UNIT 01

PHYSICS AND MEASUREMENTS

Physics:

Physics is the branch of science which observes the nature represents it mathematically and conclude with the experiments. Basically *PHYSICS IS THE STUDY OF NATURE.*

Physics is branch of science that seeks to understand and describe the fundamental principles and laws governing the natural world.

Physics is divided into several sub-disciplines, on different aspect of physical world.

I. **Classical mechanics:-**
This branch deals with the motion of object and forces that acts upon them.

II. **Electromagnetism:**-
In this branch of physics electromagnetic theory explain the behavior of electric and magnetic fields, as well as their interaction with charged particles and currents.

III. **Thermodynamics:**-
Thermodynamics deals with the study of heat energy and their transformations. Also explores the concept of temperatures and entropy.

IV. **Optics :**-
In this branch of physics we focuses on the properties, behavior and interaction of light with different material and optical systems.

V. **Quantum mechanics:**-
Quantum mechanics is the branch of physics that deals with the behavior of particles at the atomic and sub-atomic levels. It describe the phenomenon of wave-particle duality, quantum superposition and quantum entanglement.

VI. **Relativity:**-
Special relativity and general relativity both deals with the behavior of objects in extreme conditions, such as those involving high speed and strong gravitational fields. It also describe the concept like Time dilation, length contraction and curvature of space and time.

Scope of physics in science, Technology and society:

Physics play vital role in science, Technology and society in various domains.

- Physics provides the fundamental laws and principles that form the basis of understanding the natural world.
- Physics drives scientific progress to uncover new insights of universe from microscopic realm of particles to the cosmos.
- Interdisciplinary connection of physics with the chemistry, Biology, astronomy and geology. It provides the frame work for understanding complex systems and phenomenon.
- The principles of physics are applied in various field of engineering i-e electrical engineering rely on the magnetism, material engineering depend upon quantum mechanics and mechanical engineering uses the laws of motion for technology.

- Physics play a measure role in generation, transmission and utilization of energy.
- The principle of physics play a vital role to design the electronic devices, telecommunication and information technology
- Physics also contribute to the medical technology like X-rays, CT scan, MRI and ultra sound it also facilitates advancement of radiation therapy, medical surgery and medical diagnostics.
- Physics also plays the crucial role for the studying of climates, atmosphere and environmental monitoring.
- Physics education fosters critical thinking, problem solving skills and scientific mind set. Physics promotes the scientific literacy.

Physical Quantities, S.I Base, Derived and supplementary units:

PHYSICAL QUANTITIES:

All those quantities in terms of which laws of physics can be described and whose measurement is necessary to understand any problem is called Physical quantities **OR** Combination the magnitude usually expressed by the number and a unit is called physical quantity.

Physical quantities are classified in to two categories:

- Fundamental or Basic physical quantities
- Derived physical quantities.

Fundamental or Basic physical quantities:

Those physical quantities which cannot be defined in terms of other physical quantities are called Fundamental or Basic physical quantities.

There are seven fundamental physical quantities along with their units and symbols:

S.No.	**Fundamental or Basic physical quantities**	**Symbol of quantities**	**S.I Units**	**Symbol of units**
1	Length	L	meter	m
2	Mass	m or M	kilogram	Kg
3	Time	T or t	second	S
4	Temperature	θ	Kelvin	K
5	Electric current	I	Ampere	A
6	Amount of substance	n	Mole	Mol
7	Luminous intensity	I_v or λ	candela	cd

Derived Physical Quantities:

Those physical quantities which are expressed in terms of other physical quantities are known as derived physical quantities.

S.No.	Derived physical quantities	S.I units	Symbol
1	Force	Newton	N
2	Volume	Cubic meter	m^3
3	velocity	meter/second	m/s
4	Density	Kilogram/cubic meter	Kg/m^3

The S.I. Supplementary Quantities And Units:

There are two supplementary quantities and their units in S.I.

	QUANTITY	NAME OF UNIT	SYMBOL
1.	Plane Angle	Radian	rad
2.	Solid Angle	Steradian	Sr

Radian: A unit of measurement of unit equal to 57.3°, equivalent to the angle subtended at the centre of the circle by an arc equal in length of radius.

Steradian: The solid angle subtended at the centre of sphere by an area of surface equal to the square of the radius of that sphere.

Conventions of units:-

Convention of units in physics follows the standards to ensure the consistent and standardized communication of measurement.

International system of Units (S.I units):- The S.I system is globally accepted standard for the units of measurement in physics.

Prefixes :- The range of prefixes can be used with S.I base units to represents multiples or fraction of units. Some common prefixes are Kilo (k), Mega (M), micro (μ) etc.

Consistent formatting :- when writing units, it is important to write in lower case letters except for the symbol derived from the name of scientists i-e Kelvin (K), Newton (N) etc.

Appropriate choice of units:- it is important that appropriate units are used for the quantity being measured e.g meter per second (mS^{-1}) for speed and meter per second square (mS^{-2})for the acceleration.

Measurement Techniques:-

Different techniques and instruments are used to measure the physical quantities.

- The length of an object can be measured by ruler, vernier calipers or tape measure.
- Physical balance used to measure mass of an object.
- The traditional method of measuring the time are mechanical clocks such as pendulum or rotating escapement to keep the time. For highly accurate measurement atomic clocks are used.
- Thermometer are used to measure the temperature, thermocouples are also used to measure the temperature.
- Voltmeter is used to measure the voltage or potential difference.
- Current can be measured by ammeter.

- Resistance can be measured by ohm-meter or multi-meter.
- A cathode ray oscilloscope (C.R.O) is a type of electronic instrument that can be used for a variety of measurement techniques.

DIMENSION:

It is the representation of derived physical quantity in terms of fundamental physical quantity.

OR

It represents the nature of physical quantity. A mass may be measured in Kg or in Slug but it is still a mass. We say that its dimension is Mass $[M]$.

The symbol $[L]$, $[M]$, $[T]$ are used to specify the dimension of length, mass and time respectively which are the fundamental quantity.

Physical quantity	Expression	Dimensional formula
Area	Length x Breadth	$[L^2]$
Density	Mass / Volume	$[ML^{-3}]$
Work/ Energy	Force x Displacement	$[ML^2T^{-2}]$
Electric charge	Current x Time	$[AT]$
Moment of Inertia	Mass x $(\text{distance})^2$	$[ML^2]$
Angular momentum	Linear momentum x distance	$[ML^2T^{-1}]$
Gravitational constant	Force x $(\text{distance})^2$ / Mass^2	$[M^{-1}L^3T^{-2}]$

Applications and limitation of dimension

- To check the consistency of physical equation.
- To derive the physical relation between quantities.
- To change units from one system to another system.
- It does not give the information about the dimensional constant.
- The formula containing trigonometric function, exponential function, logarithmic function etc cannot be derived.
- It gives no information about the quantity whether the quantity is scalar or vector.

Example: Show that equation $S = vit + \frac{1}{2} at^2$ is dimensionally correct.

$$S = V_i t + \frac{1}{2} at^2$$

As

$$S = [L]$$

$$V_i = \left[\frac{L}{T}\right]$$

$$T = [T]$$

$$a = \left[\frac{L}{T^2}\right]$$

Now

$$L = \frac{L}{\cancel{T}} \times \cancel{T} + \frac{L}{\cancel{T^2}} \times \cancel{T^2}$$

$$L = L + L$$

$$L = 2\,L \qquad \therefore (2 = \text{dimensionless})$$

Or $$L = L$$

As the dimensions of both sides of the equation are the same so this equation is dimensionally correct.

Using Dimension to Derive the Equation

Consider the time period of simple pendulum depends upon:

- The mass of the bob (m).
- Length of the pendulum (L)
- Gravitational acceleration (g)

The equation can be written as:

$$T = K\, m^a\, L^b\, g^c$$

Where a, b and c are unknown powers and K is the dimensionless constant.

$$[T] = [M]^a\, [L]^b [LT^{-2}]^c$$

$$[T] = [M^a]\, [L^b][L^c T^{-2c}]$$

$$[M^0 L^0 T] = [M^a L^{b+c} T^{-2c}]$$

Equating the powers of $[L]$, $[M]$, $[T]$ on both side of the equation:

a = 0, (b+c) = 0, $-2c = 1 \Rightarrow c = -\frac{1}{2}$

a= 0, $b = \frac{1}{2}$ $c = -\frac{1}{2}$

the equation becomes:

$$T = K\, m^a\, L^b\, g^c$$

$$T = K\, m^0\, L^{\frac{1}{2}}\ g^{-\frac{1}{2}}$$

$$T = K \sqrt{\frac{L}{g}}$$

Dimension not gives any information about constant by other method the value of constant K is 2π.

$$T = 2\pi \sqrt{\frac{L}{g}}$$

Error:

A measure of the estimated difference between the observed or calculated value of a quantity and its true or expected value.

Error are the common occurrences in Physics. There are two types of error.

- Systematic Error
- Random Error

Systematic Errors:

Systematic error are errors which have a clear cause and can be eliminated for future experiments. OR the error that is not determined by the chance but is introduced by an inaccuracy inherent in the system. There are different types of systematic error:

Instrumental :

If the instrument not function properly causing error this error is known as instrumental error.

Enviromental :

When surrounding causes error in the measurement such as cell phone signal, air friction etc this type of systematic error is known as environmental error.

Observational :

When the person performing experiment read reading in accurately known as observational error.

Theoretical:

When the model system being used causes the result to be inaccurate known as theoretical error.

Random Errors:

Random error are occur randomly, sometimes no source or cause.

Observational :

When the observer consistent make observational mistakes such as writing down the values which are too low or high, not reading the scale correctly etc are the observational random error.

Enviromental :

When unpredictable changes occur such as student repeatedly open or closed the door when pressure being measured are termed as the environmental random error.

Uncertainty:

Uncertainty relates to the lack of precise knowledge or the degree of doubt with the measurement, estimation or prediction. It arises due to limitation in available information.

The main difference between error and uncertainties is *error is the difference between actual and measured value.* While *uncertainty is an estimate of range between them.*

Uncertainty in measurement:

There is always a degree of uncertainty when measurements are taken; uncertainty can be thought as the difference between actual reading and standard value.

- ***Uncertainty is the range of value around a measurement with in which the true value is expected to lie and is an estimate***.

Graph:-

Graphs are the visual that shows the relationship between the quantities or graph are represents the variation of variable in comparison with one or more than other variables.

Dependent and independent variables:

An independent variable is the variable that is manipulated in an experiment to observe effect it has on dependent variables. Independent variable also called the predictor variable .

The dependent variable is the variable that being measured or observed in the experiment or in the study and expected to change in the result of manipulation of independent variables. Dependent variable also called as response variables.

Best fit line graph:

A best fit line graph is used to show the general trend in the data. The line of best fit is the straight line that is drawn in the ways that it best represents the underlying pattern in the data.

Error Bar graph:

Error bars are the graphical representation of uncertainty or variability in set of data points. They are often used in scientific plots to indicate the precision of the data being plotted. Shorter error bar gives the greater precision while longer error gives less precision. Error bars are useful in indicating the reliability and accuracy of the data and they allow the reader to assess the significance of the result.

Extrapolation graph:

It is the process of making predictions or estimation about future or unseen data based on the trends or patterns in the existing data. Extrapolation is the statistical technique that involved using observed data to estimate values beyond the range of the data that was collected.

Significant figure:

The measured value of a physical quantity is always a number usually a decimal number. We take a large number of readings on the given instrument. The average value of the length calculated on the basis of these readings will be more accurate. Let this value be **320.5 cm** in this number **(320.5)** the digit on the extreme right to the decimal point which is **(5)** is uncertain. The reason of uncertainty is that scale can read correctly up to units placed on a number where this as the digit **(5)** lies on the first place of decimal point hence uncertain. The remaining digits **3, 2, 0** are known with certainty.

The significant figures in a number are the digits which are known with certainty. OR The accurately known digits and first doubtful is called significant figure.

Sometimes the measured value of a physical quantity can be written in more than one way. But only one of them is the correct way of writing the measured value which contains correct number of significant figures.

RULES FOR FINDING SIGNIFICANT FIGURES:

1. All non-zero digits are significant. For example the number **239** has three significant figures **2, 3, 9**.
2. Zeros lying between non-zero digits are significant, for example the number **2009** has four significant figures.
3. All the zeros which locate the decimal point in a number less than (one) are not significant. For example the number **0.00786** has only three significant figures that is **7, 8, 6**.
4. The zeros which are located immediately to the right of the decimal point are significant. For example the number **78.000** has five significant figures.
5. Zeros locating the decimal point in a number greater than 1 (one) are not necessarily significant. For example, the number **500** has only **one** significant figure. In such cases scientific notations is particularly convenient for finding the significant figures. Suppose that a certain distance of **1500** m is known to **four** significant figures.
6. **Significant zeros:**

We shall explain the meaning of significant zeros by taking an example. Consider a number **1200**. This number may have two, three or four significant figure, depending upon whether the zeros represent measurements or are merely used to locate the decimal point. Scientific notation avoids this ambiguity (uncertainty). The above number can be written as:

$$\mathbf{1.2 \times 10^3,\ 1.20 \times 10^3 \text{ and } 1.200 \times 10^3}.$$

Scientific Notations:

Scientific notation or the standard form is a simple method for writing very large number or very small numbers. Numbers in scientific notation are made up of three parts: The co-efficient, the Base and the exponent.

- The co-efficient must be equal to greater than one. (not equal to zero)
- The base must be 10.
- The exponent must be negative or positive.

$$\mathrm{m} \times 10^n$$

here in the above 'm' is the co-efficient '10' is the base and 'n' is the power.

Least Count OR Resolution:

Least count or resolution of a measuring instrument is the smallest increment that can be measured by the instrument. For example if the least count of the ruler is 0.1 cm then it only measure the length nearest to 0.1 cm.

Least count is inversely proportional to the precision of the measurement equipment. The smaller the minimum value of an instrument can measure the lower will be the least count, and higher will be the precision.

