about the crystal's structure. This phenomenon is described by Bragg's Law, which relates the wavelength of the X-rays to the angle of diffraction and the spacing between atomic planes.
- iii. **High Resolution:** X-ray diffraction (XRD) can produce high-resolution data on the positions of atoms in a crystal, the symmetry of the crystal lattice, and the dimensions of the unit cell. This detailed information is crucial for understanding the material's properties and behavior.
- iv. **Non-Destructive:** X-ray analysis is a non-destructive technique, meaning it can be used to study samples without altering or damaging them. This is especially important for valuable or sensitive materials.
- v. **Quantitative Data:** X-ray diffraction provides quantitative information about the crystal structure, such as lattice parameters, atomic positions, and the presence of any structural distortions or defects.

Properties of X-rays:

- i. **Electromagnetic Radiation:** X-rays are a form of electromagnetic radiation with wavelengths ranging from about 0.01 to 10 nanometers, which is shorter than ultraviolet light but longer than gamma rays.
- ii. **Penetrating Power:** X-rays have high penetrating power, allowing them to pass through various materials, including human tissue, which makes them useful in medical imaging and industrial inspections.
- iii. **High Energy:** X-rays have high energy and can ionize atoms and molecules. This property is used in medical imaging to visualize the internal structures of the body and in industrial applications to inspect welds and materials for defects.
- iv. **Absorption by Dense Materials:** X-rays are absorbed more by dense materials (like bones or metals) compared to less dense materials (like soft tissues). This difference in absorption allows for contrast in X-ray images.
- v. **Wave-Particle Duality:** X-rays exhibit both wave-like and particle-like properties. They can be described as photons with energy, frequency, and wavelength, and they can also behave like waves in diffraction experiments.

4. What is Laser? Write the characteristics of Laser light. Can a two-level system be used for the production of Laser? Why?

Ans) **LASER (Light Amplification by Stimulated Emission of Radiation):** The electromagnetic waves experienced in daily life, ranging from the sun, stars, incandescent and fluorescent Lamps are emitted from atoms or molecules spontaneously.

- Ordinary natural and artificial, light is released by energy changes on the atomic and molecular level that occur without any outside interference.
- A second type of light exists, when an atom or molecule retains its excess energy until stimulated or induced to emit the energy in the form of light.

“Lasers are designed to produce and amplify this stimulated form of light into intense and focused beams. Compared to conventional sources of ordinary light, the light from a laser is quite intense, monochromatic, and emitted in a unidirectional beam limited by diffraction.”

The special nature of laser light has made laser technology a vital tool in nearly every aspect of everyday life including communications, entertainment, manufacturing, and medicine.

Characteristics of Laser light: Laser light has several distinct characteristics that differentiate it from ordinary light sources. These characteristics include:

- Monochromaticity:** Laser light is typically monochromatic, meaning it consists of a single wavelength or color. Unlike ordinary light, which contains a broad spectrum of wavelengths, laser light is nearly pure in its color.
- Coherence:** Laser light is highly coherent, both spatially and temporally. This means that the light waves are in phase over a significant distance (spatial coherence) and remain in phase over time (temporal coherence), allowing the light to be tightly focused or travel over long distances without significant divergence.
- Directionality:** Laser light is emitted in a very narrow beam with minimal spread. It is highly directional, unlike regular light sources that emit light in all directions. This makes lasers ideal for applications that require precise targeting or long-distance transmission.
- High Intensity:** Because laser light is concentrated into a small area, it can achieve very high intensities compared to ordinary light sources. This characteristic makes lasers useful in applications requiring high energy, such as cutting, welding, and medical surgeries.
- Polarization:** Laser light can be polarized, meaning its electric and magnetic fields oscillate in a particular direction. Polarized laser light can be advantageous in various optical applications, such as in microscopy or optical communication.

Usage of Two-level System for the production of Laser: No, a two-level system cannot be used for the production of a laser. Here's why:

For laser production, a key requirement is population inversion—a condition where more electrons are in an excited state than in the ground state. In a two-level system, it is extremely difficult, if not impossible, to achieve population inversion due to the following reasons:

- i. **Equal Probability of Absorption and Emission:** In a two-level system, when atoms are excited from the ground state to a higher energy level by absorbing photons, there is an equal probability of spontaneous emission or stimulated emission back to the ground state. As a result, it is difficult to get more atoms in the excited state than in the ground state.
- ii. **Thermal Equilibrium:** According to Boltzmann distribution, in thermal equilibrium, there are always more atoms in the ground state than in the excited state in a two-level system. To generate laser light, you need to break this equilibrium, but in a two-level system, this is not feasible.
- iii. **Decay to Ground State:** In a two-level system, excited electrons quickly decay back to the ground state through spontaneous emission. Since the electrons can't be held in the excited state long enough to create the necessary population inversion, sustained lasing cannot occur.

5. What is pumping? What are the different methods of pumping? Explain optical pumping.

Pumping: Pumping refers to the process of supplying energy to the active medium (the material inside the laser) in order to excite electrons from a lower energy state to a higher energy state. The goal of pumping is to achieve population inversion, where more atoms or molecules are in the excited state than in the ground state. This population inversion is essential for the stimulated emission of light, which is the basis of laser action.

Different Methods of Pumping in Lasers:

- i. **Optical pumping:** Optical pumping is one of the most common methods used to achieve population inversion in laser systems, especially in solid-state lasers and dye lasers. In this method, light from an external source is used to excite electrons in the active medium to higher energy states. The light source typically emits photons that are absorbed by the atoms or ions of the active medium, causing a transition to an excited state.

Key Elements of Optical Pumping:

i. Pump Light Source:

- The light source used for optical pumping can be a flashlamp, arc lamp, LED, or even another laser. These sources are chosen based on the energy levels and absorption spectrum of the active medium.
- The pump light must have a frequency (or wavelength) that matches the energy difference between the ground state and an excited state of the active medium, ensuring that the medium can absorb the photons efficiently.

ii. Active Medium:

- The active medium is the material inside the laser where the light-matter interaction occurs. For optical pumping, this medium can be a solid, such as a crystal (like ruby or Nd), or a liquid, such as a dye solution.

- The atoms, ions, or molecules in the active medium must have distinct energy levels, and the medium should be able to absorb the incoming light to excite electrons.

iii. Absorption and Excitation:

- The process begins when the pump light is absorbed by the atoms or ions in the active medium, causing their electrons to move from the ground state (lower energy level) to an excited state (higher energy level). This is a non-lasing transition.
- In most cases, the excited state is not the upper laser level but an even higher energy state. The electrons then rapidly decay to the upper laser level, from where the laser transition occurs.

iv. Decay and Population Inversion:

- Once the electrons are in the upper laser level, they are relatively stable for a certain period (depending on the medium). If enough electrons accumulate in this upper level, population inversion is achieved, which is necessary for stimulated emission and laser action.
- The electrons eventually decay from the upper laser level to a lower energy level (usually a metastable state), releasing photons that contribute to the laser beam through stimulated emission.

Application of Optical Pumping:

- **Ruby Laser:** One of the earliest lasers, the ruby laser, uses a cylindrical ruby rod as the active medium and a flashlamp as the optical pump source.
- **Nd Laser:** The Nd laser uses a neodymium-doped yttrium aluminum garnet (YAG) crystal as the active medium. In this case, a flashlamp or diode laser is often used as the optical pumping source.
- **Dye Lasers:** In dye lasers, a solution of organic dye molecules is used as the active medium. Optical pumping is usually performed by a flashlamp or another laser.

ii. Electrical Pumping:

- **Description:** In electrical pumping, an electric current is passed through the active medium to excite electrons. This method is particularly effective in gases and semiconductors.
- **Application:** Electrical pumping is used in gas lasers (e.g., Helium-Neon (He-Ne) lasers) and semiconductor lasers (e.g., diode lasers). For instance, in gas lasers, a high voltage is applied across the gas, causing electrons to collide with the gas atoms and excite them.

iii. Chemical Pumping:

- **Description:** In chemical pumping, a chemical reaction provides the energy needed to excite the atoms or molecules in the active medium. The energy released from exothermic chemical reactions drives the population inversion.

- **Application:** This method is typically used in chemical lasers, such as the hydrogen fluoride (HF) laser or deuterium fluoride (DF) laser, where a chemical reaction directly excites the laser medium.
- iv. **Direct Pumping by Electron Beams:**
- **Description:** An electron beam is used to excite the atoms or molecules in the active medium. The high-energy electrons collide with the atoms, transferring energy and causing excitation.
 - **Application:** This method is often used in excimer lasers, where electron beams ionize the active medium (such as a halogen gas mixture) and create excited states, leading to laser action.
- v. **Radiofrequency (RF) or Microwave Pumping:**
- **Description:** In RF or microwave pumping, radiofrequency or microwave energy is used to excite the gas atoms. The RF or microwave energy ionizes the gas, leading to collisions that excite atoms into higher energy states.
 - **Application:** This method is used in gas lasers, such as the CO₂ laser, where radiofrequency or microwave energy can efficiently generate the necessary population inversion.

6. What is the principle of Laser? Write the construction and working of Helium-neon laser.

Ans) **Principle of Laser:** A laser works on the principle of stimulated emission, where photons trigger excited atoms to emit identical photons, amplifying light. This requires population inversion, and mirrors in the laser cavity help produce a coherent, monochromatic, and directional beam.

Helium-neon Laser/He-Ne Gas Laser:

Helium-neon Laser Construction: The helium-neon laser consists of three essential components:

- **Pump Source (high voltage power supply):** The pump energy of the laser is provided by an electrical discharge of several hundred Volts between an anode and cathode at each end of the glass tube. A current of 5 to 100 mA is typical for laser operation.
- **Gain Medium (laser glass tube or discharge glass tube):** Figure below shows a gas discharge tube contains a low- pressure mixture of helium-neon in a ratio between 5:1 and 20:1 bound in a glass tube. The partial pressure of helium is 1 mbar whereas that of neon is 0.1 mbar.
- **Resonating Cavity:** The glass tube (containing a mixture of helium and neon gas) is placed between two parallel mirrors. These two mirrors are silvered or optically coated. Each mirror is silvered differently. The left side mirror is partially silvered and is known as output coupler whereas the right side mirror is fully silvered and is known as the high reflector or fully reflecting mirror. The fully silvered mirror will completely reflect the light whereas the partially silvered mirror will reflect most part of the light but allows some part of the light to produce the laser beam.

Laser Operation: This electrical excitation raises the helium atom into a meta-stable state with energy 20.61 eV above the ground state. Neon has a meta-stable state with energy 20.66 eV above its ground state. Collision of the excited helium atoms with the ground-state neon atoms results in transfer of energy to the neon atoms, exciting neon electrons. The difference between the energy states of the two atoms is in the order of 0.05 eV, which is supplied by kinetic energy. The number of neon atoms in the excited states builds up as further collisions between helium and neon atoms occur, causing a population inversion. Spontaneous and stimulated emission results in emission of 632.82 nm wavelength light.

7. Give some important properties of lasers. Also write the uses of lasers in the field of medicine, defense and communication.

Properties of Laser light: Laser light has several distinct properties that differentiate it from ordinary light sources. These characteristics include:

- **Monochromaticity:** Laser light is typically monochromatic, meaning it consists of a single wavelength or color. Unlike ordinary light, which contains a broad spectrum of wavelengths, laser light is nearly pure in its color.
- **Coherence:** Laser light is highly coherent, both spatially and temporally. This means that the light waves are in phase over a significant distance (spatial coherence) and remain in phase over time (temporal coherence), allowing the light to be tightly focused or travel over long distances without significant divergence.
- **Directionality:** Laser light is emitted in a very narrow beam with minimal spread. It is highly directional, unlike regular light sources that emit light in all directions. This makes lasers ideal for applications that require precise targeting or long-distance transmission.
- **High Intensity:** Because laser light is concentrated into a small area, it can achieve very high intensities compared to ordinary light sources. This characteristic makes lasers useful in applications requiring high energy, such as cutting, welding, and medical surgeries.
- **Polarization:** Laser light can be polarized, meaning its electric and magnetic fields oscillate in a particular direction. Polarized laser light can be advantageous in various optical applications, such as in microscopy or optical communication.

Uses of Lasers: Lasers are widely used in many fields some of them are

i. Communication:

- Lasers are used in Barcode scanners to convert a printed barcode into a number.
- A semiconductor laser beam helps to convert the printed pattern of data into numbers in a CD/DVD.
- In Photonics, lasers are used in fibre optic cables.

ii. Defence:

- In defence field, the military employs laser-guided guns and missiles.
- Laser range-finders use high-resolution scanning to find the distance and speed from an object that is located beyond the point-blank range.

iii. **Medicine:**

- In medicine, the laser beam is used as part of phototherapy for many procedures.
- The development of laser technology in recent decades has enabled the creation of a new field of medicine laser surgery.
- Lasers have uses in dermatology, ophthalmology, urology, rheumatology and dentistry.

8. What is wave number? Derive the expression $\frac{1}{\lambda} = R_H \left(\frac{1}{p^2} - \frac{1}{n^2} \right)$.

Ans) **Wave number:** The wave number of a photon is the number of wave cycles per unit distance and is related to the wavelength of the photon. It is commonly used in spectroscopy and is defined as:

$$\tilde{\nu} = \frac{1}{\lambda}$$

Where;

- $\tilde{\nu}$ is the wave number (in cm^{-1} , or other reciprocal length units),
- λ is the wavelength of the photon (in cm, m, etc.).

The wave number is directly proportional to the energy of the photon. Using Planck's relation for energy $E = h\nu$ (where h is Planck's constant and ν is the frequency of the photon), and the relationship between frequency and wavelength ($\nu = \frac{c}{\lambda}$, where c is the speed of light), we can also express the energy of a photon in terms of its wave number:

$$E = hc\tilde{\nu}$$

Thus, the wave number gives a measure of the energy of the photon in terms of inverse wavelength.

Derivation of $\frac{1}{\lambda} = R_H \left(\frac{1}{p^2} - \frac{1}{n^2} \right)$: The electron in a hydrogen atom can jump between quantized energy levels by emitting or absorbing Photon for some different values of wavelengths. Any such wavelength is often called a line spectrum which can be absorption or emission lines. The lines for hydrogen are said to be grouped into series, according to the level at which upward jumps start and downward jumps end. The formula for these series corresponding to the different wavelengths can be obtained from equation.

$$E = -\frac{me^4}{8\epsilon_0^2 h^2} \left(\frac{1}{n^2} \right)$$

The frequency of the photon emitted when the electron makes a transition from an orbit to an inner orbit as stated in postulate –III of Bohr's atomic model.

$$\nu = \frac{E_n - E_p}{h}$$

Putting the values of E in above equation, we get

$$v = \frac{me^4}{8\epsilon_0^2 h^3} \left(\frac{1}{p^2} - \frac{1}{n^2} \right)$$

To express above equation in terms of wavelength, it is convenient to use $c = v\lambda$.

$$\frac{1}{\lambda} = \frac{me^4}{8c\epsilon_0^2 h^3} \left(\frac{1}{p^2} - \frac{1}{n^2} \right)$$

Where $R_H = \frac{me^4}{8\epsilon_0^2 h^3} = 1.097 \times 10^7 \text{ m}^{-1}$, is called Rydberg constant.

$$\frac{1}{\lambda} = R_H \left(\frac{1}{p^2} - \frac{1}{n^2} \right)$$

The possible series associated the different wavelengths can be explained from equation above.

9. Derive the expression for Bohr's radius and develop a general relation for radii of quantized orbits of hydrogen atom.

Bohr's radius: The electron revolving around the nucleus is in uniform circular motion and thus experiences a centripetal force. Here centripetal force is provided by electrostatic force between electron and proton.

$$F_c = \frac{mv^2}{r} \dots\dots\dots(i)$$

where v is the speed of electron. The magnitude of electrostatic force $\left(F_e = \frac{kq_1q_2}{r^2}\right)$ between the electron ($q_1 = e$) and the proton ($q_2 = e$) separated by the orbital radius r is given as.

$$F_e = \frac{kq^2}{r^2} \dots\dots\dots(ii)$$

Where $k = 9.0 \times 10^9 \text{ Nm}^2/\text{C}^2$ is called Coulomb's constant.

The electron can only move in a particular orbit if the above two forces are balanced each other. Comparing equation (i) and (ii)

$$\frac{kq^2}{r^2} = \frac{mv^2}{r} \dots\dots\dots(iii)$$

From Bohr's postulate-II,

$$L = mvr = n \frac{h}{2\pi}$$

Substituting the value of $V = \frac{nh}{2\pi mr}$

Where

$$k = \frac{1}{4\pi\epsilon_0}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$m = 9.1 \times 10^{-31} \text{ kg}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$$

$$r = \frac{n^2 \epsilon_0 h^2}{\pi m e^2}$$

$$r_0 = \frac{\epsilon_0 h^2}{\pi m e^2}$$

$$r = r_0 n^2$$

where $r_0 = 5.29 \times 10^{-11} \text{ m} = 0.53 \text{ \AA}$ is called Bohr's first orbit.

As the values of r is depending only on the principle quantum number n . Therefore, the radii have the quantized values as shown in figure below.

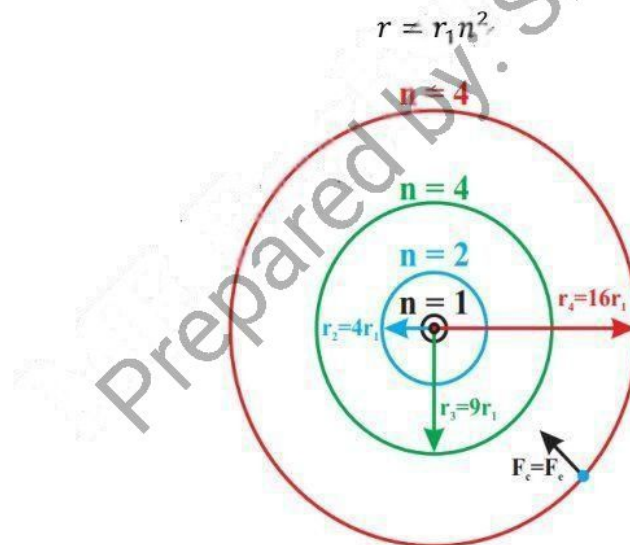


Fig: Quantized radii for hydrogen atom

Section (D): Numerical:

1. Calculate the energy of an electron in the $n = 2$ orbit of a hydrogen atom according to the Bohr model.

Data:

$$E_2 = ?, n = 2$$

Solution:

$$E_n = -\frac{13.6}{n^2}$$

$$E_2 = -\frac{13.6}{2^2}$$

$$E_2 = -3.4 \text{ eV} = -3.4 \times 1.602 \times 10^{-19} = 5.447 \times 10^{-19} \text{ J}$$

2. Calculate the speed of the electron if it orbits in (a) the smallest allowed orbit and (b) the second smallest orbit? (c) If the electron moves to larger orbits, does its speed increase, decrease, or stay the same?

Data:

$$(a) v_1 = ?, (b) v_2 = ?,$$

(c) What happens to the speed if the electron moves to larger orbits?

Solution:

(a) The Coulomb force between the electron and the proton provides the centripetal force that keeps the electron in circular orbit about the proton:

$$\frac{ke^2}{r_n^2} = \frac{m_e v_n^2}{r_n}$$

The smallest orbital radius is $r_1 = 52.9 \times 10^{-12} \text{ m}$. The corresponding speed of the electron is obtained as follows:

$$v_1 = \sqrt{\frac{ke^2}{m_e r_1}}$$

$$\therefore k = 9 \times 10^9, e = 1.6 \times 10^{-19}, m_e = 9.11 \times 10^{-31} \text{ kg}$$

$$v_1 = \sqrt{\frac{(9 \times 10^9)(1.6 \times 10^{-19})^2}{(9.11 \times 10^{-31})(52.9 \times 10^{-12})}}$$

$$v_1 = 2.19 \times 10^6 \text{ m/s}$$

(b) The radius of the second smallest orbit is $r_2 = n^2 r_1 = (2)^2 (52.9 \times 10^{-12})$
 $= 2.116 \times 10^{-10} \text{ m}$

$$v_2 = \sqrt{\frac{ke^2}{m_e r_2}}$$

$$v_2 = \sqrt{\frac{(9 \times 10^9)(1.6 \times 10^{-19})^2}{(9.11 \times 10^{-31})(2.116 \times 10^{-10})}}$$

$$v_2 = 1.09 \times 10^6 \text{ m/s}$$

(c) Since the speed is inversely proportional to $r^{\frac{1}{2}}$, the speed of the electron will decrease if it moves to larger orbits.