Accuracy	Precision
Accuracy is the level of agreement between the actual measurement and absolute measurement.	Precision suggest the level of variation that happen in the values of several measurement.
It represents the how close the result agree with the standard values.	Represents the how closely the values with one another.
Single factor or measurement is required.	Multiple measurements are needed to comment

	about precision.
Occasionally measurement may happen to be accurate by chance while consistent accuracy and precision are required for measurement to be reliable.	Result can be precise without being accurate

Significance of Resolution

Precision: The smallest resolution of a measuring instrument determines its precision, which is the degree of reproducibility of measurement. Instrument with smaller precision ensures that the measurement is precise and consistent.

Accuracy: The smallest resolution of a measuring instrument also effects its accuracy, which is the degree of closeness of measurement to its true value.

Detail: The smallest resolution of a measuring instrument determines the level of detail that can be obtained from the measurement. Using an instrument of smallest resolution allows for the measurement of finer details and features.

Reduced uncertainty: The smallest resolution of a measuring instrument is directly proportional to the reduced uncertainty in a measurement. The smaller the resolution the smaller the uncertainty, and more accurate the measurement.

Interpreting data from graphs:

Linear and non linear graphs are graphical representations of mathematical relationships between variables. By measuring variables slopes and intercepts, you can intercept important information about the nature of the relationship between the variables represented on the graph.

UNIT 02 **KINEMATICS**

Physical Quantity:

The Property of matter which is related to its measurement is called the physical quantity. The physical quantities are used in study of physics are classified into two group.

- Scalar Quantities.
- Vector Quantities.

Scalar Quantities:

The physical quantity which completely specified by their magnitude (number) with proper unit is known as Scalar physical quantities.

Examples:

Mass, Distance, Speed, Energy, Work, Volume, Temperature, Time, Electric charge, Atmospheric pressure are all scalar quantities.

The scalars can be compared only when they have same physical dimension (units). Two or more than two scalars measured in same system of units are equal only if they have the same magnitude and sign. Scalars can be denoted by ordinary letter and scalars are divide, multiple, subtracted and added by simple arithmetical rule.

Vector Quantities:

The physical quantity which specified by their magnitude as well as direction with proper units is called vector quantities.

Examples:

Displacement, Velocity, Acceleration, Force, Weight, Momentum, Torque, Electric intensity, Angular velocity are all vector quantities. Hence vectors are completely expressed by Magnitudes (Number x units) specific direction.

Vectors due to having direction cannot be added, subtracted, multiplied and divided by ordinary arithmetical rules, but we used methods for additions of vector, subtraction and vector multiplication called Vector algebra or vector analysis. When vectors are parallel (in the same direction) or anti-parallel (in opposite direction) then they can be added and subtracted by simple arithmetic rules.

Vector Representation:

There are two ways in which vector can be represented.

1. Symbolic (Analytical) representation
2. Graphical (Geometrical) representation

Representation of vector symbolically:

Vector represented symbolically in two ways.

Vector usually represented by bold face letter as **A**. OR Vector represented by letter with arrow is drawn above it as $\vec{A}$

Magnitude:

Magnitude of vector $\vec{A}$ is represented by simple letter A or by drawn a modulus as $|\vec{A}|$
the magnitude of vector also known as modulus.

Graphical Method:

In order to represent a vector graphically a straight line is drawn in the direction of vector with an arrow head at the end. The length of the line shows the magnitude and arrow head shows the direction. Initial point of the line is called tail and final point where arrow head is placed is called head of the vector.

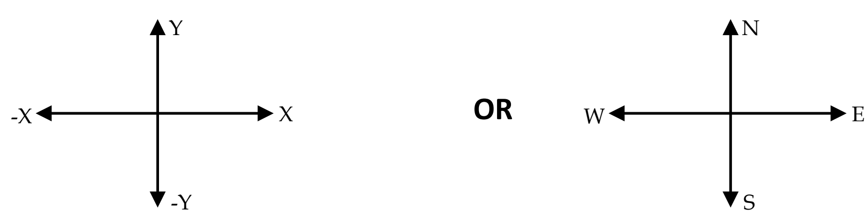
To represent vector graphically we require two things.

i. A suitable scale so that the vector should have length according to size of vector.
ii. Direction indicator which are two mutually line pointed East-west and North-South, by comparing the arrow head shows the direction.

Rectangular Co-ordinate system:

Co-ordinate axes (Frame of Reference):

Two lines drawn at right angles to each other are called co-ordinate axes. The point where two lines intersect each other is called origin.



this axes is called as Cartesian or rectangular co-ordinate system.
The horizontal line is called X-axis while vertical line is called Y-axis. These line form two dimensional co-ordinate system.

Three-Dimensional Co-ordinate system:

The axes which perpendicular to both x and y axis called z-axis.
In space three mutually perpendicular lines ox, oy and oz taken as co-ordinate axes (or reference axes) their also called three dimensional system.

Multiplication of vector by number (Scalar):

Vector can be multiplied by a number or scalar. When vector $\vec{A}$ is multiplied by a number or scalar 'n', it results into a new vector n$\vec{A}$. Where magnitude equal to 'n' times the magnitude of $\vec{A}$ and it direction remain same.

Magnitude of resultant vector = |n $\vec{A}$|= n$\vec{A}$

If number is negative then the magnitude becomes 'n' time the magnitude of $\vec{A}$ by direction becomes opposite

- n|$\vec{A}$| = -n A

Resultant vector:

The resultant (or sum) of a number of similar vectors is that single vector which would have the same effect as the combined effect of all the original vectors taken together.

Example:

Suppose $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ and $\overrightarrow{F_3}$ are similar vectors their resultant will be

$$\vec{F} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3}$$

Here$\vec{F}$ is the resultant vector having the combined effect of $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ & $\overrightarrow{F_3}$.

Negative of a vector:

Vector having same magnitude as that of vector $\vec{A}$ but opposite in direction is known as the negative of vector $\vec{A}$. Its is denoted by -$\vec{A}$.

Types of vectors:

There are four types of vectors.

i. Unit vector
ii. Position vector
iii. Free vector
iv. Null vector

Unit vector:

A vector whose magnitude equal to one is called unit vector and it just represent the direction of vector. In three dimensional space the unit vector along X, Y and Z axis are $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ respectively.

A unit vector is represented by drawing a cap or hat or circumflex above letter as '$\hat{a}$' or $\widehat{r_A}$.

If we want to find unit vector , then it is obtained by dividing the vector with its magnitude mathematically we write unit vector as.

$$\hat{a} = \frac{\vec{A}}{|\vec{A}|}$$

From the above equation we can find vector $\overrightarrow{A}$ in terms of unit vector $\hat{a}$. multiply magnitude with unit vector.

$$\vec{A} = |\vec{A}|\hat{a}$$

Other uses of unit vector:

Unit vector can be used for any direction in different ways and not confined to Cartesian co-ordinate. With the help of unit vectors the position vector in x-y plane can be written :

$$\vec{r} = x\hat{i} + y\hat{j}$$

the unit vector of vector $\vec{r}$ can be represent as $\hat{r}$.

Position vector:

A vector which starts from origin or fixed point is called position vector.

OR The position vector is that vector which describe the location of the object with respect to origin. It is denoted by $\vec{r}$. In three dimensionally space it is written as

$$\vec{r} = X\hat{i} + y\hat{j} + Z\hat{k}$$

and its magnitude can be given as

$$r = |\vec{r}| = \sqrt{x^2 + y^2 + Z^2}$$

Free vector:

Such vector which can be displayed anywhere in space parallel to itself is called free vector. In this case magnitude and direction remain same. All vector excepts positions vectors are free vectors.

Rectangular components of vectors:

Two components of vector which are at right angle to each other is known as rectangular components, one of them along horizontal axis and other along vertical axis these two are called x-component and y-component respectively.

Resolution of vector:

Consider a vector quantity $\vec{A}$ represented by the line $\overline{OP}$

making an angle '∅' with x-axis as shown in figure.

Draw a perpendicular from the terminal point of vector $\vec{A}$

$A_x\hat{i}$ = projection of vector $\vec{A}$ on x-axis

$A_y\hat{j}$ = Projection of vector $\vec{A}$ on y-axis

- **Geometrically:**

$$\overline{OP} = \overline{OQ} + \overline{QP}$$

$$\vec{A} = A_x\hat{i} + A_y\hat{j} \text{ ----------- (i)}$$

Thus '$A_x\hat{i}$' and '$A_y\hat{j}$' are the components of resultant vector $\vec{A}$, and these are right angle to each other so they called rectangular components of vector $\vec{A}$.

By the trigonometric we can find the magnitude of rectangular components of vector $\overrightarrow{A}$.

Trigonometrically:

In ΔOPQ

$$\text{Cos}\theta = \frac{Ax}{A}$$

or $$Ax = A\text{Cos}\theta \text{ ----------- (i)}$$

and $$\text{Sin}\theta = \frac{Ay}{A}$$

or $$Ay = A\text{Sin}\theta \text{ ------------ (iii)}$$

Putting in eq (i)

$$\vec{A} = (A\text{Cos}\theta)\hat{i} + (A\text{Sin}\theta)\hat{j}$$

The above equators, shows the vector resolve in to its component.

Composition of vector from its components:

If the rectangular components of vector are known we can find the magnitude and direction of the vector in lines of rectangular components.

Suppose 'Ax' and 'Ay' are the components of vector $\vec{A}$ represent by the line OM and ON shown in fig:

As ΔOPM is right angled triangle so by applying pathagoroustheorem

$(OP)^2 = (OM)^2 + (PM)^2$

$$A^2 = A_x^2 + A_y^2$$

or $A = \sqrt{A_x^2 + A_y^2}$ ------ This equation gives the magnitude of required vector $\vec{A}$.

Now direction can be determined by

$$\text{Tan}\theta = \frac{Ay}{Ax}$$

$$\text{or } \theta = \tan^{-1}\left(\frac{Ay}{Ax}\right)$$

Addition of vector:

A vector can be added into another vector only which result into a new vector of same kind. Addition of vector can be done by two ways.

i. Geometrical Method

ii. Analytical Method

Graphical (Geometrical) Method:

Head to tail Rule (Graphical Method)

Number of vectors can be added by joining the tail of successive vector with head of previous vector and resultant can be obtained by joining the tail with the first and head with the last vector.

$$\vec{R} = \vec{A} + \vec{B}$$

The above equation gives resultant vector and it is also known as triangular law of vector addition.

"After knowing the triangular law we can define vector as the quantities having magnitude as well as direction and following the law of vector addition."

Parallelogram Law:

If two vectors from the two adjacent side of the parallelogram the diagonal gives the resultant vector as shown in figure.

$$\overline{OC} = \overline{OA} + \overline{OB}$$

Diagonal = resultant = $\vec{R} = \vec{A} + \vec{B}$

Properties of vector Addition:

Commutative property of vector addition:

Consider two vectors $\vec{A}$ and $\vec{B}$ these two vectors represent the two adjacent side of Parallelogram then the diagonal $\overline{OC}$ represents their resultant vector $\vec{R}$shown in figure.

In triangle OAC

OC = OA + AC

$\vec{R} = \vec{A} + \vec{B}$--------- 1

Now again taken ΔOBC

$\overline{OC} = \overline{OB} + \overline{BC}$

$\vec{R} = \vec{A} + \vec{B}$-------- 2

Comparing eq 1 and 2

$\vec{A} + \vec{B} = \vec{\mathrm{B}} + \vec{A}$

Hence addition of vector obey commutative law.

Associative law of vector addition:

Suppose these vectors $\vec{A}, \vec{B}$ and $\vec{C}$ and join them by head to tail rule as shown an figure.

Consider ΔOPQ

$\overline{OP} = \overline{OQ} + \overline{QP}$

$\vec{R} = (\vec{A} + \vec{B}) + \vec{C}$ --------- 1)

Now suppose ΔOSP

$\overline{OP} = \overline{OS} + \overline{SP}$

$\vec{R} = \vec{A} + (\vec{B} + \overrightarrow{C})$ ----------- 2)

Comparing eq 1 and 2

$\overrightarrow{(A+B)}+\vec{C}=\vec{A}+\overrightarrow{(B+C)}$

Hence vector addition obeys Associative law

1) **Analytical Method of vector addition:**

Addition of vector by

Law of cosine:-

R = $\sqrt{A^2 + B^2 + 2AB\,Cos\theta}$ -------- This equation gives the magnitude of vector $\vec{R}$

$\frac{A}{SinA}=\frac{B}{SinB}=\frac{C}{SinC}$ this equation gives direction of resultant $\vec{R}$

This method is restricted to the addition of two vectors only.

Addition of vector by Rectangular components Methods:

Consider two vectors $\overrightarrow{A_1}$ and $\overrightarrow{A_2}$ making ample 'θ_1' & 'θ_2' with positive x-axis respectively as shown in figure:

Now join them by head to tail rule to get the resultant vector $\vec{A}$.

Resolve $\overrightarrow{A_1}$ and $\overrightarrow{A_2}$ in to its rectangular components as shown in figure:

Step- 1

X - Components

$A_{1x} = A_1Cos\theta_1$

$A_{2x} = A_2Cos\theta_2$

Y - Components

$A_{1y} = A_1Sin\theta_1$

$A_{2y} = A_2Sin\theta_2$

Step- 2

Resultant of x-components

$A_x = A_{1x} + A_{2x}$

or A_x = A, Cosθ, + $A_2Cos\theta_2$

Resultant of y-components

$A_y = A_{1y} + A_{2y}$

or A_y = A, Cosθ, + $A_2Cos\theta_2$

Step- 3

Magnitude:

A = $\sqrt{A_x^2 + A_y^2}$

A = $\sqrt{(A_1Cos\theta_1 + A_2Cos\theta_2)^2 + (A_1Sin\theta_1 + A_2Sin\theta_2)^2}$

Step- 4

Direction:

$\tan\theta = \frac{Ay}{Ax}$

$\theta = \tan^{-1}\frac{A_1 Sin\theta_1 + A_2 Sin\theta_2}{A,Cos\theta, + A_2 Cos\theta_2}$

Vector Product:

Different vectors can be multiplied with each other but the result of multiplications of two vectors may be a scalar or new vector quantity. Therefore there are two types of product of two vectors depending upon the result.

i. Scalar product **OR** Dot product of two vectors.
ii. Vector product **OR** cross product of two vectors.