3. What are the (a) energy, (b) magnitude of the momentum, and (c) wavelength of the photon emitted when a hydrogen atom undergoes a transition from a state with $n = 3$ to a state with $n = 1$?

Data:

$$(a) E = ?, (b) p = ?, (c) \lambda = ?, n_1 = 1, n_3 = 3$$

Solution:

(a)

$$\Delta E = -13.6 \left(\frac{1}{n_3^2} - \frac{1}{n_1^2} \right)$$

$$\Delta E = -13.6 \left(\frac{1}{3^2} - \frac{1}{1^2} \right)$$

$$\Delta E = 12.1 \text{ eV}$$

(b)

$$p = \frac{\Delta E}{c}$$

$$\therefore c = 3 \times 10^8 \text{ m/s}$$

$$p = \frac{12.1 \times 1.602 \times 10^{-19}}{3 \times 10^8}$$

$$p = 6.46 \times 10^{-27} \text{ N.s}$$

(c)

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_1^2} - \frac{1}{n_3^2} \right)$$

$$\frac{1}{\lambda} = (1.097 \times 10^7) \left(\frac{1}{1^2} - \frac{1}{3^2} \right)$$

$$\lambda = 102 \text{ nm}$$

4. What is the energy of the photon emitted by hydrogen atom when the hydrogen atom changes directly from the $n = 5$ state to the $n = 2$ state?

Data:

$$\Delta E = ?, n_5 = 5, n_2 = 2$$

Solution:

$$\Delta E = -13.6 \left(\frac{1}{n_5^2} - \frac{1}{n_2^2} \right)$$

$$\Delta E = -13.6 \left(\frac{1}{5^2} - \frac{1}{2^2} \right)$$

$$\Delta E = 2.85 \text{ eV}$$

5. How much work must be done to pull apart the electron and the proton that make up the hydrogen atom if the atom is initially in (a) its ground state and (b) the state with $n=3$?

Data:

$$\text{Work done for ground state} = ?, \text{Work done for state (when } n = 3) = ?$$

Solution:

$$E_n = -\frac{13.6}{n^2}$$

$$E_1 = -\frac{13.6}{1^2}$$

$$E_1 = -13.6 \text{ eV}$$

Hence, the value of work done to pull apart the electron and the proton when the atom is initially in its ground state is 13.6 eV.

$$E_3 = -\frac{13.6}{3^2}$$

$$E_3 = -1.51 \text{ eV}$$

Hence, the value of work done to pull apart the electron and the proton when the atom is initially in its state when $n=3$ is 1.51 eV.

6. (a) What is the wavelength of light for the least energetic photon emitted in the Balmer series of the hydrogen atom spectrum lines?

(b) What is the wavelength of the series limit?

Data:

(a) Wavelength of light for the least energetic photon emitted in the Balmer series =?

(b) Wavelength of the series limit =?

Solution:

(a)

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_2^2} - \frac{1}{n_3^2} \right)$$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{3^2} \right)$$

$$\lambda = 656.333 \text{ nm}$$

(b)

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_2^2} - \frac{1}{n_{\infty}^2} \right)$$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{\infty^2} \right)$$

$$\lambda = 364.63 \text{ nm}$$

7. A laser emits light with a wavelength of 632.8 nm and has a power output of 55 mW. Calculate the energy of one photon emitted by this laser.

Data:

$$\lambda = 632.8 \text{ nm} = 632.8 \times 10^{-9} \text{ m}, P = 55 \text{ mW}, \Delta E = ?$$

Solution:

$$\Delta E = \frac{hc}{\lambda}$$

$$\therefore h = 6.63 \times 10^{-34} \text{ Js}, c = 3 \times 10^8 \text{ m/s}$$

$$\Delta E = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{632.8 \times 10^{-9}}$$

$$\Delta E = 3.14 \times 10^{-19} \text{ J}$$

8. Calculate the wavelength of X-rays if the energy of one photon emitted by the X-ray machine is 1.9878×10^{-15} joules.

Data:

$$\lambda = ?, \Delta E = 1.9878 \times 10^{-15} \text{ J}$$

Solution:

$$\Delta E = \frac{hc}{\lambda}$$

$$\therefore h = 6.63 \times 10^{-34} \text{ Js}, c = 3 \times 10^8 \text{ m/s}$$

$$1.9878 \times 10^{-15} = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{\lambda}$$

$$\lambda = 1 \times 10^{-10} \text{ m} = 0.1 \text{ nm}$$

Prepared by: Sir Ahad

UNIT 27: NUCLEAR PHYSICS

MCQ'S

KEY

1. c	2. d	3. a	4. a	5. b
6. d	7. d	8. d	9. a	10. d

Section (B): CRQs (Short Answered Questions):

1. Why are protons and neutrons necessary for the stability of an atomic nucleus?

Ans) Protons and neutrons are both necessary for the stability of an atomic nucleus. Protons contribute positive charge, and while they experience repulsive electrostatic forces, they also engage in the strong nuclear force that binds the nucleus together. Neutrons, on the other hand, contribute only to the strong nuclear force without adding repulsive charge, helping to balance the forces and maintain nuclear stability. Both particles work together to ensure the nucleus remains stable against the forces that would otherwise cause it to break apart.

2. How do isotopes of an element differ and why are these differences significant?

Ans) **Difference:** Isotopes of an element differ in the number of neutrons in their nuclei, while having the same number of protons and electrons. This results in different atomic masses but identical chemical properties, since chemical behavior is determined by the number of protons and electrons, not neutrons.

Significant: The differences in neutron count affect nuclear stability, leading to some isotopes being stable and others radioactive. These differences are significant because radioactive isotopes have important applications in fields like medical imaging, cancer treatment, carbon dating, and nuclear energy. Stable isotopes, meanwhile, help in understanding elemental behavior.

3. Name the two methods of controlling a chain reaction.

Ans) **The two methods of controlling a chain reaction are:**

- i. Control Rods
- ii. Moderators

4. Why is nuclear decay described as spontaneous and random?

Ans) Nuclear decay is described as spontaneous because it occurs without any external influence or trigger; the nucleus of an unstable atom breaks down on its own. It is considered random because it is impossible to predict exactly when a particular atom will decay. While the overall decay rate of a large sample can be calculated statistically (via half-life), the decay of individual atoms happens unpredictably.

5. Explain what happens during the nuclear fusion process.

Ans) **Nuclear Fusion Process:** The process of combining two light nuclei to form a heavy nucleus with the release of huge amount of energy due to mass defect is known as nuclear fusion.

This mass defect results in the release of a huge amount of energy according to the relation $E = mc^2$. When two nuclei of heavy hydrogen or deuterium (${}^2_1\text{H}$) are combined, the following reaction is possible:

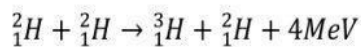
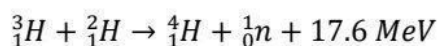


figure below shows the nucleus of tritium (${}^3_1\text{H}$) so formed can again fuse with a deuterium nucleus (${}^2_1\text{H}$) to give the following reaction



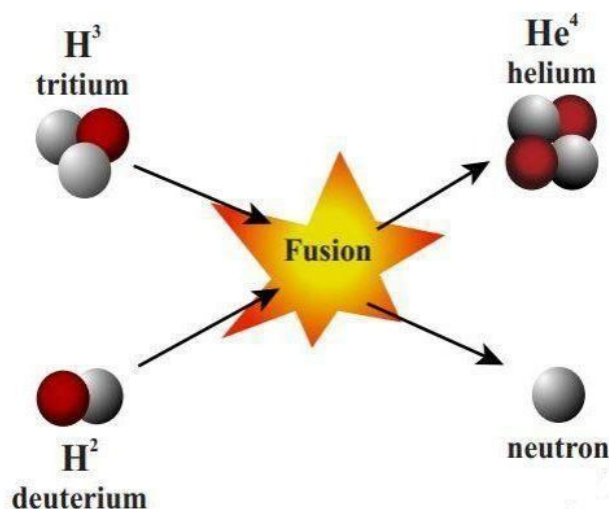


Fig: Process of nuclear fusion

The net result of these two nuclear reactions is that three deuterium (2_1H) nuclei fuse together to form a helium nucleus (4_2He) and a neutron with the release of 21.6 MeV ($4 \times 0 + 17.6 = 21.6$ MeV). This energy of 21.6 MeV is obtained in the form of kinetic energy of proton (1_1H) and a neutron 1_0n .

Note that energy released in the fusion reaction is 21.6 MeV which is very much less than the energy of about 200 MeV released in the fission of ${}^{235}_{92}U$ nucleus. But this does not mean that fusion is a weaker energy source than fission. The sun and other stars are very hot so nuclei are moving fast enough for fusion to take place and the energy released keeps the temperature high so that further fusion reactions can occur. But on earth, such high temperatures are not attained in a controlled manner. However, the temperature produced by a fission bomb (atom bomb) is close to 108 K. Therefore, fission bomb can be used to cause the fusion process.

6. How are activity and decay constant related in radioactive materials?

Ans) **Activity (A):** Sometimes we are more interested in the decay rate A ($A = -\frac{\Delta N}{\Delta t}$) than in N itself. The activity A of a radioactive sample is the number of disintegrations (decays) occurring per unit of time.

If a radioactive sample contains N atoms at any time t , then its activity at time t is given as

$$A = -\frac{\Delta N}{\Delta t}$$

where negative sign shows that activity decreases with time. According to law of radioactive decay

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

Therefore,

$$A = \lambda N$$

Since $N = N_0 e^{-\lambda t}$, so

$$A = \lambda N_0 e^{-\lambda t}$$

Putting $A_0 = \lambda N_0$, in above equation, we get

$$A = A_0 e^{-\lambda t}$$

The equation $A = \lambda N$ and $A = A_0 e^{-\lambda t}$ are alternative forms of the law of radioactive decay.

7. What is meant by binding energy and binding energy per nucleon?

Ans) **Binding Energy:** Binding Energy is the energy required to break apart the nucleus of an atom into its constituent protons and neutrons as shown in figure.

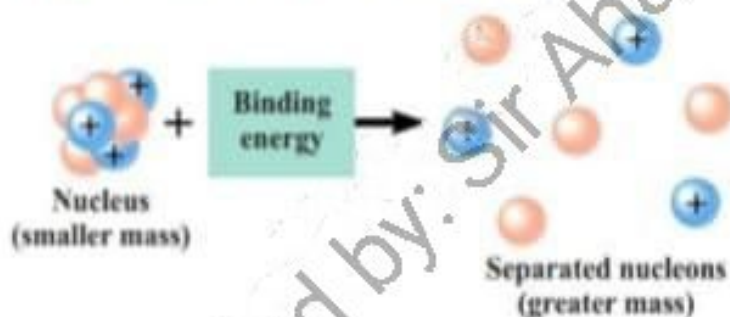


Fig: Mass defect and binding energy of an atom

Binding Energy Per Nucleon: Binding energy per nucleon is a measure of the stability of a nucleus and is defined as the amount of energy required to disassemble a nucleus into its individual nucleons (protons and neutrons) divided by the total number of nucleons. It reflects how tightly bound the nucleons are within the nucleus.

Mathematically, it is expressed as:

$$\text{Binding Energy per Nucleon} = \frac{\text{Total Binding Energy}}{\text{Number of Nucleons}}$$

A higher binding energy per nucleon indicates a more stable nucleus. Typically, this value increases with the size of the nucleus up to iron and nickel, after which it decreases for heavier elements.

8. Why is the concept of half-life important in studying radioactive substances?

Ans) Half-life helps in

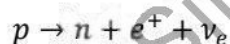
- i. **Predicting Decay Rates:** Measuring the rate at which a radioactive material decays.
- ii. **Dating:** Estimating the age of archaeological and geological samples based on the remaining quantity of radioactive isotopes.
- iii. **Safety and Handling:** Managing and planning for the safe storage, disposal, and use of radioactive materials.
- iv. **Nuclear Medicine:** Determining the appropriate timing for medical treatments and diagnostic procedures that use radioactive isotopes.

9. Explain the process of positron emission.

Ans) Positron emission, also known as beta plus (β^+) decay, is a type of radioactive decay where a proton in the nucleus of an atom is converted into a neutron. The process involves the following steps:

- i. **Proton Conversion:** A proton in the nucleus transforms into a neutron. This transformation is accompanied by the emission of a positron and a neutrino.

The reaction can be represented as:



Where;

- p represents the proton
 - n represents the neutron
 - β^+ is the positron (the antimatter counterpart of the electron),
 - ν_e is the neutrino (a nearly massless, chargeless particle).
- ii. **Emission of Positron:** The positron is emitted from the nucleus. Being the antimatter counterpart of the electron, it has the same mass as an electron but a positive charge.
 - iii. **Neutrino Emission:** A neutrino is also emitted to conserve lepton number and energy in the decay process.

The result of positron emission is that the atomic number of the element decreases by one, converting the original element into a new element with one fewer proton but with the same mass number (since a proton is replaced by a neutron). This process often occurs in proton-rich nuclei, leading to a change in the element's identity while emitting energy.

Section (C): DRQs (Long Answered Questions):

1. What is radioactivity? State the law of radioactive disintegration. Show that radioactive decay is exponential in nature.

Ans) **Radioactivity:** Radioactivity is the process by which an unstable atomic nucleus loses energy by radiation.

Law of radioactive disintegration: The law of radioactive disintegration states that the rate of decay of a radioactive sample at any instant is directly proportional to the number of atoms present at that instant.

Radioactive Decay is Exponential in Nature: Radioactive decay is described as exponential in nature because the rate of decay of a radioactive substance is proportional to the number of undecayed atoms present at any given time. The mathematical expression for this can be derived as follows:

According to law of radioactive disintegration, the rate of decay is proportional to the number of undecayed atoms $N(t)$ at time t :

$$\frac{dN(t)}{dt} \propto \lambda N(t)$$
$$\frac{dN(t)}{dt} = -\lambda N(t)$$

Where

- λ is the decay constant (a positive value),
- The negative sign indicates that the number of atoms decreases over time.

Integrating both sides

$$\int \frac{dN(t)}{dt} = -\lambda \int N(t)$$

The result of the integration is:

$$\ln(N(t)) = -\lambda t + C$$

- Where C is the integration constant.

At $t = 0$, let the initial number of undecayed atoms be N_0 , so

$$\ln N_0 = C$$

Thus, the equation becomes:

$$\ln(N(t)) = -\lambda t + \ln N_0$$

Rearranging this:

$$\ln\left(\frac{N(t)}{N_0}\right) = -\lambda t$$

Exponentiating both sides:

$$\frac{N(t)}{N_0} = e^{-\lambda t}$$

Finally, multiply by N_0 to get:

$$N(t) = N_0 e^{-\lambda t}$$

This equation shows that radioactive decay is exponential in nature.

2. Explain the main differences between alpha, beta, and gamma emissions.

Ans) **Main Differences Between Alpha, Beta, and Gamma Emissions:**

- i. **Nature of the Particle/Emission:**
 - **Alpha (α) Emission:** Involves the emission of an alpha particle, which consists of 2 protons and 2 neutrons (essentially a helium nucleus).
 - **Beta (β) Emission:** Involves the emission of a beta particle, which can be either an electron (β^-) or a positron (β^+) depending on the type of beta decay.
 - **Gamma (γ) Emission:** Involves the emission of high-energy photons (electromagnetic radiation) and does not consist of particles, but pure energy.
- ii. **Mass and Charge:**
 - **Alpha Emission:** Alpha particles are relatively heavy (mass = 4 atomic mass units) and positively charged (+2 charge).
 - **Beta Emission:** Beta particles (electrons or positrons) have a very small mass (about 1/1836 of a proton) and carry a single negative (β^-) or positive (β^+) charge.
 - **Gamma Emission:** Gamma rays are massless and electrically neutral (no charge).
- iii. **Penetrating Power:**
 - **Alpha Emission:** Low penetrating power; alpha particles can be stopped by a sheet of paper or the outer layer of skin.
 - **Beta Emission:** Moderate penetrating power; beta particles can penetrate paper but are stopped by materials like plastic or a few millimeters of aluminum.
 - **Gamma Emission:** High penetrating power; gamma rays can penetrate several centimeters of lead or meters of concrete before being absorbed.

iv. **Ionizing Power:**

- **Alpha Emission:** High ionizing power; alpha particles can ionize atoms in a short distance but lose energy quickly due to their large mass and charge.
- **Beta Emission:** Moderate ionizing power; beta particles ionize atoms, but less effectively than alpha particles due to their lower mass and charge.
- **Gamma Emission:** Low ionizing power; gamma rays ionize atoms indirectly and over long distances.

v. **Effect on the Nucleus:**

- **Alpha Emission:** Reduces the mass number by 4 and the atomic number by 2, transforming the original nucleus into a new element.
- **Beta Emission:** Either increases (for β^-) or decreases (for β^+) the atomic number by 1, without changing the mass number. This transforms the nucleus into a different element.
- **Gamma Emission:** Does not change the atomic number or mass number. It simply results from the nucleus releasing excess energy after undergoing alpha or beta decay.

vi. **Speed:**

- **Alpha Emission:** Alpha particles travel at relatively slow speeds, typically 5-10% of the speed of light.
- **Beta Emission:** Beta particles travel at higher speeds, close to the speed of light (up to 99%).
- **Gamma Emission:** Gamma rays travel at the speed of light.

3. A fission reactor produces energy to drive a generator. Describe briefly how this energy is produced.

Ans) **Nuclear Reactor (Fission Reaction):** A nuclear reactor is a device in which controlled fission chain reaction takes place. A nuclear reactor is also known as nuclear pile or atomic pile.

Such a system was first achieved with uranium as the fuel in 1942 by Enrico Fermi. He used uranium-235 isotope that releases energy through nuclear fission. The schematic diagram of nuclear reactor is shown in figure below.

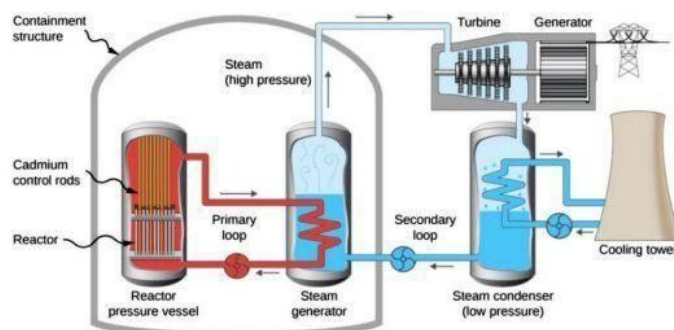


Fig: Schematic Diagram of Nuclear reactor

The following are the main components of the nuclear reactor:

Fissionable Substance: Nuclear reactors use fuel, typically enriched uranium or plutonium, to sustain the fission chain reaction. The U-235 is fissionable, but uranium from ore typically contains only about 0.7 percent of U-235, with the remaining 99.3 percent being the U-238 isotope. Because uranium-238 tends to absorb neutrons, reactor fuels must be processed to increase the proportion of U-235 so that the reaction can sustain itself. This process is called enrichment.

Moderator: The function of the moderator is to slow down the highly energetic neutrons produced in the process of fission of U-235 to thermal energies. Heavy water (D_2O), graphite, beryllium, etc., are used as moderators. Ideally, moderators have low atomic weight and low absorption cross-section for neutrons.

Control Rods: Control rods are made of materials like boron or cadmium that absorb neutrons, regulating the rate of the fission chain reaction. By adjusting the position of control rods within the reactor core, operators can control the power output and maintain stability.

Coolant: Coolant circulates through the reactor core to transfer heat away from the fuel and other reactor components. Common coolants include water, heavy water, or gases like helium or carbon dioxide. The heated coolant then transfers its thermal energy to a secondary loop containing water, which turns into steam. This steam drives turbines connected to generators, producing electricity.

Protective Shield: In a nuclear reactor, there are many types of harmful radiations emitted which are dangerous for all living things. In order to protect from these radiations, the reactor is surrounded by a massive biological shield.