Scalar Product OR Dot Product:

Whenever the product of multiplications of two vector quantities is a new scalar quantity then this types of product is called scalar product of two vectors. We usually placed a Dot (·) in between these two vector so it is also known as dot product of two vectors.

Mathematically,

Dot product is defined as the product of magnitude of two vectors and cosine of the angle between them is called scalar or Dot product. Or the product of two parallel vector are scaler so it is also called scalar product.

$\vec{A}\cdot\vec{B}$ = AB Cosθ

Scalar product =$\vec{A}\cdot\vec{B}$

So $\vec{A}\cdot\vec{B}$ = ABCosθ

$\vec{A}\cdot\vec{B}$ = A(BCosθ)

As, BCosθ is the projection on A so, B_A

$\vec{A}\cdot\vec{B}$ = AB_A

Example:

We known that force $\vec{F}$ and displacement $\vec{d}$ both vector quantities but their product is work which is scalar quantity we can say work done is the Dot or scalar product of force and displacement.

When there ample between force and displacement are in the same $\vec{F}$ direction them,

Work done = (F) (d)

When there source ample between them,

Would = F_xd

Work done = FCosθd

Work done = FdCosθ

Work done = $\vec{F}\cdot\vec{d}$

Other example of Dot product are electric force power etc.

Properties of Dot Product:

Commutative Property of Dot Product:

Consider two vectors $\vec{A}$ and $\vec{B}$ having ample 'θ' between them. we know that scalar product is equal to the product of magnitude of $\vec{A}$ and projection of $\vec{B}$ on to the direction of $\vec{A}$ as shown in figure I

$\vec{A}\cdot\vec{B}$ = (A)(B_A)

$\vec{A}\cdot\vec{B}$ = A(BCosθ)

$\vec{A} \cdot \vec{B}$ = ABCosθ--------- i

∵ B_A = Projection of vector $\vec{B}$ on to the direction of vector $\vec{A}$.

Now from fig: II

A_B = projection of vector $\vec{A}$ on to the direction of vector $\vec{B}$

$\vec{B} \cdot \vec{A}$ = (B) (A_B)

= B (ACosθ)

$\vec{B} \cdot \vec{A}$ = BACosθ

or $\vec{B} \cdot \vec{A}$ = ABCosθ ------- ii

By composing eq i and ii

$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

Hence showed that Dot product obey commutative law.

Dot product obey distributive property.

$\vec{A} \cdot (\vec{B} + \overrightarrow{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$

Characteristics of Dot product:

i. If $\vec{A}$ is parallel to $\vec{B}$ i-e 'θ' = 0° then $\vec{A} \cdot \vec{B}$ = AB
ii. If $\vec{A} = \vec{B}$ i-e $\vec{A}$ is parallel and equal to $\vec{B}$then $\vec{A} \cdot \vec{B} = \vec{A} \cdot \vec{C}$= AACosθ = A^2 ∵ θ = 0°
iii. If $\vec{A}$ is perpendicular to $\vec{B}$ i-e θ = 90° or one of them is a null vector them$\vec{A} \cdot \vec{B}$= o
iv. If unit vectors $\hat{i}, \hat{j}, \hat{k}$ are perpendicular to each other them

$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i}$ = o and if are parallel them

$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k}$ = 1

v. The projection of vector $\vec{A}$ on to $\vec{B}$ = A_B

∵ A_B = ACosθ

projection of $\vec{A}$on to $\vec{B}$ = ACosθ = A(1)Cosθ

projection of $\vec{A}$ on to $\vec{B}$ = $\vec{A} \cdot \hat{b}$

vi. If vectors are in components from the same components are multiplied in each other. eg:

$\vec{A} = A_x\hat{i} + A_y\hat{i} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{i} + B_z\hat{k}$ then

$\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$

Vectors product OR Cross product:

Two vectors are multiplied in such as way that new quantity is also a vector then the product is called vector product or cross product we put cross (x) between two vector so it is also called cross product mathematically the product of magnitude of two vectors and sine of the angle between them is called cross product.

$\vec{A} \times \vec{A}$ = ABSinθ or $\vec{A} \times \vec{B}$ = ABSinθ $\hat{n}$

Where ($\hat{n}$) is unit vector which is perpendicular to both $\vec{A}$ and $\vec{B}$ and represent the direction of new vector $\vec{C}$.

∵ $\vec{C} = \vec{A} \times \vec{B}$

$\vec{C}$ = ABSinθ $\hat{n}$

The direction of new vector quantity which results in to the multiplication of then two vectors is determine by right and rule from $\vec{A}$ to $\vec{B}$.

If vector $\vec{A}$ grip in right hand and rotate towards vector $\vec{B}$ then thumb points the direction of new vector $\vec{C}$ that is ($\hat{n}$) unit normal is.

Example:

We know that force $\vec{F}$ and moment arm $\vec{d}$ or $\vec{r}$ are both vector quantities but their product is torque which also a vector quantity so we can say that torque or the cross product of force and moment.

Torque = (r)(F)

But in general

Torque = (r)(F_y)

$\vec{T}$ = rFsinθ

$\vec{T}$ = rFsin$\hat{n}$

$\vec{T} = \vec{r} \times \vec{F}$

other, flow of electromagnetic energy, angular momentum and force acting or charge in magnetic field examples of vector or cross product.

Properties of Cross product:

There are some important properties of vector product.

i. $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

ii. $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$

iii. $(\vec{A} + \vec{B}) \times \vec{C} = \vec{A} \times \vec{C} + \vec{B} \times \vec{C}$

iv. If $\vec{A} \neq o$, $\vec{B} \neq o$ and $\vec{A} \times \vec{B} = o$ then $\times \vec{A}$ and $\times \vec{B}$ are parallel

v. $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$

$\because \hat{i} \times \hat{i} = |i||i|\ Sin0° \hat{n}$

$= (1)(1) \times O\hat{n}$

$= 1 \times O\hat{n}$

$\hat{i} \times \hat{i} = O$

vi. But

$\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$

or

$\hat{j} \times \hat{i} = -\hat{k}$

$\hat{k} \times \hat{j} = -\hat{i}$

$\hat{i} \times \hat{k} = -\hat{j}$

If vectors are given in component from

$\vec{A} = A_x + A_y + A_z$

$\vec{B} = B_x + B_y + B_z$

The cross product or vector product will be written as.

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

DISPLACEMENT:

"The change of Position of a body in a particular direction is called its DISPLACEMENT".

OR

"The shortest distance between two points in a straight line is called DISPLACEMENT."

The displacement is a vector quantity; the magnitude of displacement is independents of the path of the body. The S.I. unit of displacement is **meter (m)**.

VELOCITY:

*"The displacement Covered by a body in a unit time is called **VELOCITY**."*

OR

*The rate of change of displacement is called **VELOCITY**.*

Suppose a body is in motion. The path of its motion is represented by AB. At some instant (t_1), the body is at C after time Δt the body is at D. as the body moves from point C and D in time $\Delta t = t_2 - t_1$, it undergoes a change in position $\Delta \vec{r} = \vec{r}_2 - \vec{r}_1$

If a body is displaced through a distance d in certain direction in time t then velocity is given by

$$\vec{V} = \frac{\vec{d}}{t}$$

UNITS:

The S.I unit of velocity is **meter per second (m/s)**. The dimension of velocity is **[L T $^{-1}$]**

AVERAGE VELOCITY:

It is obtained by dividing the total displacement by total time taken by the body

$$V_{avg} = \frac{\Delta \vec{r}}{\Delta t} \qquad \because \Delta r = \vec{r}_2 - \vec{r}_1$$

$$\Delta t = t_2 - t_1$$

INSTANTANEOUS VELOCITY:

Instantaneous velocity of a body is obtained by finding the distance covered by the body in an extremely small interval of time.

$$\therefore V_{ins} = \lim_{\Delta t \to o} \frac{\overrightarrow{\Delta r}}{\Delta t}$$

UNIFORM VELOCITY:

When a body covers equal displacements in equal intervals of time, it is said to be moving with a uniform velocity.

NON- UNIFORM VELOCITY:

When a body undergoes different displacement in equal intervals of time, its velocity is called non-uniform or variable velocity.

DISPLACEMENT – TIME GRAPH:

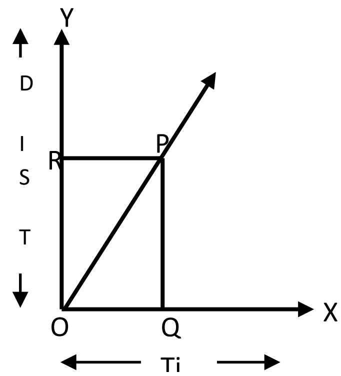
The velocity of a body can be obtained by distance time graph

UNIFORM VELOCITY:

Suppose a body moving with uniform velocity. A graph of distance against time will be a straight line.

If we take any point "P" on graph and draw perpendiculars $\vec{P}Q$ and $\vec{P}R$ on time and distance axis respectively.

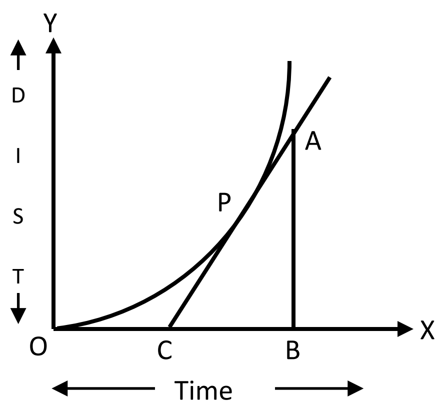
From fig



$$V = \frac{\overline{P}Q}{\overline{O}Q}$$

NON- UNIFORM VELOCITY:

If a body moves with non- uniform velocity, the graph of its motion is not a straight line. It is curved as shown in figure



The velocity of a body at any point is equal to the slop of the curve at the point which is obtained by drawing tangent line at that point

From Figure

$$V = \frac{\overline{A}B}{\Sigma B} \qquad \because \left(\frac{\overrightarrow{A}\overrightarrow{B}}{\overrightarrow{C}\overrightarrow{B}} = \text{Slope of curve} \right)$$

ACCELERATION:

The change in velocity per unit time is called ACCELERATION.

OR

The rate of change of velocity is called ACCELERATION.

UNITS:

The S.I unit of acceleration is m/s^2 and its dimension is LT^{-2}

TYPES:

Average Acceleration:

Average acceleration is defined as the ratio between the change in velocity (Δv) of a body and the time (Δt) in which this change takes place.

$$\vec{a}_{av} = \frac{\Delta \vec{V}}{\Delta t}$$

INSTANTANEOUS ACCELERATION

The acceleration at any particular time is called INSTANTANEOUS ACCELERATION. It is given by

$$\vec{a}_{ins} = \lim_{\Delta t \to 0} \frac{\Delta \vec{V}}{\Delta t}$$

POSITIVE ACCELERATION: If the velocity of moving body is increasing with time, then acceleration is said to be POSITIVE ACCELERATION.

NEGATIVE ACCELERATION: Velocity of moving body is decreasing with time, then acceleration is said to be NEGATIVE ACCELERATION is also called DECELERATION OR RETARDATION.

Derivation of First Equation of motion by graphical method:-

The body has an initial velocity *u* at point *A* and then its velocity changes at a uniform rate from *A* to *B* in time *t*. In other words, there is a uniform acceleration 'a' from *A* to *B*, and after time *t* its final velocity becomes '*v*' which is equal to *BC* in the graph. The time *t* is represented by *OC*. To complete the figure, we draw the perpendicular *CB* from point *C*, and draw *AD* parallel to *OC*. *BE* is the perpendicular from point *B* to *OE* .

Now, Initial velocity of the body, $u = OA$ ------- (1)

And, Final velocity of the body, $v = BC$ -------- (2)

But from the graph $BC = BD + DC$

Therefore, $v = BD + DC$ -------- (3)

Again $DC = OA$

So, $v = BD + OA$

Now, From equation (1), $OA = u$

So, $v = BD + u$ --------- (4)

We should find out the value of *BD* now.
We know that the slope of a velocity – time graph is equal to acceleration, *a*
Thus, Acceleration, *a* = slope of line *AB*
or *a* = BD/AD
But $AD = OC = t$,
so putting *t* in place of *AD* in the above relation, we get:
$a = BD/t$
or $BD = at$
Now, putting this value of *BD* in equation (4) we get
$v = at + u$
This equation can be rearranged to give:
$v = u + at$
And this is the first equation of motion.

Derivation of Second Equation of motion by graphical method:-

It has been derived by the graphical method.
Suppose the body travels a distance *s* in time *t*. In the above Figure, the distance travelled by the body is given by the area of the space between the velocity – time graph *AB* and the time axis *OC*, which is equal to the area of the figure *OABC*. Thus

Distance travelled = Area of figure *OABC*

= Area of rectangle *OADC* + Area of triangle *ABD*

We will now find out the area of the rectangle *OADC* and the area of the triangle *ABD*.

(i) Area of rectangle *OADC* = $OA \times OC$

$= u \times t$

$= ut$ ...... (5)

(ii) Area of triangle *ABD* = (1/2) × Area of rectangle *AEBD*

$= (1/2) \times AD \times BD$

= (1/2) × *t* × *at* (because *AD* = *t* and *BD* = *at*)

= (1/2) at^2 ------ (6)

So, Distance travelled, *s* = Area of rectangle *OADC* + Area of triangle *ABD*

or $s = ut + (1/2)\, at^2$

This is the second equation of motion.

Derivation of Third Equation of motion by graphical method:-

We have just seen that the distance travelled *s* by a body in time *t* is given by the area of the figure *OABC* which is a trapezium.
In other words,
Distance travelled, *s* = Area of trapezium *OABC*

Now, putting this value of *t* in equation (7) above, we get:

or $2as = v^2 - u^2$ [because $(v + u) \times (v - u) = v^2 - u^2$]
or $v^2 = u^2 + 2as$
This is the third equation of motion.

PROJECTILE MOTION

When a body with certain velocity is allowed to move under the force of gravity, such that it posses horizontal and vertical components of velocity. The body moves along a curved path. Such a body in motion is called a **PROJECTILE** and its motion is known as **PROJECTILE MOTION.**

EXAMPLES:

I. The motion of a football kicked off by a player
II. The motion of a missile shot from a gun
III. The motion of a player making a long jump.