4. Define Q-value of a nuclear reaction and its significance.

Ans) **Q-value of a nuclear reaction:** The Q-value of a nuclear reaction is a measure of the energy released or absorbed during the reaction. It is defined as the difference between the total mass of the reactants and the total mass of the products, converted into energy using Einstein's mass-energy equivalence principle ($E = mc^2$).

Mathematically, the Q-value can be expressed as:

$$Q = (\text{total mass of reactants} - \text{total mass of products}) \times c^2$$

Where:

- c is the speed of light in a vacuum.

Significance of Q-value:

- Energy Release or Absorption:** A positive Q-value indicates that the reaction releases energy, making it exothermic. Conversely, a negative Q-value indicates that the reaction requires energy input, making it endothermic.

- ii. **Reaction Feasibility:** The Q-value helps determine whether a nuclear reaction will occur spontaneously. Reactions with a positive Q-value are more likely to occur spontaneously under normal conditions.
- iii. **Energy of Particles:** In particle physics and nuclear engineering, the Q-value is crucial for calculating the kinetic energy of the products. This information is essential for understanding the dynamics of the reaction and the behavior of the resulting particles.
- iv. **Design of Nuclear Reactors:** For practical applications such as in nuclear reactors, knowing the Q-value helps in the design and optimization of reactor performance, including energy output and fuel consumption.

5. Describe the construction and working of GM counter.

Construction:

- The GM (Geiger Muller) counter consists of a hollow metallic chamber as shown in the figure that acts as a cathode.
- A thin wire anode is also placed along its axis.
- The chamber has a sealed window, through which the radiation enters the chamber.
- The chamber is filled with an inert gas at low pressure.
- There is a counter connected to this system to measure the radiation.

Working: The chamber is filled with an inert gas (helium, neon, or argon) at low pressure. A high voltage is applied to this chamber. The metallic chamber will conduct electricity. When radiation enters the chamber through the window, the photons in the radiation will ionize the inert gas inside the chamber. This will make the gas conductive. The electrons produced due to ionization are accelerated due to the potential that we applied and these electrons cause even more ionization. The ionized electrons travel towards the anode. The anode is connected to a counter. The counter counts the electrons reaching the anode. This is how we measure radiation.

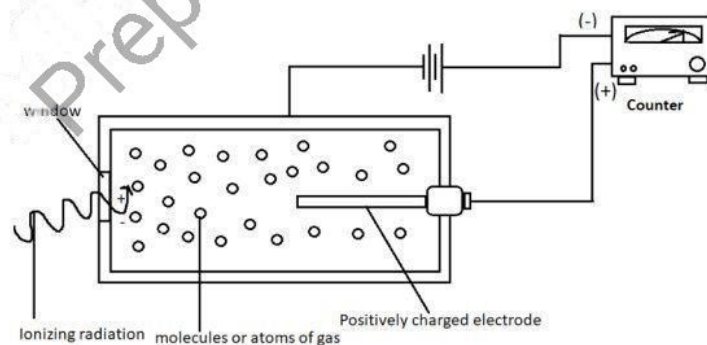


Fig: GM counter

6. Discuss nuclear reactions induced by neutrons. Why are neutrons preferred to other particles?

Ans) **Nuclear reactions induced by neutrons:** Nuclear reactions induced by neutrons are

Nuclear Fission: The splitting of a heavy nucleus ($A > 230$) into two medium-mass nuclei in a nuclear reaction with the release a huge amount of energy due to mass defect is called nuclear fission. For example, when a uranium nucleus (U-235) is bombarded by a slow moving neutron (called thermal neutron), the U-235 nucleus splits into two medium-mass nuclei with the release of huge amount of energy as shown in figure below.

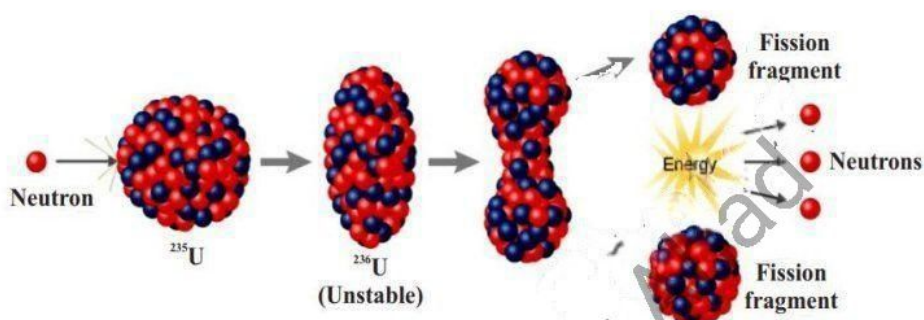


Fig: Heavy nuclei splits into two light nuclides (nuclear fission process)

This fission reaction is give given below:



Where on ${}_{92}^{236}\text{U}$ represents that U-236 is in excited state. Two things are worth noting in this fission reaction. First, a huge amount of energy (about 200 MeV per U-235 nucleus) is released in the process.

Secondly, on the average 2 to 3 neutrons are released in the process. The released neutrons can further cause splitting of ${}_{92}^{235}\text{U}$ nuclei and lead to self-sustaining nuclear fission.

When nuclear fission takes place, it is found that the sum of the masses of fission products is very slightly less than the sum of the masses of reactant products. As a result, there occurs a mass defect (m) in nuclear fission. This mass defect is converted into energy according to the relation $E = mc^2$. The energy released in the above fission can be determined from the mass defect that occurs in the process.

Total mass before fission:

mass of U-235 = 235.043933 a.m.u

mass of neutron = 1.008665 a.m.u

sum of masses before fission = 236.052598 a.m.u

Total mass after fission:

mass of two fragments = 232.812000 a.m.u

mass of 3 neutrons = 3.025995 a.m.u

sum of masses after fission = 235.837995 a.m.u

Mass defect $\Delta m = 236.052598 - 235.837995 = 0.214603$ a.m.u

Therefore, energy released per fission of U-235 = $0.214603 \times 931.5 = 200$ MeV

Nuclear Fusion Process: The process of combining two light nuclei to form a heavy nucleus with the release of huge amount of energy due to mass defect is known as nuclear fusion.

This mass defect results in the release of a huge amount of energy according to the relation $E = mc^2$. When two nuclei of heavy hydrogen or deuterium (${}^2_1\text{H}$) are combined, the following reaction is possible:

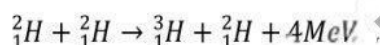


figure below shows the nucleus of tritium (${}^3_1\text{H}$) so formed can again fuse with a deuterium nucleus (${}^2_1\text{H}$) to give the following reaction

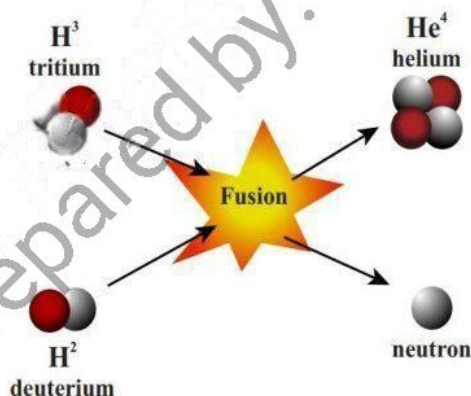
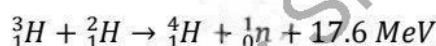


Fig: Process of nuclear fusion

The net result of these two nuclear reactions is that three deuterium (${}^2_1\text{H}$) nuclei fuse together to form a helium nucleus (${}^4_2\text{He}$) and a neutron with the release of 21.6 MeV ($4.0 + 17.6 = 21.6$ MeV). This energy of 21.6 MeV is obtained in the form of kinetic energy of proton (${}^1_1\text{H}$) and a neutron ${}^1_0\text{n}$.

Note that energy released in the fusion reaction is 21.6 MeV which is very much less than the energy of about 200 MeV released in the fission of ${}^{235}_{92}\text{U}$ nucleus. But this does not mean that fusion is a weaker energy source than fission. The sun and other stars are very hot so nuclei are moving fast enough for fusion to take place and the energy released keeps the temperature high so

that further fusion reactions can occur. But on earth, such high temperatures are not attained in a controlled manner. However, the temperature produced by a fission bomb (atom bomb) is close to 10⁸ K. Therefore, fission bomb can be used to cause the fusion process.

Preference of Neutrons Over Other Particles:

i. Neutral Charge:

- Neutrons have no electric charge, which allows them to penetrate the positively charged nucleus without experiencing Coulomb repulsion. Other particles, like protons or alpha particles, face strong electrostatic repulsion when approaching the nucleus, making it harder to induce nuclear reactions.
- Neutrons can penetrate deeper into the nucleus and have a higher probability of interacting with nuclear matter.

ii. High Penetration Power:

- Due to their lack of charge, neutrons can penetrate through thick layers of material, which makes them ideal for inducing reactions in materials that are otherwise difficult to bombard with charged particles.

iii. Fission Chain Reactions:

- In neutron-induced fission, the reaction releases additional neutrons, which can further induce fission in neighboring nuclei. This leads to a self-sustaining chain reaction, essential for both nuclear reactors and nuclear bombs.
- This is not possible with charged particles, which do not produce multiple particles capable of continuing the reaction.

iv. Lower Energy Requirement:

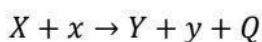
- Neutron-induced reactions typically require lower kinetic energy to initiate than reactions with charged particles, which need to overcome electrostatic barriers. This makes neutron sources more efficient for certain types of nuclear reactions.

v. Versatility in Nuclear Reactions:

- Neutrons can induce a wide variety of nuclear reactions (e.g., capture, fission, scattering) in a broad range of isotopes, making them highly versatile for applications in reactor design, isotope production, and materials testing.

7. What is a nuclear reaction? Explain nuclear fission and fusion.

Ans) **Nuclear Reactions:** Any process that involves a change in the nucleus of an atom is called a nuclear reaction. Mathematically,



where X is the target, x are the projectiles, y are the ejectiles and Y is called the residual(product) nucleus.

Nuclear Fission: The splitting of a heavy nucleus ($A > 230$) into two medium-mass nuclei in a nuclear reaction with the release a huge amount of energy due to mass defect is called nuclear fission. For example, when a uranium nucleus (U-235) is bombarded by a slow moving neutron (called thermal neutron), the U-235 nucleus splits into two medium-mass nuclei with the release of huge amount of energy as shown in figure below.