Consider the motion of a football kicked off by a player with initial velocity $\vec{V}_o$ at an angle θ with the horizontal as shown in fig. which is given below.

From diagram.

$V_{ox} = V_o \cos\theta$ ______________(i)

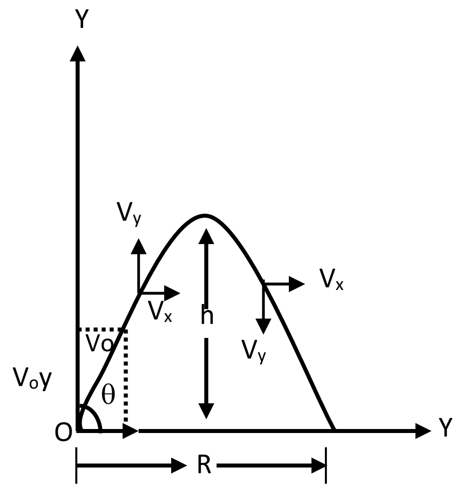
And $V_{oy} = V_o \sin\theta$ __________(ii)

During the projectile motion the velocity component V_x along x-axis at any time is equal to initial velocity component V_{ox}

i.e. $V_{ox} = V_x$

$V_x = V_o \cos\theta$ _______________(iii)

The calculate V_y we consider vertical motion



a = -g

t = t

$V_{oy}=V_o \sin\theta$

$V_y = ?$

Using equation

$\therefore V_y = V_o \sin\theta - gt.$ ___________(iv)

At the highest point $V_y = o$

$\therefore\ 0 = V_o \sin\theta - gt$

$gt = V_o \sin\theta$

$$T = \frac{V_o \sin\theta}{g} \text{ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ } (v)$$

Where T is the time to reach the highest point

I. MAXIMUM HEIGHT
II. TIME OF FLIGHT
III. RANGE OF PROJECTILE
IV. MAXIMUM RANGE

MAXIMUM HEIGHT (h):

It is the maximum distance covered by projectile along y-axis. In order to calculate ‘h’ we consider the vertical motion.

$V_i = v_{oy} = V_o \sin\theta$

a = -g

$t = T = \frac{V_o \sin\theta}{g}$

s=h=?

$V_y = 0$ (at height)

using formula

$2aS = V_f^2 - V_i^2$

$2(-g)H = V_y^2 - V_{0y}^2$

$H = \frac{V_0^2 \sin^2\theta}{2g}$

TIME OF FLIGHT (t)

The total time in which projectile is remains in air is called TIME OF FLIGHT.

Mathematically we can write

$t = T + T$

Where T = time to reach the maximum height

$\therefore t = 2T$

As $t = \frac{2V_o \sin\theta}{g}$ ___________ (vii)

RANGE OF PROJECTILE (R).

The horizontal distance from the origin to point, where the projectile return is called the RANGE OF THE PROJECTILE (R). In order to calculate R, let us consider horizontal motion

$V = V_x = V_o \cos\theta$

$t = \frac{2V_o \sin\theta}{g}$

$S = R = ?$

Using formula

$S = vt$

$R = (V_o \cos\theta)\ \frac{2V_o \sin\theta}{g}$

$R = \frac{V_o^2\ (2\cos\theta \sin\theta)}{G}$

$R = \frac{V_o^2 Sin2\theta}{g}$ __________ $(viii)$

$(\because \sin 2\theta = 2\cos\theta \sin\theta)$

It is from equation (Viii) that the range of projectile depends on initial velocity and on angle θ.

MAXIMUM RANGE (R_{MAX})

From equation (Viii) it is clear that if v_o and g are constant then "R" is maximum when $\sin 2\theta$ is maximum. The maximum value of $\sin 2\theta$ =

$\sin 2\theta = 1$

$2\theta = 90^0$

$\theta = 45^o$

$R_{max} = \frac{V_o^2}{g}$ $\quad\quad (\because \sin 90 = 1)$

Hence the projectile must be launched at an angle of 45^o with the horizontal to attain maximum range. For all other angles greater or smaller then 45^o the range will be less then $R_{max.}$.

UNIT 03 **DYNAMICS**

NEWTON'S LAWS OF MOTION:

Isaac Newton born in the year Galileo died (1642) restated and generalized Galileo's finding in the form of laws of motion and represented these in his world's famous book "Mathematical principles of Natural Philosophy" in the year 1687.

Newton's three laws express the relationship among force, Mass and Motion of a body in Mathematical form. These are considered to be still true and valid for bodies moving with velocities fairly small as compared to the velocity of light.

i. FIRST LAW OF MOTION:

Statement: This law states that "An object remains at rest or continues to move with uniform velocity unless acted upon by some unbalance force.

Explanation:

Newton's first law consist of two parts

(a) According to the first part a body remains at rest unless acted upon by an unbalanced force. For example: A table lying in class room does not change its place by itself. It will change its place, only if some unbalanced force act upon it.

(b) The second part of this law gives the definition of the force according to which "force is an agency which when applied to a body changes or tends to change its state of rest or of uniform motion".

This Law is also called the law of INERTIA

INERTIA:

The property of matter by virtue of which if it is in state of rest or motion it tries to remains in that state. The mass of body is a direct measure of its inertia.

ii. SECOND LAW OF MOTION:

Statement: This law states "If we apply a force on a body, the body will accelerate in the direction of the force. The magnitude of the acceleration will be directly proportional to the applied force and inversely proportional to the mass of the body".

Explanation:

According to 2nd law of motion.

$$a \propto F$$

And

$$a \propto \frac{1}{m}$$

OR

$$a \propto \frac{F}{m}$$

$$a = K \times \frac{F}{m}$$

OR

$$a = \frac{F}{m} \qquad (\because K = 1)$$

OR

F = ma

In vector form (Where "m" is the mass, "a" is the acceleration of the object and "F" is applied force.)

$$\vec{F} = m\vec{a}$$

UNITS OF FORCE:

Newton's first law of motion gives us the definition of force but Newton's 2nd law of motion provides us a method for measuring force and its unit.

The S.I unit of force is called **NEWTON** (N).

F= ma

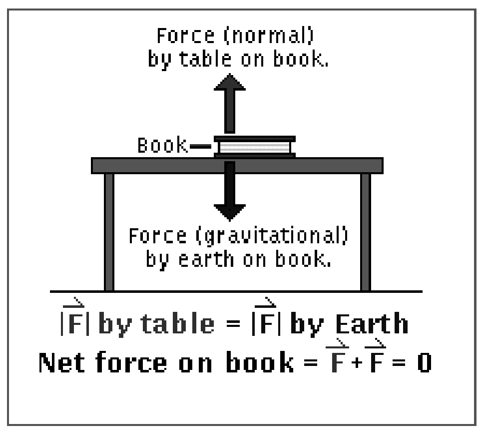
$$\therefore \quad 1N = 1\ kg \times 1m/s^2$$

DEFINE ONE NEWTON:

One Newton (1N) is the force which produces an acceleration of $1m/s^2$ is a body of mass 1kg.

iii THIRD LAW OF MOTION:

Statement: This law states that "To every action there is an equal and opposite reaction".



Explanation: When a body A exerts a force on another body B ($\vec{F}_{AB}$) , it is called action. The body B will also magnitude but opposite in direction. This force ($\vec{F}_{BA}$) is called reaction

Mathematically we can write

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

Another example of 3rd law

When a bullet is fired from a gun equal and opposite forces are exerted on the bullet and gun. Since, therefore both bullet and gun are acted upon by equal forces for the same time, they will in accordance with the 2nd law, acquire equal and opposite momentum. The backward momentum of gun itself is shown by its recoil.

MOMENTUM:

The product of mass and velocity of a body is called "LINEAR MOMENTUM" or MOMENTUM .

Mathematically we can write

$$\vec{P} = m\vec{V}$$

Where

$\vec{P}$ = Momentum of the body

M = mass of the moving body

$\vec{V}$ = Velocity of the body.

The direction of momentum is along the direction of velocity.

UNITS

The S.I unit of momentum is Kg-m/s or N- S

Show that unit of momentum is N.S:

Solution:

According to Newton's 2nd law of motion.

F = ma

F = $\frac{mv}{t} (\because a = v/t)$

OR

F = $\frac{P}{t}$

OR

P = F x t

$\therefore$ S.I unit of momentum is N x S

IMPULSE:

The impulse of a force $\vec{J}$ is the product of the force and the time interval Δt during which it acts.

$$\vec{J} = \sum \vec{F} \cdot \Delta t$$

$$\vec{J} = \sum \vec{F} \cdot \Delta t = \Delta\vec{P}$$

$\vec{J} = \Delta\vec{P}$

<u>LAW OF CONSERVATION OF MOMENTUM:</u>

Statement:

The law of conservation of momentum states that "If no external force act on a system of colliding objects, the total momentum before and after collision remains constant. "

Explanation:

Consider two bodies A and B of masses m1 and m_2 moving with velocities u1 and u_2 respectively before collision and V_1 and V_2 are the velocities of the bodies after collision along the same line and direction as shown in fig:

Total momentum of the system before collision= $m_1 u_1 + m_2 u_2$

And

Total momentum of the system after collision= $m_1 v_1 + m_2 v_2$.

According to Newton's 2^{nd} law of motion

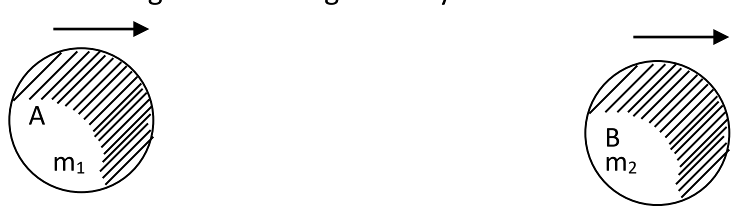
F = Rate of change of momentum

The average force acting on body B

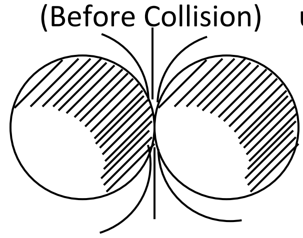
$$F_{AB} = \frac{m_2 v_2 - m_2 u_2}{t} \text{\_\_\_\_\_\_\_\_\_\_\_\_}(1)$$

Similarly

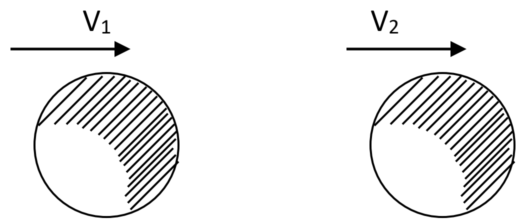
The average force acting on body A



(Before Collision) $u_1>u_2$



(Collision)



(After Collision)

$$F_{BA} = \frac{m_1V_1 - m_1u_1}{t}$$

According to Newton's 3[rd] Law of motion.

$F_{AB} = - F_{BA}$

$$\left(\frac{m_2v_2 - m_2u_2}{t} = -\frac{m_1v_1 + m_1u_1}{t}\right)$$

OR

$m_2 v_2 – m_2u_2 = - m_1v_1 + m_1u_1$

OR

$m_1 u_1 + m_2u_2 = m_1v_1 + m_2 v_2$

Hence

Total momentum before = Total momentum after collision

COLLISION:

If two bodies A and B of masses m_1 and m_2 respectively come close enough so that there is some kind of interaction between them, then we say that collision has taken place between A and B .

ELASTIC COLLISION:

An Elastic collision is that in which the momentum of the system as well as the Kinetic energy of the system before and after collision remains constant.

IN-ELASTIC COLLISION:

In this type of collision the momentum of the system remain constant but the Kinetic energy of the system before and after collision not remains constant.

ELASTIC COLLISION IN ONE DIMENSION:

Consider two spherical bodies A and B of masses m_1 and m_2 moving initially with the velocities u_1 and u_2, Such that $u_1 > u_2$ So they collide with one another and after elastic collision start moving with velocities V_1 and V_2 as shown in fig.

Applying Law of conservation of momentum.

$$m_1 U_1 + m_2 U_2 = m_1 V_1 + m_2 V_2$$

OR

$$m_1(U_1 - V_1) = m_2 (V_2 - U_2) \text{\_\_\_\_\_\_\_\_\_\_\_\_\_ (i)}$$

(Before collision)

(After collision)

Now applying law of Conservation of K.E

$$\frac{1}{2} m_1 u_1^{\ 2} + \frac{1}{2} m_1 u_2^{\ 2} = m_1 v_1^{\ 2} + \frac{1}{2} m_2 v_2^{\ 2}$$

OR

$$m_1 (u_1^2 - v_1^2) = m_2 (V_2^2 - u_2^2)$$

OR

$$m_1 (u_1 - v_1) (u_1 + v_1) = m_2(v_2 - u_2) (v_2 + u_2) \text{ \_\_\_\_\_\_\_\_\_\_\_ (ii)}$$

Dividing equation (ii) by equation (i), we have

$$\frac{m_1(u_1 - v_1)(u_1 + v_1)}{m_1(u_1 + v_1)} = \frac{m_2(v_2 - u_1)(v_2 + u_2)}{m_2(v_2 - u_2)}$$

OR

$$u_1 + v_1 = v_2 + u_2 \qquad \text{\_\_\_\_\_\_\_\_\_\_\_ (iii)}$$

Value of V_1:

From equation (iii)

$V_2 = (u_1 + v_1) – u_2$

Put value of v_2 in equation (i), we get $m_1(u_1-v_1) = m_2 [(u_1 + V_1) – u_2 – u_2)]$

OR

$$V_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{(m_1 + m_2)} \_\_\_\_\_\_\_\_\_\_\_\_\_\_ (iv)$$

Value of V_2:

Again from equation (iii)

$V_1 = (V_2 + u_2) – u_1$

Put this value of v_1 in equation (i)

$M_1 [u1 – \{(v_2 + u_2) - u_1\}] = m_2 (v_2 – u_2)$

$M_1 (u_1 – v_2 – u_2 + u_1) = m_2 (V_2 - u_2)$

OR

$$V_2 = \frac{2m_1u_1}{(m_1 + m_2)} + \frac{(m_2 - m_1)u_2}{(m_1 + m_2)} \_\_\_\_\_\_\_\_\_\_\_\_\_\_ (v)$$

With the help of equation (IV) and (v) we can find the velocities of the bodies A and B after collision.

FRICTION:

When two surfaces are in contact and slides over one another a force is setup between the surface which oppose the relative motion of both the surface is known as Friction.

Types of Friction

Statics Friction:

The friction force that comes into play between the two surfaces when one body tends to move on the surface of other body is called static friction.

$$f \leq \mu_s N$$

The maximum value of static friction is called limiting friction.

Kinetic Friction:

The friction force that comes into play between the two surfaces when one body move on the surface of other body is called kinetic friction.

$$f \leq \mu_k N$$

There are two types of kinetic friction.

a. **Sliding Friction:**

The friction that exists between two surfaces when one body is sliding on the other is called sliding friction.

b. **Kinetic Friction:**

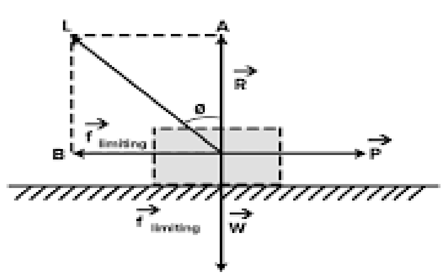
The friction that exists between two surfaces when one surface is rolling over the surface called rolling friction.

Laws of Friction:

- The frictional force opposes the relative motion of two surfaces.
- The frictional force is parallel to the surfaces in contact.
- The frictional force is directly proportional to the normal reaction.
- The frictional force is independent of the area of contact.
- The frictional force depends upon the nature of the two surfaces in contact and their state of roughness.
- The kinetic friction is independence of the relative velocities of the surface.

Angle of friction

Angle of friction is defined as the angle made by the resultant of frictional force and the normal reaction with the normal reaction.



$$\tan \emptyset = \frac{AL}{AC} = \frac{f}{NR}$$

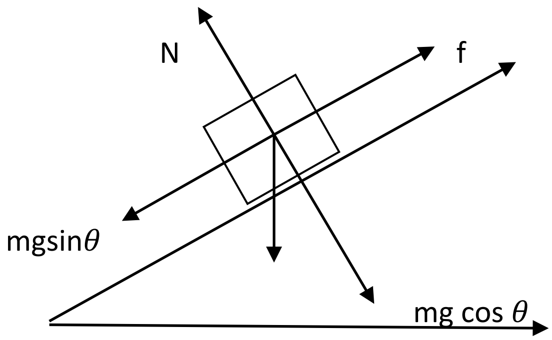
$$\therefore \frac{f}{NR} = \mu$$

$$\tan \emptyset = \mu$$

Therefore the tangent of angle of friction is called co efficient of friction.

Relationship between angle of Friction and angle of Repose.

It is defined as the minimum angle made by the inclined plane with the horizontal such that object on inclined plane just about to move.



Let 'N' be the normal and f be the friction in the figure:

mg sinθ = f ⟶ (i)

mg cosθ = N ⟶ (ii)

dividing eq: (i) by (ii)

$$\frac{mg\ sin\theta}{mg \cos \theta} = \frac{f}{N}$$

$$\tan \theta = \frac{f}{N}$$

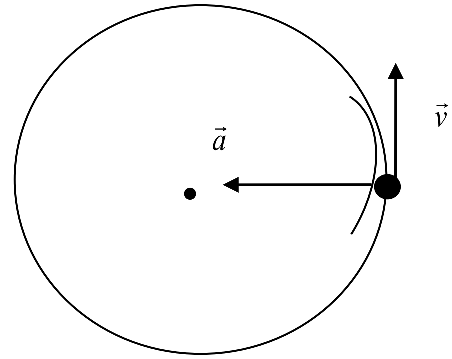
$\therefore \frac{f}{NR} = \mu$

$$\tan \theta = \mu$$

$$\theta = \tan^{-1} \mu$$

UNIT 04 ROTATIONAL & CIRCULAR MOTION

Uniform circular motion When an object moves along a circular path in such a way that its speed is uniform or constant. This type of motion is known as UNIFORM CIRCULAR MOTION.



Following terms are important to describe the uniform circular motion.

I. Angular Displacement (θ)
II. Angular velocity (ω)
III. Angular Displacement (∝)
IV. Time period (T)

Angular Displacement (θ)

The angle through which a body moves, while moving along a circular path is called its ANGULAR DISPLACEMENT (θ)

The unit of angular displacement is radian or degree.

Relationship between linear and angular displacement

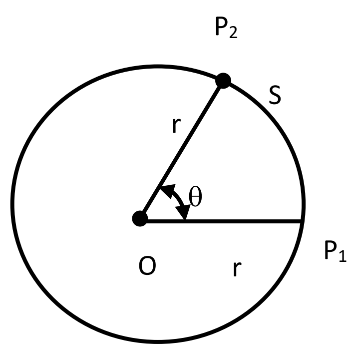
Suppose an object moving along a circular path of radius 'r' consider further that the object initially is at the point P_1 on the circumference of the circle after a small interval of time "t", it moves to the position P_2 as shown in fig.

Mathematically we can write

$$\theta = \frac{|\vec{P_1 P_2}|}{|\overrightarrow{OP_1}|}$$

OR

$$\theta = \frac{S}{r}$$



Relationship between degree and radian

For one complete revolution θ =360^0, when the arc length becomes the circumference of the circle i.e. s= 2 π r.

Then,

$$\theta = \frac{2\pi\text{ r}}{\text{r}}$$

θ = 2 π radian

OR

$360^0 = 2\pi$ radian

Thus

$$1 \text{ rad} = \frac{360}{2\pi} = \frac{360}{2\times3.14} = 57.3 \text{ degree}$$

$\therefore$ 1 rad = 57.3^0

Similarly

$$1^0 = \frac{2\pi}{360} = \frac{2\times3.14}{360}$$

1^0 =0 .01745 rad

Radian:

It is the angle subtended to the centre of a circle by an are equal in length to the radius of the circle known as radian. (S =r)

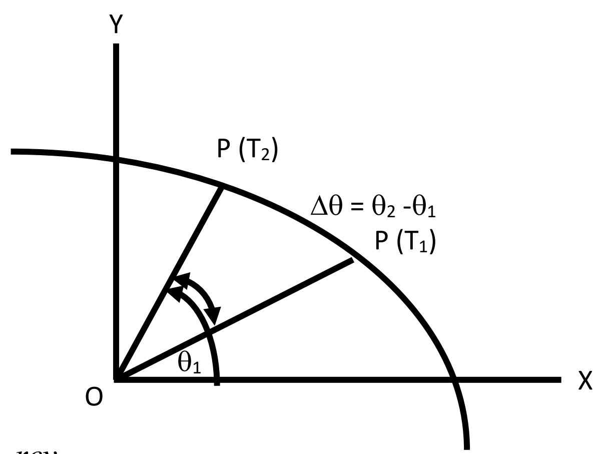
Angular velocity (ω):

The rate of change of angular displacement is called ANGULAR VELOCITY.

Consider a body moves anti-clock wise in a circle of radius 'r'. The angular position of P is θ_1 at time t_1 and at time t_2 , it angular position is θ_2.

The magnitude of average angular velocity is given by.

$$\omega_{avg} = \frac{\Delta\theta}{\Delta t}$$



The S.I unit of angular velocity is $\frac{rad}{Sec}$ $\frac{\deg ree}{Sec}$, $\frac{rad}{Sec}$ and $\frac{rev}{Sec}$.

Direction:

The angular velocity $\vec{\omega}$ is taken to be directed along the axis of rotation.

It is directed out of the page parallel to the axis of rotation, if the rotation is anti-clock wise.

If the rotation is clockwise $\vec{\omega}$ is directed into the page.

Angular acceleration (α)

The rate of change of angular velocity is called ANGULAR ACCELERATION.

Consider ω_1 and ω_2 be the magnitude of angular velocities at time t_1 and t_2 respectively. The average acceleration is given by:

$$\alpha_{avg} = \frac{\Delta\omega}{\Delta t}$$

and

$$\alpha_{inst} = \lim_{0} t \frac{\Delta\omega}{\Delta t}$$

OR

$$\alpha = \frac{d\omega}{dt}$$

The S.I unit of angular acceleration is rad/Sec^2.

When the average and instantaneous acceleration are equal, then the body is said to be moving with uniform acceleration

Direction:

When angular velocity increases, $\vec{\alpha}$ has same direction as $\vec{\omega}$. When angular velocity decreases $\vec{\alpha}$ has a direction opposite to $\vec{\omega}$.

Time Period

The time required for one complete revolution is called TIME PERIOD. The time period is inversely proportional to angular velocity

$$T = \frac{1}{\omega}$$

$$T = \frac{2\pi}{\omega} \qquad T = \frac{1}{F}$$

$$T = \frac{2\pi}{2\pi f}$$

Unit of T is second (s)

Relationship between linear and angular Quantities:

Relationship between linear and angular velocities:

If Δs is the length of an arc subtending an angle $\Delta\theta$ at the centre of a circle of radius r, than

$$\Delta\theta = \frac{\Delta s}{r}$$

OR

$$\Delta s = r\Delta\theta$$

Dividing both sides by Δt, we have

$$\frac{\Delta s}{\Delta t} = r\frac{\Delta\theta}{\Delta t}$$

OR

$$\lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = r \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

$$\left(\because \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} v \text{ and } \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = w\right)$$

OR

$V = r\omega$ OR $V_t = r\omega$

Relationship between linear and angular acceleration:

The tangential velocity V_t of a particle moving in a circular path is given by the product of radius and angular velocity i.e.

$$V_t = r\omega$$

OR

$$\Delta V_t = r\Delta\omega$$

Dividing both side by Δt

$$\frac{\Delta V_t}{\Delta t} = r\frac{\Delta\omega}{\Delta t}$$

$$a_t = r\propto$$

Where a_t is tangential acceleration

Centripetal acceleration:

The acceleration produced by virtue of the changing direction of the velocity of an object moving in a circular path is called CENTRIPETAL ACCELERATION

Consider an object moving in a circular path of radius "r" with constant speed v. suppose the object takes a time $\Delta t = t_2 - t_1$, to go from position A to B.

It follows from geometry that triangle OAB and OCD are isosceles triangles, and the angles $\Delta\theta$ a , e the same

Hence

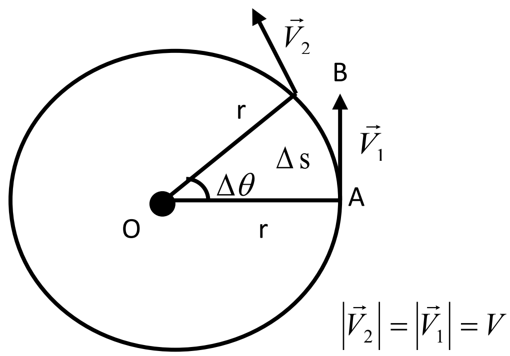


Fig ___(i)

$$\frac{\Delta v}{v} = \frac{\Delta s}{r}$$

OR

$$\Delta v = \frac{v}{2} \times ds$$

Dividing both side by Δt

$$\frac{\Delta v}{\Delta t} = \frac{v}{r} \times \frac{\Delta s}{\Delta t}$$

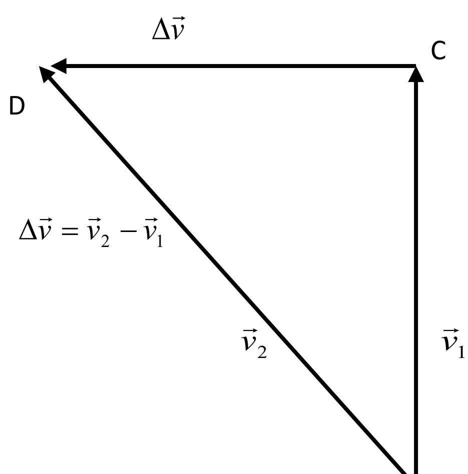


Fig (ii)

$$\frac{\lim}{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{v}{r} \times \frac{l\,\text{int}}{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$

$$a_c = \frac{v}{r} \times v$$

OR

$$a_c = \frac{v^2}{r} \_\_\_\_\_\_\_\_\_\_\_\_ (i)$$

Equation (i) gives the magnitude of the centripetal acceleration. The direction of a_c is always towards the centre of circle put v=rw in equation (i), we have

$$a_c = \frac{(rw)^2}{r}$$

$a_c = rw^2$ ___________(ii)

As = $w = 2\pi f = \frac{2\Pi}{T} (\because f = \frac{1}{f})$

$$\therefore a_c = r(\frac{2\pi}{T})^2$$

$a_c = \frac{4\pi^2 r}{T^2}$ _____________(iii)

Special case:

If the speed of the object is increasing or decreasing (motion is not uniform) . it also has a tangential component of acceleration. Both a_c and a_t exist perpendicular to each other.

It is clear from fig (ii) that

$\vec{a} = \vec{a}_c + \vec{a}_t$ _____________(i)

The magnitude of total acceleration is

$$a = \sqrt{a_c^{\ 2} + a_t^{\ 2}}$$

The direction of $\vec{a}$ is

$$\theta = \tan \frac{a_t}{a_c}$$

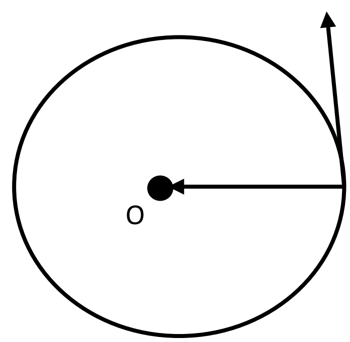


Fig ___(i)

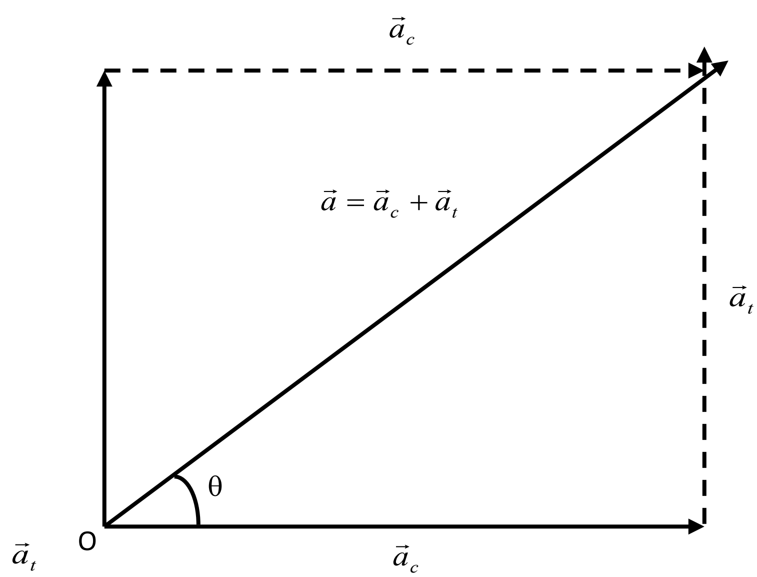


Fig (ii)

Centripetal Force:

A force that causes an object to move in a curved path rather than continuing in a straight line is called CENTRIPETAL FORCE"

Consider an object of mass 'm' moving in a circular path of radius 'r' with a constant speed as shown in fig.