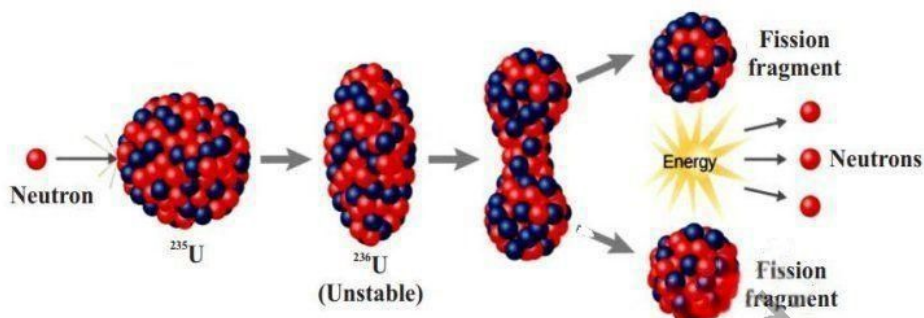
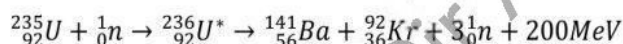


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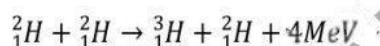


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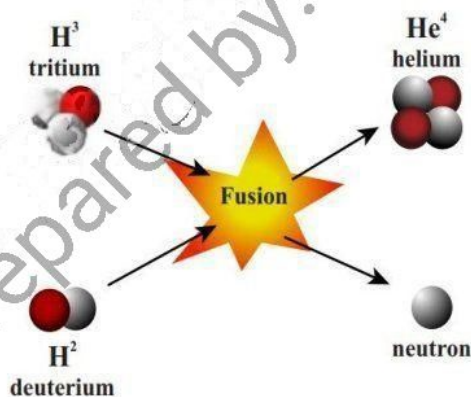
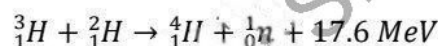


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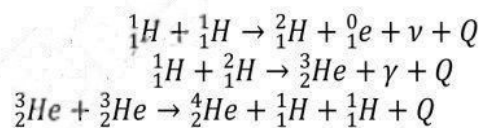
8. How do the Sun and stars produce energy? What is the proton-proton cycle? Explain with details.

Ans) **Nuclear Fusion in Sun and Stars:** Every second, the sun fuses around 500 million metric tons of hydrogen in its core. The core of the sun is incredibly hot, with temperatures reaching about 20 million degrees Celsius, while its surface temperature is around 5 million degrees Celsius.

The sun is a star which is primarily made up of hydrogen (about 75%), helium (about 25%), and trace amounts of other elements. It produces energy through a process called nuclear fusion, where hydrogen atoms combine to form helium atoms, releasing light and heat in the process.

The fusion in the sun can take place in two different reaction sequences, the most common of which, the Proton-Proton (PP) Cycle and the other one is Carbon-Nitrogen-Oxygen (CNO) Cycle.

The Proton-Proton Cycle involves the fusion of protons (hydrogen nuclei) to form helium as shown in figure below. Hans Bethe was the first to work out the detailed steps of the PP cycle in 1938. The PP cycle is a very efficient way to generate energy in the Sun. In the p-p chain, two protons first fuse to produce a deuterium nucleus which combines with another proton to yield ${}^3_2\text{He}$. Two ${}^3_2\text{He}$ nuclei fuse and form ${}^4_2\text{He}$ and two protons. These reactions can be represented by the equations.



The net Q value of the chain reactions is about 26 MeV.

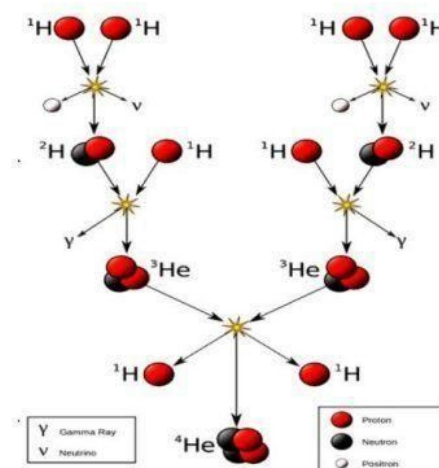
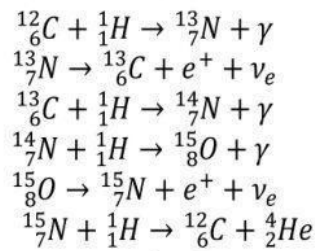
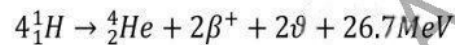


Fig: Proton-proton cycle to form helium

The Carbon-Nitrogen-Oxygen (CNO) cycle was independently suggested by Carl von Weizsacker and Hans Bethe in the late 1930s. The CNO cycle is a series of nuclear fusion reactions that convert hydrogen into helium as predicted in figure below, and it is the primary source of energy in stars that are more than 1.3 times as massive as the Sun. This cycle uses carbon, nitrogen, and oxygen as catalysts to convert hydrogen to helium.



In the CNO cycle, four protons fuse, using carbon, nitrogen and oxygen isotopes as a catalyst, to produce one alpha particle, two positrons and two electron neutrinos. Combining all the above reactions, the net reaction for the CNO cycle comes out to be



We see that the net energy release is nearly same for both cycles.

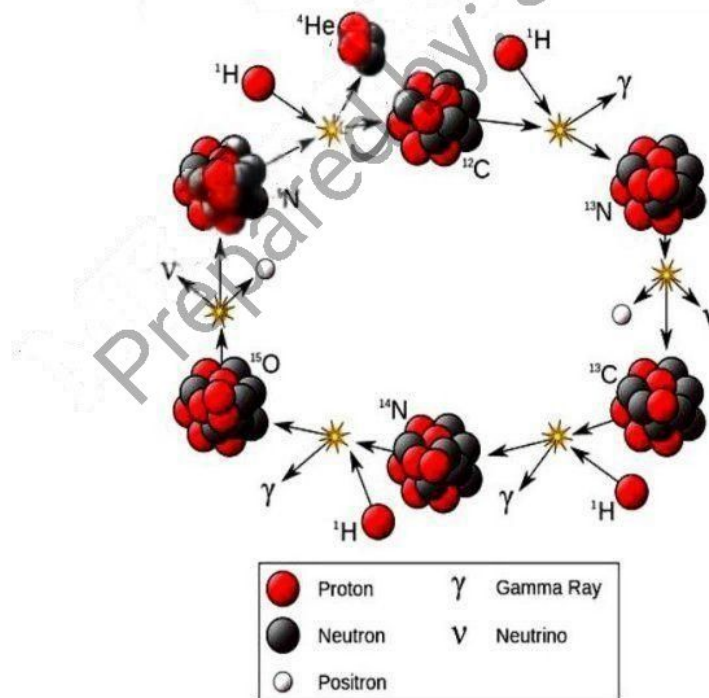


Fig: The Carbon-Nitrogen-Oxygen (CNO) cycle

Section (D): Numerical:

1. In 9.0 days the number of radioactive nuclei decreases to one-eighth the number present initially. What is the half-life (in days) of the material?

Data:

$$N(9) = \frac{1}{8} N_0, T_{\frac{1}{2}} = ?$$

Solution:

$$N(t) = N_o \times \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

$$N(9) = N_o \times \left(\frac{1}{2}\right)^{\frac{9}{T_{1/2}}}$$

$$\frac{1}{8} N_0 = N_o \times \left(\frac{1}{2}\right)^{\frac{9}{T_{1/2}}}$$

$$\frac{1}{8} = \left(\frac{1}{2}\right)^{\frac{9}{T_{1/2}}}$$

$$\log \frac{1}{8} = \log \left(\frac{1}{2}\right)^{\frac{9}{T_{1/2}}}$$

$$\log \frac{1}{8} = \frac{9}{T_{1/2}} \log \frac{1}{2}$$

$$T_{1/2} = \frac{9}{\log \frac{1}{8}} \times \log \frac{1}{2}$$

$$T_{1/2} = 3 \text{ days}$$

2. The $^{32}_{15}\text{P}$ isotope of phosphorus has a half-life of 14.28 days. What is its decay constant in units of s^{-1} ?

Data:

$$T_{1/2} = 14.28 \text{ days} = 1233792 \text{ s}, \lambda = ?$$

Solution:

$$T_{1/2} = \frac{\ln 2}{\lambda}$$

$$1233792 = \frac{\ln 2}{\lambda}$$

$$\lambda = 5.62 \times 10^{-7} \text{ s}^{-1}$$

3. Find the binding energy (in MeV) for lithium ${}^7_3\text{Li}$ (atomic mass = 7.016 003 u).

Data:

mass of lithium = 7.016 003 u, $E_B = ?$, No. of protons = 3,

No. of neutrons = 7 – 3 = 4

Solution:

Mass of protons = $3 \times 1.0073 = 3.0219 \text{ u}$,

Mass of neutrons = $4 \times 1.0087 = 4.0348 \text{ u}$

The combined mass of protons and neutrons = $3.0219 + 4.0348 = 7.0567 \text{ u}$

$\Delta m = 7.0567 - 7.016\ 003 = 0.040697 \text{ u}$

$E_B = \Delta m \times 931.5$

$E_B = 0.040697 \times 931.5$

$E_B = 37.9 \text{ MeV}$

4. The binding energy of a nucleus is 225.0 MeV. What is the mass defect of the nucleus in atomic mass units?

Data:

$E_B = 225.0 \text{ MeV}$, $\Delta m = ?$

Solution:

$E_B = \Delta m \times 931.5$

$225.0 = \Delta m \times 931.5$

$\Delta m = 0.2415 \text{ u}$

5. A copper penny has a mass of 3.0 g. Determine the energy (in MeV) that would be required to break all the copper nuclei into their constituent protons and neutrons. Ignore the energy that binds the electrons to the nucleus and the energy that binds one atom to another in the structure of the metal. For simplicity, assume that all the copper nuclei are ${}^{63}_{29}\text{Cu}$ (atomic mass = 62.939 598 u).

Data:

Mass of penny = 3.0 g, $E_B = ?$, No. of protons = 29,

No. of neutrons = 63 – 29 = 34, mass of copper = 62.939 598 u.

Solution:

Calculate the Binding Energy per Nucleus:

Mass of protons = $29 \times 1.0073 = 29.2117 \text{ u}$,

Mass of neutrons = $34 \times 1.0087 = 34.2958 \text{ u}$

The combined mass of protons and neutrons = $29.2117 + 34.2958 = 63.5075 \text{ u}$

$\Delta m = 63.5075 - 62.939 598 = 0.567902 \text{ u}$

$E_B = \Delta m \times 931.5$

$E_B = 0.567902 \times 931.5$

$E_B = 529.000713 \text{ MeV}$

Calculate the Total Energy Required

Mass of the penny = $3.0 \times 6.022 \times 10^{23} = 1.8066 \times 10^{24} \text{ u}$

No. of atoms = $\frac{\text{Mass of the penny}}{\text{Atomic mass of copper}}$

No. of atoms = $\frac{1.8066 \times 10^{24}}{62.939 598}$

No. of atoms = 2.8704×10^{22}

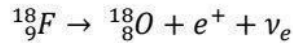
Total Binding Energy = $2.8704 \times 10^{22} \times 529.000713$

Total Binding Energy = $1.518 \times 10^{25} \text{ MeV}$

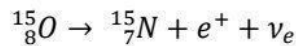
6. Write the β^+ decay process for each of the following nuclei with their proper chemical symbols including Z and A for each daughter nucleus: (a) ${}^{18}_9F$ (b) ${}^{15}_8O$.

Solution:

(a)



(b)



7. A device used in radiation therapy for cancer contains 0.50 g of cobalt ${}^{60}_{27}Co$ (59.933 819 u). The half-life of ${}^{60}_{27}Co$ is 5.27 years. Determine the activity of the radioactive material.

Data:

$$m = 0.50 \text{ g} = 0.5 \times 6.022 \times 10^{23} = 3.011 \times 10^{23} \text{ u},$$

$$\text{mass of cobalt} = 59.933 \text{ 819 u},$$

$$T_{1/2} = 5.27 \text{ years} = 5.27 \times 365.25 \times 24 \times 60 \times 60 = 166308552 \text{ s}$$

Solution:

$$A = \lambda N$$

$$\therefore \lambda = \frac{\ln(2)}{T_{1/2}} = \frac{\ln(2)}{166308552} = 4.168 \times 10^{-9} \text{ s}^{-1}$$

$$\therefore N = \frac{3.011 \times 10^{23}}{59.933 \text{ 819}} = 5.024 \times 10^{21} \text{ atoms}$$

$$A = (4.168 \times 10^{-9})(5.024 \times 10^{21})$$

$$A = 2.094 \times 10^{13} \text{ Bq}$$

UNIT 28: PARTICLE PHYSICS

MCQ'S

KEY

1. b	2. c	3. c	4. a	5. b
6. b	7. c	8. b	9. b	10. c

Section (B): CRQs (Short Answered Questions):

1. Explain the difference between bosons and fermions and their roles in mediating fundamental forces.

Ans) **Difference between bosons and fermions:**

Property	Bosons	Fermions
Spin	Integer spin (0, 1, 2, ...)	Half-integer spin (1/2, 3/2, ...)
Role	Mediate fundamental forces	Constitute matter
Statistical Behavior	Follow Bose-Einstein statistics	Follow Fermi-Dirac statistics
Pauli Exclusion Principle	Do not obey (multiple bosons can occupy the same quantum state)	Obey (no two fermions can occupy the same quantum state)

Roles in Mediating Fundamental Forces: Bosons are particles that mediate the fundamental forces of nature. The primary bosons associated with the fundamental forces are:

- i. **Photon (γ):**
 - **Force Mediated:** Electromagnetic Force
 - **Role:** The photon is the gauge boson responsible for electromagnetic interactions, including those between charged particles like electrons and protons.
- ii. **W and Z Bosons (W^+ , W^- , Z^0):**
 - **Force Mediated:** Weak Nuclear Force

- **Role:** W and Z bosons mediate the weak nuclear force, which is responsible for processes such as beta decay in radioactive materials.
- iii. **Gluons (g):**
- **Force Mediated:** Strong Nuclear Force
 - **Role:** Gluons are the gauge bosons of the strong nuclear force, which holds quarks together within protons, neutrons, and other hadrons.
- iv. **Graviton:**
- **Force Mediated:** Gravitational Force
 - **Role:** The hypothetical particle associated with gravity is the graviton, although it has not been observed yet. The Standard Model doesn't explain gravity.

2. Compare and contrast the properties of quarks and leptons, the two main categories of fermions in the Standard Model.

Ans) **Compare and Contrast Table:**

Property	Quarks	Leptons
Fundamental Particles	Yes	Yes
Subtypes	6 flavors: Up, Down, Charm, Strange, Top, Bottom	6 flavors: Electron, Muon, Tau, and their respective neutrinos (electron neutrino, muon neutrino, tau neutrino)
Electric Charge	Fractional charges: $+\frac{2}{3}$ (up-type), $-\frac{1}{3}$ (down-type)	Integer charges: -1 (electron, muon, tau), 0 (neutrinos)
Interaction via Strong Force	Yes (Quarks experience the strong nuclear force)	No (Leptons do not experience the strong force)
Interaction via Weak Force	Yes	Yes

3. Define the term "lepton" and provide examples of leptons. Explain their fundamental properties and role in the Standard Model of particle physics.

Ans) **Lepton:**

Definition: Leptons are a group of elementary particles that do not experience the strong nuclear force. There are six types or flavors of leptons, which come in three pairs.

Examples: The pairs are made up of three charged particles named electron, muon, and tau, along with their Partners called neutrinos (charge less).

Properties & Roles:

- **Electron:** Negatively charged; commonly found in atoms.
- **Muon and Tau:** Heavier counter parts of the electron.
- **Neutrinos:** Electrically neutral; they interact very weakly with matter.

- Leptons interact via weak and electromagnetic forces but not through the strong force.
- Leptons are stable particles and do not undergo decay under normal circumstances.
- Leptons exist alone and do not form groups.

4. Explain the concept of color charge in quarks and its significance in the strong nuclear force. How does the combination of quarks contribute to the color-neutral nature of protons and neutrons?

Ans) **Color Charge:**

- Electric charge comes in only one type: positive (with its opposite negative). But strong charge (that deals with strong nuclear force) comes in three types, red, green, and blue. These color names are just labels and do not correspond to actual colors in the visual spectrum.
- Quarks carry either one of the three color charges, and they can change their colors during particle interactions. Quarks of different colors are attracted to one another due to the strong nuclear force, it means red attracts green, blue attracts red, and so on. On the other hand, quarks of the same color repel one another.
- Quarks always combine in ways that result in "color-neutral" or "white-color" particles. For example, a proton consists of three quarks: one red, one green, and one blue, making it color-neutral. Only white color combinations are permitted. This is why isolated quarks do not exist in nature. This is known as quark confinement. Therefore, all free particles (electrons, protons and neutrons) have a color charge of zero.

5. Describe the structure of a proton and neutron in terms of its quark composition. How do quarks combine to form a proton and a neutron, and what are the specific types of quarks involved?

Ans) **Proton:** It consists of two up and one down quarks ($\bar{u}\bar{u}\bar{d}$). Up (\bar{u}) quark has $+2/3$ charge and down quark has $-1/3$ charge. Therefore, net charge on proton has $+1$ charge:

$$Proton = \bar{u}\bar{u}\bar{d} = \frac{2}{3} + \frac{2}{3} - \frac{1}{3} = +1$$

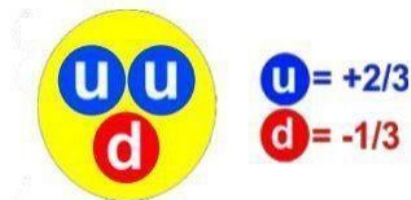


Fig: Proton Quarks

Neutron: It consists of three quarks: one up and two down quarks ($\bar{u}dd$). The net charge on neutron is zero:

$$\text{Neutron} = \bar{u}dd = \frac{2}{3} - \frac{1}{3} - \frac{1}{3} = 0$$

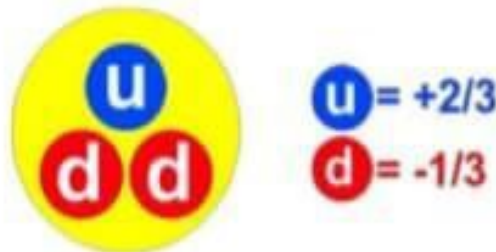


Fig: Neutron Quarks

Section (C): ERQs (Long Answered Questions):

1. Explain the structure of the Standard Model, including the different types of particles and their relationships. How does the model classify fundamental forces?

Ans) **The Standard Model:** Our universe is made up of two things: matter and energy (radiation). To understand them better, scientists have divided particles into two main groups: matter particles and force particles. Scientists have identified many elementary particles belonging to these categories. These particles are categorized and explained in detail in the Standard Model of Particle Physics, which is the best-known theory to date. It is a framework that explains three of the four fundamental forces (electromagnetism, the weak nuclear force, and the strong nuclear force) and all known elementary particles. The Standard Model classifies all known elementary particles into two main classes:

Fermions: These are the matter particles, which make up everything in the universe.

Bosons: These are the force-carrying particles, which mediate the interactions between fermions.

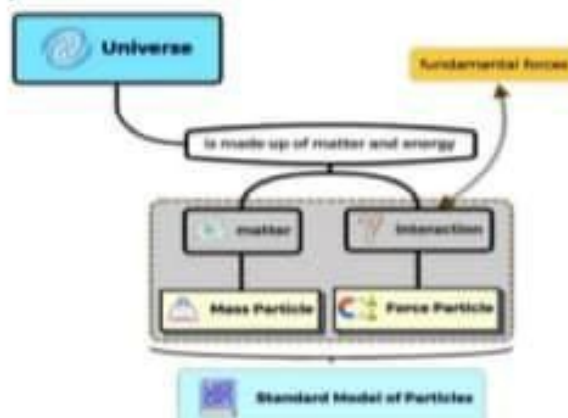


Fig: The universe is made up of matter and energy

2. Describe in detail all fundamental forces and their associated field particles. What is a boson, and why are bosons referred to as field particles?

Ans) **Boson:** The Standard Model has kept all field particles in a class of 'Boson'. The term "field particles" refers to particles associated with force fields.