According to the first law of maintain motion in straight line however, the string does not allows this to happen by exerting a force on the ball such that the ball follow its circular path. This force (The force of tension) is directed along the string towards the centre of the circle. This force is called centripetal force ($\vec{F}_c$)

According to Newton's 2nd law of motion

F = ma

OR

$F_c = ma_c$ ___________(i)

But

$a_c \frac{V^2}{r}$

$\therefore F_c \frac{mv^2}{r}$ _____________(ii)

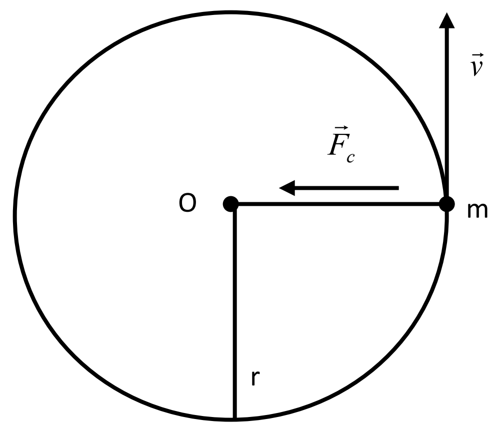


Fig.

Similarly

$F_c = mr\omega^2$

And

$F_c = \frac{4m\pi r^2}{T^2}$

Linear and angular equations of motion:

Linear Motion	Rotational Motion
$S = Vt$	$\theta = \frac{S}{r}$ or $\theta = \omega t$
$S = V_i t + \frac{1}{2}at^2$	$\theta = \omega_i t + \frac{1}{2}\alpha t^2$
$V_{avg} = \frac{V_f + V_i}{2}$	$\omega_{avg} = \frac{\omega_f + \omega_i}{2}$
$V_f = V_i + at$	$\omega_f = \omega_i + \alpha t$
$2aS = V_f^2 - V_i^2$	$2\alpha\theta = \omega_f^2 - \omega_i^2$

Centripetal acceleration caused by the Tension force:

Any force or combinations of forces can cause centripetal or radial acceleration for example gravitation, electrostatics and tension in the string can cause the centripetal force or acceleration. If a ball is tied to the end of the string and whirling in a circle, accelerates towards the centre of the circle. The centripetal force on the object in this situation is the sum of weight and tension force.

$$F_c = F_t + F_w$$

$$F_t = F_c - F_w$$

$$F_t = \frac{mV^2}{r} - mg$$

Banking of roads

When a moving car turns round a corner the man (occupants) inside the car tends to fall outward direction. On level curve (unbanked) when car is going, three forces are acting on it, its weight vertically downward, the upward push of the road and force of friction between roads and tires which provides necessary centripetal force to cause the car to maintain the acceleration for normal speed. If the speed of car exceeds the normal limit, there will be insufficient force to cause the car to maintain acceleration and the car is unable to go around the curve and will skid in a curve of greater radius.

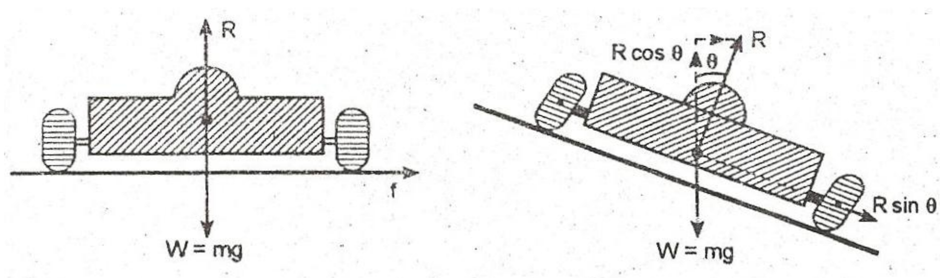


Fig. 7.6 Fig. 7.7

To provide sufficient force the curves on highways are banked, thus car going properly on banked curve at some particular speed. When the force exerted by the road perpendicular to its surface has a horizontal component equal to the required centripetal force and vertical component equal and opposite to the weight of the car (vehicle), From figure:

R Cos θ = mg ∴ R = N

N Cos θ = mg ⟶ (i)

N Sin θ = $\frac{mV^2}{r}$ ⟶ (ii)

Dividing Equation: (ii) by (i)

$$\frac{N\ Sin\ \theta}{N\ Cos\ \theta} = \frac{\frac{mV^2}{r}}{mg}$$

$$\tan\theta = \frac{V^2}{rg} \quad => \quad \theta = \tan^{-1}\left(\frac{V^2}{rg}\right)$$

Orbital Velocity:

The velocity required to keep the satellite into its orbit is called orbital velocity.
Consider a satellite of mass 'm' moving in an orbit of radius 'R' with velocity 'v' around the earth. The gravitational force of attraction between the satellite and earth provides necessary centripetal force.

If "M_e" is the mass of earth, then:

$$\text{Gravitational force} = \frac{G\,Me\,m}{R^2}$$

$$\text{Centripetal force} = \frac{m\,v^2}{R}$$

$$\frac{m\,v^2}{R} = \frac{G\,Me\,m}{R^2} \text{ or } v^2 = \frac{G\,Me}{R} \text{ ------------------------- } (i)$$

$$v = \sqrt{\frac{GMe}{R}}$$

Time period:

Planet or satellite revolve in orbit, its orbital time period is T to travel the circumference of the orbit is $2\pi R$.

$$V = \frac{2\pi R}{T}$$

$$V^2 = \left(\frac{2\pi R}{T}\right)^2$$

$$\therefore\ v^2 = \frac{G\,Me}{R}$$

$$\left(\frac{2\pi R}{T}\right)^2 = \frac{G\,Me}{R}$$

$$T^2 = \frac{4\pi^2 R^3}{GM_e}$$

Moment of inertia

Moment of inertia is that property of body by virtue of which its resist the angular acceleration. Moment of inertia is also known as the rotational inertia.

$$I = mr^2$$

S. I unit of moment of inertia $is\ Kgm^2$ or $M^1L^2T^0$.

The moment of inertia depends upon the following factors:

- Shape and size of the body
- The density of body

- Axis of rotation (Distribution of mass relative to the axis).

Moment of inertia of various bodies:

- *Moment of inertia of solid cylinder:*

$$I = \frac{1}{2} M R^2$$

- *Moment of inertia of Hollow cylinder:*

$$I = \frac{1}{2} M (a^2 + b^2)$$

- *Moment of inertia of sphere:*

$$I = \frac{2}{5} M R^2$$

Angular Momentum:

A body having translation motion possesses linear velocity and linear momentum. In the same way a body having rotatory motion possesses angular velocity and angular momentum. The angular momentum of the particle about the origin "O" is defined as the vector product of $\vec{r}$ and $\vec{p}$, and denoted by $\vec{L}$.

Mathematically we can write

$$\boxed{\vec{L} = \vec{r} \times \vec{p}} \longrightarrow \text{(i)}$$

Where $\vec{r}$ is position of particle from origin and $\vec{p}$ is the linear momentum if m is the mass of the particle and $\vec{v}$ its velocity then

$$\vec{L} = \vec{r} \times m\vec{v}$$

OR

$$\boxed{\vec{L} = m\left(\vec{r} \times \vec{v}\right)} \longrightarrow \text{(ii)}$$

The direction of angular momentum $\vec{L}$ is perpendicular to the plane of $\vec{r}$ and $\vec{p}$. In other words its direction is perpendicular to the plane of rotation of the particle and can be determined by right hand rule.

The magnitude of $\vec{L}$ is given by

$\left|\vec{L}\right|$ = r P Sin θ = m v r Sin θ (iii)

Where θ is the angle between $\vec{r}$ and $\vec{p}$

For circular motion $\vec{r}$ is ⊥ to $\vec{p}$

L = rPSin90^{O} = r P(1)

$\boxed{L = r P}$ = L = m v r

If $r = x\hat{i} + y\hat{j} + z\hat{k}$

and

$$\vec{P} = Px\hat{i} + Py\hat{j} + Pz\hat{k}$$

Then angular momentum $\vec{L}$ is defined as

$$\vec{L} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ Px & Py & Pz \end{vmatrix}$$

$$Lx\hat{i} + Ly\hat{j} + Lz\hat{k} = (Pzy - Pyz)\hat{i} + (zP_x - XPz)\hat{j} + (XP_y - YPx)\hat{k}$$

By using condition of equality of two vectors

$$Lx = YPz - zPy$$

$$Ly = zPx - xPz$$

$$Lz = YPy - zPx$$

Unit

The S.I. unit of angular momentum is J.S.

Dimension

The dimensions of angular momentum are given below

$$L = (r)(p) = (r)(m)(v)$$

Dimension = $LM\frac{L}{T}=\frac{ML^2}{T}$

Law Of Conservation Of Momentum

According to the law of conservation of angular momentum "When the total external torque acting on a system or particle is zero, the total angular momentum of the system or particle is remains constant."

According to Newton's 2nd law of motion

$$\vec{F}=\frac{d\left(\vec{P}\right)}{dt}$$

Taking the vector product of the both the sides with $\vec{r}$ from the left we get

$$\vec{r}\times\vec{F}=\vec{r}\times\frac{d\vec{P}}{dt}$$

$$\tau=\vec{r}\times\frac{d\vec{P}}{dt} \longrightarrow \text{(i)} \qquad \left(\because \&\vec{\tau}=\vec{r}\times\vec{F}\right)$$

As e know that

$$\vec{L}=\vec{r}\times\vec{p}$$

Differentiate w.r.t time t.

$$\frac{d\vec{L}}{dt}=d\frac{\left(\vec{r}\times\vec{p}\right)}{dt}$$

$$\frac{d\vec{L}}{dt}=\frac{d\vec{r}}{dt}\times P+\vec{r}\times\frac{d\vec{P}}{dt}$$

OR

$$\frac{d\vec{L}}{dt}=\vec{v}\times m\vec{v}+\tau \qquad \left(\because \frac{d\vec{r}}{dt}=\vec{v}, P=m\vec{v}\ and\ r\times\frac{dP}{dt}=\vec{\tau}\right)$$

OR

$$\boxed{\frac{d\vec{L}}{dt} = \tau} \longrightarrow \text{(ii)} \qquad \left(\because \vec{v} \times \vec{v} = 0\right)$$

If $\tau = 0$

Then $\frac{d\vec{L}}{dt} = 0$

OR

$\vec{L} = Constant$

Thus the angular momentum of particle is constant if the net torque acting on it is zero.

Now consider a system n particle

Then $\vec{L} = \vec{L}_1 + \vec{L}_2 + ... + \vec{L}_n$

$$\overline{L} = \vec{r}_1 \times \vec{P}_1 + \vec{r}_2 \times \vec{P}_2 + ... + \vec{r}_n \times \vec{P}_n$$

OR

$$\frac{d\vec{L}}{dt} = r_1 \times \frac{d\vec{P}_1}{dt} + r_2 \times \frac{d\vec{P}_2}{dt} + ... + r_n \times \frac{d\vec{P}_n}{dt}$$

OR

$$\frac{d\vec{L}}{dt} = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + ... + \vec{r}_n \times \vec{F}_n$$

$$\frac{d\vec{L}}{dt} = \vec{\tau}_1 + \vec{\tau}_2 + ... + \vec{\tau}_n$$

OR

$$\frac{d\vec{L}}{dt} = \sum_{1=1}^{n} \vec{\tau}_i = \tau$$

If the net external torque $\tau = 0$

Then $\frac{d\vec{L}}{dt} = 0$ $\qquad \therefore \vec{L} = Constant$

Torque:

Consider a force $\vec{F}$ is applied on a particle of mass 'm' whose position vector with respect to origin 'O' is $\vec{r}$ as shown in figure:

The force F_{11} can pull the mass be cannot rotate. Therefore the magnitude of the torque $\tau$is defined as

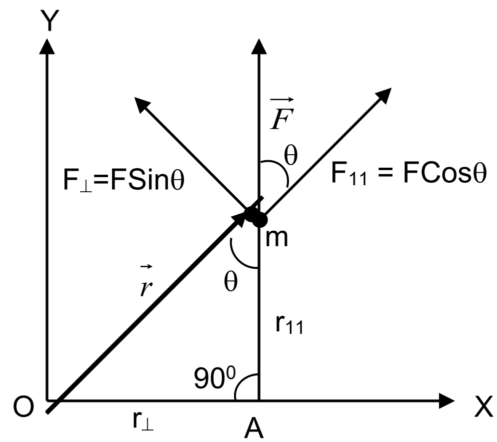
$\vec{\tau}= rF_{\perp} = r\ F\ Sin\theta$

$= rF\ Sin\theta \longrightarrow$ (i)

$|\vec{\tau}| = |\vec{r} \times \vec{F}|$

OR

$\vec{\tau} = \vec{r} \times \vec{F} \longrightarrow$ (ii)



Hence

The Torque τ (tau) is defined as cross product of Position vector (moment arm) and applied force.

Direction:

Torque is a vector quantity. Its direction is perpendicular to the plane containing $\vec{r}$ and $\vec{F}$. It is determined by right hand screw rule.