Reason to refer bosons as field particles: According to the Standard Model, bosons are often considered field particles because they are linked to force fields. For example, the photon is a field particle associated with the electromagnetic field.

Fundamental forces and their field particles: Scientists have grouped all fundamental forces into four basic types. In order of increasing strength, these forces and their associated field particles are described as:

- i. **Gravitational Force:** It is the force that attracts two masses towards each other. It is the weakest of the four fundamental forces but acts over long distances.
Field Particle: The hypothetical particle associated with gravity is the graviton, although it has not been observed yet. The Standard Model doesn't explain gravity.
- ii. **Weak Nuclear Force:** The weak force is responsible for radioactive decay, where unstable atomic nuclei break down into smaller, more stable nuclei. It's responsible for processes like beta decay and neutrino emission. This force is weaker than electromagnetic and strong nuclear forces but stronger than gravitational force.
Field Particle: There are three field particles associated with weak nuclear force. These are W^+ , W^- and Z bosons. These short-lived bosons carry the force over very small distances, explaining the limited range of the weak force.
- iii. **Electromagnetic Force:** This force is responsible for the interactions between charged particles, such as electrons and protons. It includes both electric and magnetic forces. The electromagnetic force is stronger than both gravitational and weak nuclear forces. It is also a long-range force, similar to the gravitational force.
Field Particle: The field particle of electromagnetic force is photon. It is mass less and chargeless particle. When ever charged particles interact, they exchange photons, causing the attractive or repulsive forces we observe.
- iv. **Strong Nuclear Force:** The strong force binds quarks together to form protons and neutrons, and it holds atomic nuclei together. It is the strongest force among all forces and acts at subatomic levels.
Field Particle: Gluons are the field particles that mediate the strong force between quarks.

3. Describe the operating principle of a Geiger-Muller counter. How does it detect and quantify ionizing radiation, and what are its limitations in terms of measurement range and types of radiation detected?

Ans) Geiger-Muller Counter:

Operating Principle: A Geiger-Muller (GM) counter operates by detecting ionizing radiation through the ionization of gas within a sealed tube.

Construction:

- The GM (Geiger Muller) counter consists of a hollow metallic chamber as shown in the figure that acts as a cathode.
- A thin wire anode is also placed along its axis.
- The chamber has a sealed window, through which the radiation enters the chamber.
- The chamber is filled with an inert gas at low pressure.
- There is a counter connected to this system to measure the radiation.

Working: The chamber is filled with an inert gas (helium, neon, or argon) at low pressure. A high voltage is applied to this chamber. The metallic chamber will conduct electricity. When radiation enters the chamber through the window, the photons in the radiation will ionize the inert gas inside the chamber. This will make the gas conductive. The electrons produced due to ionization are accelerated due to the potential that we applied and these electrons cause even more ionization. The ionized electrons travel towards the anode. The anode is connected to a counter. The counter counts the electrons reaching the anode. This is how we measure radiation.

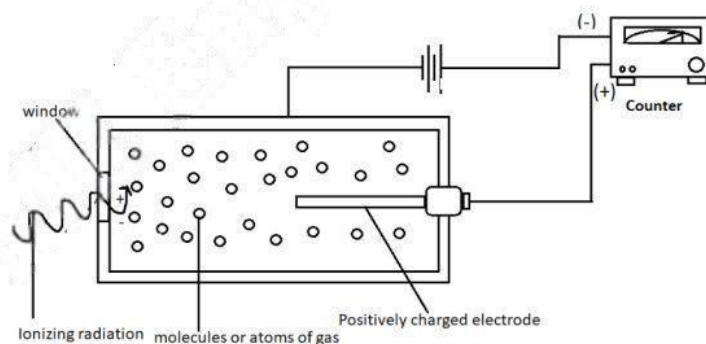


Fig: GM counter

Limitations:

- **Measurement Range:** GM counters can become saturated at very high radiation levels, making them unable to provide accurate readings in intense radiation fields. They also lack the ability to measure radiation intensity linearly, as the number of counts may not accurately reflect dose rate beyond certain limits.
- **Radiation Types Detected:** GM counters are primarily sensitive to beta and gamma radiation but have limited efficiency for detecting alpha particles due to the low penetration of alpha particles. Additionally, they cannot distinguish between different types of radiation (alpha, beta, gamma), as all ionization events are recorded in the same way.

4. Discuss the uses of a Wilson cloud chamber in particle physics experiments. How does it visualize charged particle tracks, and what factors can affect its performance?

Wilson cloud chambers:

Working Principle: The Wilson Cloud Chamber consists of a sealed container filled with a supersaturated vapor, typically water or ethanol. When a particle passes through the chamber, it ionizes the vapor, creating a trail of droplets that condense around the ionized path. This creates a visible cloud-like track that can be photographed and analyzed.

Construction: The schematic diagram of Wilson cloud chamber is shown in figure below, which consists of a large cylindrical chamber A, with walls and a ceiling made of glass. It contains dust-free air saturated with water vapor. P is a piston working inside the chamber. When the piston moves down rapidly, adiabatic expansion of the air inside the chamber takes place. The piston is connected to a large evacuated vessel F through a valve V. When the valve is opened, the air under the piston rushes into the evacuated vessel F, thereby causing the piston to drop suddenly. The wooden blocks WW reduce the air space inside the piston. Water at the bottom of the apparatus ensures saturation in the chamber. The expansion ratio can be adjusted by altering the height of the piston.

As soon as the gas in the expansion chamber is subjected to sudden expansion, the ionizing particles are shot into the chamber through a side window. A large number of extremely fine droplets are formed on all the ions produced by the ionizing particles. These droplets form a track of the moving ionizing particles. At this stage, the expansion chamber is profusely illuminated by a powerful beam of light L and two cameras CC are used to photograph the tracks as shown in figure below. The process of expansion, shooting of the ionizing particles into the expansion chamber, illuminating the chamber and clicking the camera must all be carried out in rapid succession in order to get satisfactory results. The type of ionizing particle can be identified by its track in the cloud chamber. Alpha particles, being relatively massive, travel in straight, thick, and clearly defined paths. Beta particles, being lighter, are easily deflected and create thin, curved paths. The cloud chamber has been instrumental in discovering many elementary particles, such as the positron and meson.

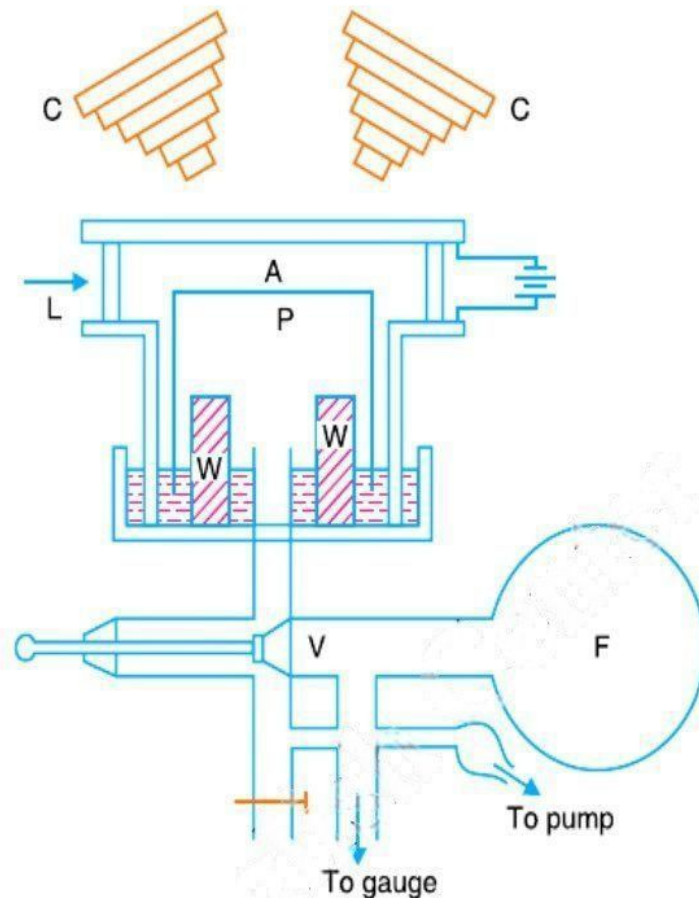


Fig: Schematic diagram of Wilson Cloud Chamber

Use:

- **Particle Identification:** Wilson cloud chambers were historically instrumental in the identification and study of subatomic particles. By observing the curvature of particle tracks in a magnetic field and the nature of the tracks themselves, scientists could identify and classify various particles.
- **Nuclear Physics Research:** Cloud chambers have been used to study the behavior of particles in nuclear reactions and to investigate the structure of atomic nuclei.
- **Cosmic Ray Studies:** Wilson cloud chambers are also used in cosmic ray research. These instruments can detect and track the passage of cosmic rays, which are high-energy particles originating from space.
- **Education and Outreach:** Cloud chambers are often used as educational tools in physics classrooms and science museums to help students and the general public visualize the behavior of subatomic particles.

Factors affecting performance:

- i. **Temperature and Pressure:** The chamber relies on the rapid expansion of saturated vapor, so maintaining the correct temperature and pressure is crucial. If the temperature is too high or the pressure is unstable, the super-saturated environment required for vapor condensation on ionizing particles may not be achieved.
- ii. **Humidity/Vapor Saturation:** A high level of humidity is essential for the operation of the chamber. Insufficient vapor saturation results in fewer or no visible condensation trails, while excessive humidity can lead to cloud formation that obscures particle paths.
- iii. **Ionization Source:** The presence of an adequate source of ionizing radiation is required for particle tracks to form. Without sufficient ionization, fewer particles will be detected, reducing the chamber's sensitivity.
- iv. **Expansion Timing:** The timing and speed of the chamber's expansion is critical. The expansion must occur quickly enough to create the super-saturated vapor condition but should not be so rapid that it disrupts the particle tracks.
- v. **Magnetic Fields:** Applying a magnetic field can help visualize particle trajectories, but if it's too weak or strong, it can distort the tracks or prevent clear detection of the particle's properties.
- vi. **External Vibrations or Disturbances:** Mechanical stability is important, as vibrations or movements can disturb the cloud, causing artifacts or misinterpretation of particle tracks.
- vii. **Contaminants:** Impurities or dust particles within the chamber can serve as unintended nucleation points for condensation, leading to unclear or false tracks.