If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

and $\vec{F} = F_1\hat{i} + F_2\hat{j} + F_3\hat{k}$

then torque is defined as

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ F_1 & F_2 & F_3 \end{vmatrix} \longrightarrow \text{(iii)}$$

From equation (i), we can write

$\tau = F\,(r\sin\theta) = Fr_{\perp}$ ⟶ (iv)

Where

$r_{\perp}$ = OA = moment of arm

Thus

Magnitude of torque = moment of arm × magnitude of force

The S.I unit of torque is N - m

Positive and Negative Torque:

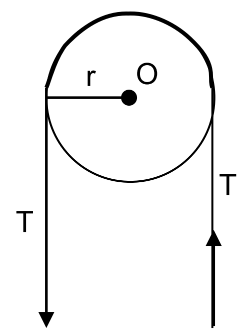
The Torque which rotates a body in anticlockwise direction is called a POSITIVE TORQUE and the torque which rotates the body in clockwise direction is called a NEGATIVE TORQUE.

The most common example of torque is the string passing over a pulley of radius 'r'. It can rotate the pulley about its centre. The magnitude of the torque produced by T is give by

$\tau = rT\sin 90^{o}$

$\tau = rT\,(1)$ $(\because \sin 90^{o} = 1)$

$\boxed{\tau = rT}$



UNIT 05 **WORK, ENERGY & POWER**

WORK

When a force acts on a body and moves it through some distance, work is said to be done.

Or

If a body covers some distance due to the application of force. Then it is said that work has done.

Mathematically

From the definition of work it is clear that work depends upon two factors i.e. Force and Displacement. Hence quantity of work is equal to the product of force and displacement. If we represent work by "W" force by "F" and displacement by "d" then the mathematical expression for work is as under:

$$\text{Work} = \text{Force} \cdot \text{Displacement}$$

$$W = F \cdot d$$

$$W = F\, d$$

Example -1

Many time it happens that a force is acting on a body and the body moves under its influence but the direction of motion does not coincide with the direction of the force as shown in a wooden block is pulled by means of a rope, it moves along a level ground but the force acts along the rope.

If the force make an angle θ with the direction of motion then the work done can be found by taking product of the component of force along the direction of motion and the distances. Therefore,

$W = F \cos\theta \times d$ or $W = Fd \cos\theta$

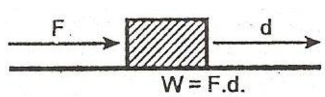


Fig. 8.1

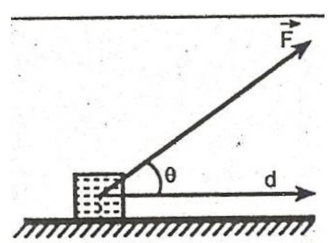


Fig. 8.2

Example 2

A person carrying a suit case covers a certain distance. He applies a force on the suit case perpendicular to distance. Here the angle between force and distance is 90° and work done is zero. Since,

$W = Fd \cos\theta$ or $W = Fd \cos 90^o$ or $W = Fd \times 0 = 0$

Unit of work We know that

$W = Fd$

$W = 1N \times 1m$

$W = 1\text{Joule}$

Hence the unit of work is joule and denoted by "J".

ONE JOULE

If one Newton force produces a displacement of 1m then the amount of work is one (1) joule.

$$1\text{Joule} = 1N \times 1m$$

One dyne:

If one dyne force produces a displacement of 1cm then the amount of work is 1erg.

$$1 \text{ Joule} = 10^7 \text{ erg}$$

$$1J = 1N \times 1m$$
$$= 10^5 \text{ Dyne} \times 100cm$$
$$= 10^5 \times 10^2$$
$$1J = 10^7 \text{ Erg}$$

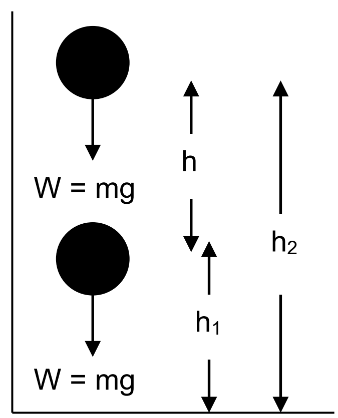
One Pound:

If one pound force produces a displacement of one foot in the direction of force then the amount of work is one foot pound.

Work done against gravitational field:

The gravitational force can do positive or negative work. When the body moves in the direction of the gravitational force, the work done is positive, where as when the body moves against the direction of the gravitational force, the corresponding work done is negative.

Consider a ball of mass 'm', which is initially at height h_1 from the surface of earth. The ball is moving upward and finally it reaches a height 'h_2' as shown in figure.



The total displacement covered by the ball ($d = h = h_2 - h_1$)

Force acting on the ball = $\vec{F}$ = W = mg.

As W = $\vec{F}.\vec{d} = Fd$ Cos180 = -Fd ($\because \theta = 180$)

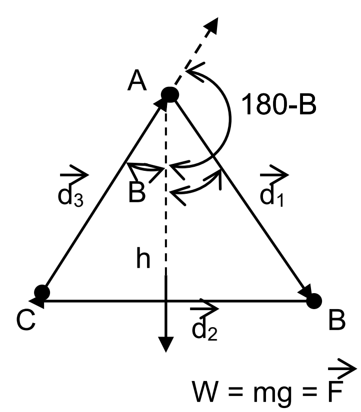
$$\boxed{W = - mgh}$$

Equation (i) shows that work done depends upon initial and final positions.

To Show That Workdone Between Two Points Independent Of Path.

OR

The Gravitational Field is Conservative Field



Consider a triangular path ABCA such that.

$$\vec{AB} = \vec{d}_1$$

$$\vec{BC} = \vec{d}_2$$

$\vec{C}A = \vec{d}_3$

The amount of work done in carrying the body from A to B, B to C and from C to A are represented by $W_{A \rightarrow B}$, $W_{B \rightarrow C}$ and $W_{C \rightarrow A}$ respectively.

Thus $W_{A \rightarrow B} = \vec{F}.\vec{D}_1 = Fd_1 \cos \propto mgh$ ($\because h = d_1 \cos\propto$)

$$\boxed{W_{A \rightarrow B} = mgh} \longrightarrow \text{(i)}$$

and

$W_{B \rightarrow C} = \vec{F}.\vec{d}_2 = Fd_2 \cos 90^O = O$

$$\boxed{W_{B \rightarrow C} = O} \longrightarrow \text{(ii)}$$

When the body is displaced from point C to A then the work done in the gravitational field is.

$W_{C \rightarrow A} = \vec{F}.\vec{d}_3 = Fd3 \cos(180 - B) = - F (d_3 \cos\beta)$

$$\boxed{W_{C \rightarrow A} = -mgh} \longrightarrow \text{(iii)} \qquad (\because d_3 \cos\beta = h)$$

Total work done along the closed path ABCA is.

$W_{A \rightarrow B \rightarrow C \rightarrow A} = W_{A \rightarrow B} + W_{B \rightarrow C} + W_{C \rightarrow A} = mgh + o - mgh = O$

$W_{A \rightarrow B \rightarrow C} + W_{C \rightarrow A} = O$

OR

$W_{A \rightarrow B \rightarrow C} - W_{C \rightarrow A}$

OR

$$\boxed{W_{A \rightarrow B \rightarrow C} = W_{A \rightarrow C}} \longrightarrow \text{(iv)}$$

Equation (iv) shows that the work done in the gravitational field is independent of the path followed.

OR

Equation (iv) shows that the gravitational field is conservation field.

Energy:

Anything which is able to do work is said to posses energy, and therefore, "Energy is the capacity to perform work."

The S.I unit of ENERGY is Joule (J)

Mechanical energy is divided into two types.

i. KINETIC ENERGY (K.E)

ii. POTENTIAL ENERGY (P.E)

Kinetic Energy

"The energy possessed by a body by virtue of its motion is called Kinetic energy.

A car moving along a highway has K.E of translation and a rotating machine has K.E of rotation.

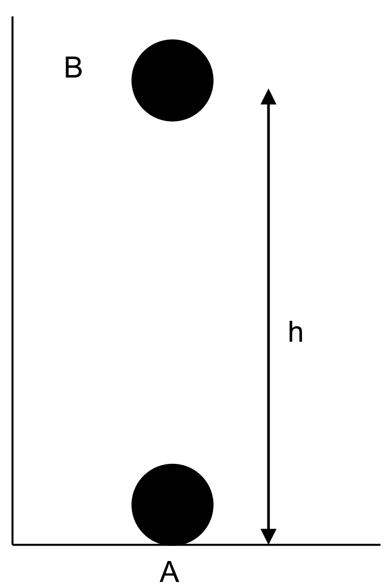
Consider a body of mass "m" which is thrown vertically upward with velocity "V" from position A. The body stops at a higher position B after covering a vertical distance "h" against the gravitational force.

The amount of work done by the body due to its motion = $\vec{F}.\vec{h}$ - Fh Cosθ

$\therefore$ K. E of the body = Fh ⟶ (i)

OR

K. E acquired by body = mgh



Now consider the motion of the body from A to B.

$a = -g$

$v_i = v$

$v_f = o$

$h = ?$

Using formula

$2as = V_f^2 - V_i^2$

$2(-g)h = (o)^2 - v^2$

$+ 2gh = \cancel{-}V^2$

$$\boxed{h = \frac{V^2}{2g}}$$

Now by substituting this value of h in equation (i)

We have

$$\boxed{K.E = mg\left(\frac{V^2}{2g}\right)}$$

OR

$$K.E = \frac{1}{2}mv^2$$

OR

$$K.E = \frac{1}{2}m\,(\vec{V}.\vec{V})$$

This shows that K.E is a scalar quantity.

Potential Energy

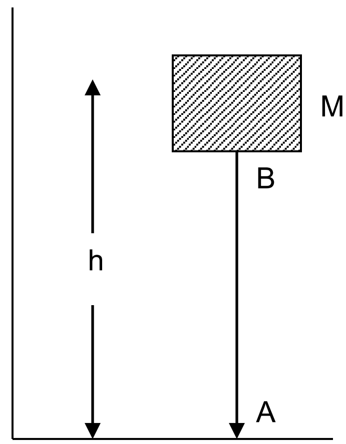
When a body is being moved against a field of force, as energy is stored in it. This energy is called **POTENTIAL ENERGY.**

If we compress a spring, an elastic potential energy is developed init. This energy is stored in it because a work is done in compressing the spring against the elastic force.

Suppose a body of mass is situated at port A. If it is lifted with very small speed vertically through a height ‘h’ the force required to rise the body is just equal and opposite to the weight (W = mg) of the body.

Thus work done on the body is

$W = \vec{F}.\vec{d}$ = Fd Cosθ = Fd



$W = mgh$ (∵ F = W and d = h)

This work is done in changing the position of the body, hence it is stored in the body as gravitational Potential energy.

$P.E = mgh$

Absolute Gravitational Potential:

The amount of work required in lifting a body fro a point ot infinity against the gravitational field is known as **Absolute Gravitational Potential**.

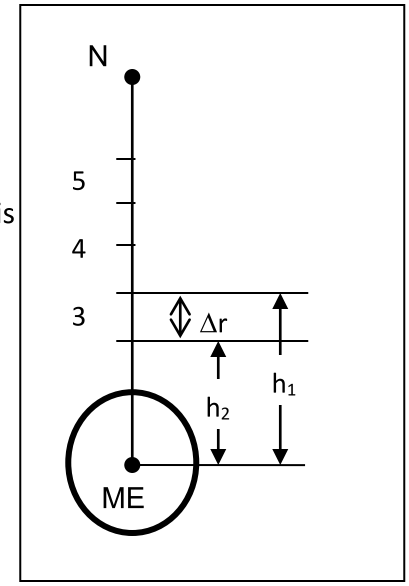
Consider a body of mass ‘m’ which is lifted from point 1 to point N in the gravitational field. The distance between points 1 and N is so large that the value of ‘g’ is not remains constant.

Divide the distance between the points 1 and N into small steps of equal length Δr. The internal Δr is taken so small that the value of ‘g’ remains constant during this interval.

The total work done in all these steps.

Lets us calculate the work done during the first step i.e. in displacing the body from point 1 to 2. If r_1 and r_2 are the distances of the points 1 and 2 respectively from the centre of the earth, then

$$r = \frac{r^1 + r^2}{2}$$



If ME is the mass of the earth, the gravitational force at the centre of this step is

$$F = \frac{GMEm}{r^2} \qquad \leftarrow (4)$$

But

$$r^2 = \left(\frac{r_1 + r_2}{2}\right)$$

As $\Delta r = r^2 - r^1$

$r2 = \Delta r + r^1$

$$\therefore\ r^2 = \left(\frac{2r1 + \Delta r}{2}\right)^2 = \left(\frac{4r_1^{\ 2} + 4r_1\Delta r + \Delta r^2}{4}\right)$$

As $(\Delta r)^2 < <r_1^2$, this term can be neglected as compound to $r1^2$ then

$$r2 = \frac{\cancel{4}r_1(r_1 + \Delta r)}{\cancel{4}} = r_1 r_2$$

$$r_2 = r_1\ r_1$$

Hence the equation (i) becomes

$$F = \frac{GM_E m}{r_1 r_2}$$

As this force is assumed to be constant during the interval Δr, the work done is

$$W_1 \rightarrow 2 = F\,\Delta\, r = \frac{GM_E m}{r_1 r_2}(r2 - r1)$$

$$W1 \propto = GM_E m \left(\frac{1}{r1} - \frac{1}{r2}\right)$$

Similarly, the work done in lifting the body from point 2 to 3, 3 to 4 and so on is W_{23} = GmME $\left(\frac{1}{r2}-\frac{1}{r3}\right)$

$W_{(n-1)n}$ = GMEm $\left(\frac{1}{r_{n-1}}-\frac{1}{rn}\right)$

Hence the total word done by the applied force M lifting the body from point 1 to N is.

$W = W_{12} + W_{23} + W_{34} + --- + W_{(n-1)n}$

$$W = GmM_E\left(\frac{1}{r1}-\frac{1}{r2}+\frac{1}{r2}---+\frac{1}{rn}\right)$$

$$\boxed{W = GmM_E\left(\frac{1}{r1}-\frac{1}{rn}\right)}$$

If the point N is situated at an infinite distance from the earth, rn = $\propto$ and $\frac{1}{rn}=0$

$$\boxed{W = \frac{GmME}{r1}}$$

When this work done, the body attains to infinity and the potential energy of the body becomes zero. Thus at point 1, the body had negative P.E. This P.E is called absolute P.E. Thus.

$$(P.E)_{abs} = -\frac{GmME}{r1}$$

OR $\boxed{(P.E)_{abs} = -\frac{GmME}{r}}$

If the point lies at the surface of earth then

$$(P.E)_{abs} = -\frac{GmME}{RE}$$

An approximate value of $(P.E)_{abs}$ at height 'h' abs the surface of earth is

$$(P.E)_{abs} = -\frac{GmME}{RE}\left(1-\frac{h}{RE}\right)$$

Escape velocity

Escape velocity is the minimum velocity required by a body to be projected to overcome the gravitational pull of the earth. It is the minimum velocity required by an object to escape the gravitational field that is, escape the land without ever falling back.

$$v = \sqrt{\frac{2Gm}{R}}$$

$$V_{esc} = \sqrt{2gR}$$

Power:

Power is defined as "the rate of doing work" OR "The rate at which work is being done."

$$\text{Power} = \frac{Work}{Time}$$

Average power (P_{avg})

Average power of a body is defined as the total work done divided by the total time taken to perform the work.

Mathematically we can write

$$P_{avg} = \frac{W}{t}$$

Instantaneous power is defined as "Power of a body at certain instant of time."

Mathematically we can write

Expression for the average power
As

$$P_{ins} = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t}$$

Power is Dot product of force and velocity

We know that

$$W = \vec{F} . \vec{d}$$

$$P = \frac{\vec{F}.\vec{d}}{t} = \vec{F}.\frac{\vec{d}}{t}$$

$$P = \vec{F}.\vec{V} \qquad \left(\because \frac{\vec{d}}{t} = \vec{V}\right)$$

$\therefore$ Power is the scalar product of force $\vec{F}$ and velocity, there force power is scalar quantity.

S.I unit of work is Joule and that of time is second. So the corresponding unit of power is joule / second, which is called Watt, doing one joule of work in one second."

| Watt = 1J/1S

The bigger units of power are

1 KW = 10^3 Watt
1 MW = 10^6 Watt
1 GW = 10^9 Watt

Horse Power (hp)

In the British Engineering system, the unit of power is Ft. Lb. Since this unit is quite small, therefore second a bigger unit horse power has been adopted.

1hp = 550 Ft x Lb/Sec and

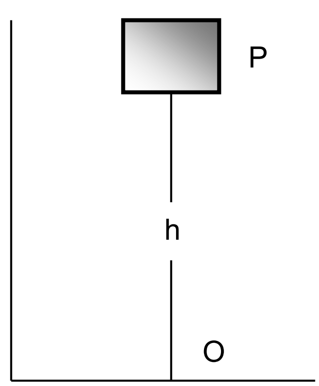
1hp = 746 Watt.

INTERCONVERSION OF POTENTIAL ENERGY AND KINETIC ENERGY

A swinging pendulum bob is an example of a body whose energy can be either Kinetic or potential or a mixture of both.

Now consider a body of mass 'm' at a height 'h' above the ground level. The body at point P has zero K.E because it is a rest i.e. v = o. During it downward motion, its velocity increases and so there is an increase in K.E. If there is no friction, then

Loss of P.E = Gain in K.E ←(i)



In Practice there is always a force of friction f opposing the downward motion of the body. Here a fraction of the P.E is used up in doing work against the force of friction Now equation (i) can be written as

Loss of P.E = Gain in K.E + Work done against friction

OR

Gain in K.E = loss of P.E – fh ← (ii) ($\because Workdone = fh$)

Equation (ii) is called work energy equation.

LAW OF CONSERVATION OF ENERGY

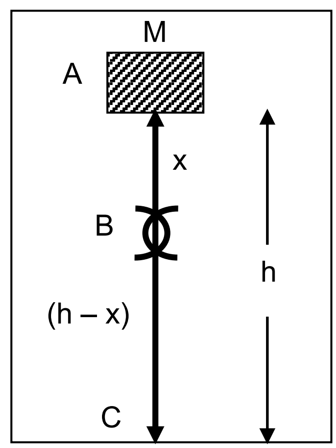
According to this Law

"The energy can neither be created nor can it be destroyed. But it can be changed from one form to another."

OR

The total energy of the system is always remains constant.

K.E + P.E = Constant



Consider a body of mass 'm' at point A at a height 'h' above the ground.

CASE I

AT POINT 'A'

The P.E of the body at A = mgh It K.E at point A = O

$\therefore$ Total E at A = P.E + K.E

Hence

E = mgh ← (i)

CASE II

AT POINT '**B**'

Now the body is allowed to fall freely under the gravity. Let the body b at point 'B' at any instant.

The P.E at point 'B' = mg (h – x)

$P.E = mgh - mgx$

Consider the motion of body from A to B

$V_i = O$

$V_f = v$

$a = g$

$s = x$

Using formula

$V_f^2 - V_i^2 = 2as$

$V^2 - O^2 = 2gx$

$\boxed{V^2 = 2gx}$

Now K.E at point B = $\frac{1}{2}mv^2 = \frac{1}{2}mx2gx = mgx$

$K.E = mgx$

∴ Total E at 'B' = P.E + K.E

$E = mgh - mgx + mgx$

$E = mgx \leftarrow$ (ii)

CASE III

AT POINT '**C**'

The P.E of the body at point C = O (∵ h = o) If the velocity of the body at C is V, then $V^2 = 2gh$

∴ K.E at C = $\frac{1}{2}mv^2 = \frac{1}{2}m \times 2gh = mgh$

Hence

The total energy 'E' at C = P.E + K.E

$\boxed{E = mgh}$ ← (iii)

Equation (i), (ii) and (iii) shows that the energy of a closed system is always remains constant.

UNIT 06 FLUID STATICS

FLUID:-

A Substance which capable of flowing from place to another and is always confined to the boundaries of any container is known as fluids.

Liquid and gases both are included in fluids, they are present in auto mobiles tire, gas tank, radiator, combustion system, air condition system, wiper reservoir lubricating system.

Fluids play a vital role in our life, we breathe and drink them rather vital fluids circulates in our body as blood in our cardiovascular system.

All the fluids exert a force perpendicularly on the walls of the container in which they are kept.

It must keep in mind that glass and pitch, are classified as fluids and they are known as highly cooled fluids.

PASCAL'S LAW:-

"If a pressure is exerted on a liquid, the liquid will transmit this pressure equally in all directions".

This law can be explained experimentally by taking a vessel which is connected with four similar pistons as shown in the diagram. Fill it with water and note down the position of every piston. Apply some pressure in one of the piston; we will observe that pistons will go out equally. This shows that water transmits the pressure equally in all directions.

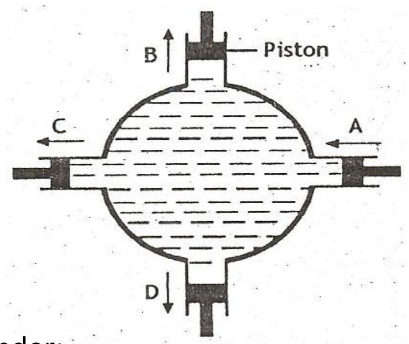


Fig. 10.6: Pascal Principle

Applications of Pascal's Law

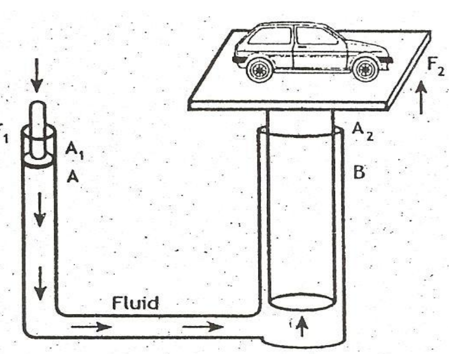
Some important and useful applications of Pascal's Law are given as under:

HYDRAULIC LIFT

It consists of two cylinders one is of smaller area and other is of greater area. Both the cylinders are fitted with air tight pistons and connected with each other with the help of the tube filled with incompressible fluids.

A pressure is applied downward on a narrow piston which transmits the pressure equally according to Pascal's Law on wider piston. Hence to keep the pressure equal, force will increase and wider piston rises up along with the car resting on it.

Fig. 10.7: A hydraulic lift is based on the fact that the pressure due to a small force applied at a narrow tube is equal to the pressure caused by a large force applied at a wide tube.



- It is used to lift heavy vehicles such as cars and buses for repairing purpose.
- It is used to lift vehicles for service purpose.

HYDRAULIC BRAKES

Hydraulic brakes are based on the principles of Pascal's Law. It consists of a master cylinder joined by tubes to four smaller cylinders. These are called brake cylinders, all the cylinders are provided with oil tight pistons.

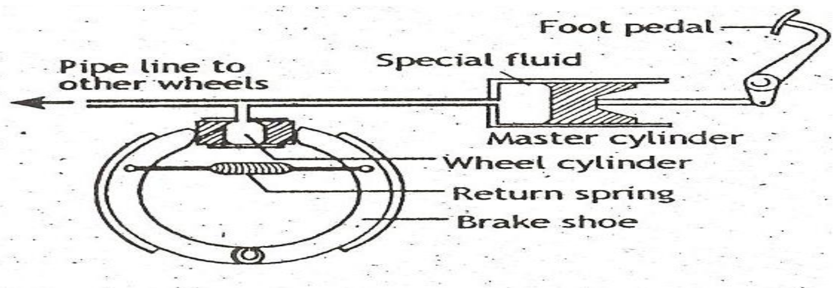


Fig. 10.8: Essential parts of a hydraulic brakes and their working

A forward push on the brake pedal applies a force on the piston of master cylinder. This pressure is transmitted to every brake piston equally according to Pascal's Law.

This force increases considerably because area of the brake piston is greater, this greater force opens the brake shoes and due to friction in between brake shoe and drum of wheel the automobile stops. When the pressure on the brake is released the spring which connects the two brake shoes contracts and pulls them away from the rotor. The wheel is then free to rotate.

- It is used to stop the vehicles.
- It is used to control the speed of the vehicles.

ARCHIMEDES PRINCIPLE

"When an object is immersed in any liquid, an up thrust acts upon it which is equal to the weight of the liquid displaced by the object".

Up thrust = weight of liquid displaced.

Verification and analytical treatment of Archimedes Principle:

To verify the above statement we consider a uniform solid cylinder of cross section 'A' submerged in a liquid of density 'ρ'. Let its upper face is at a depth 'h_1' and the lower one at 'h_2'. If 'Pa' is the atmospheric pressure then,

One force acting downward on the upper face = $Ah_1\rho g + A\,Pa$

One force on the lower face = $Ah_2\,\rho g + A\,Pa$

Up thrust = Total upper thrust – Total lower thrust

$= Ah_2\rho g - Ah_1\rho g$

$= A\rho g\,(h_2 - h_1)$

$= A\, h\, \rho g\ (h_2 - h_1 = h)$

$= V\rho g\ [Ah = V]$

$= Mg\ [\ \text{Vol} \times \text{density} = \text{mass}\ ]$

$= W\ (w = mg)$

'W' is the weight of liquid displaced by the cylinder which is equal to the net up thrust. Hence Archimedes Principle is verified.

Apparent weight:

Due to up thrust, an object immersed in a liquid has an apparent weight which is given by following formula.

Apparent weight = Actual weight – Weight of the liquid displaced by the object.

This means an object loses its weight in a liquid. This is the reason, why it is easier to lift an object while in water than when it is above water.

BUOYANCY:

When an object is immersed in a liquid, it displaces liquid equal to the volume of the object immersed in the liquid and the object experiences a force up-ward to the weight of displaced liquid. This upward force is called up thrust or Buoyancy.

LAW OF FLOATATION

A floating body must displace liquid equal to its own weight.

According to the law of floatation:

Buoyancy = weight of displaced liquid.

Hence we can define the above law as follows.

"If Buoyancy is equal to the weight of the body then it will float".

Submarine can float and can sink. This is because of large hollow ballast tanks. Normally these tanks are empty and weight of sub marine is less and it can displace liquid equal to its weight hence it floats.

When it wants to sink then water is allowed to enter into tanks which make the submarine heavy, volume remains the same so it cannot displace liquid equal to its weight and sinks down.

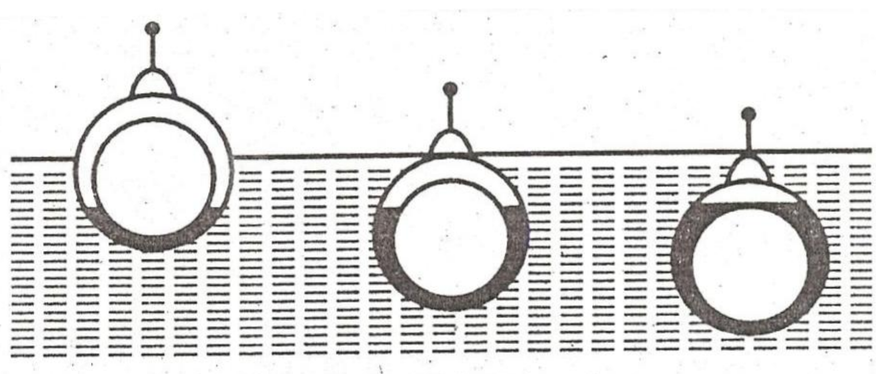


Fig. 10.11: Diving Submarine

Conditions for floatation:

When a body is immersed in liquid, its weight acting downward and the up thrust of the liquid acts upward.

1. If the weight of the body is more than the up thrust, it sinks.
2. If the up thrust is more than the weight of the body, it floats.
3. If the up thrust and the weight of the body become equal, it will neither sink nor float but remain in liquid.

SURFACE TENSION

Surface tension is the property of a liquid by virtue of which the free surface of the liquid behaves like a stretched-membrane tending to decrease the surface area.

Surface tension can also be defined as:

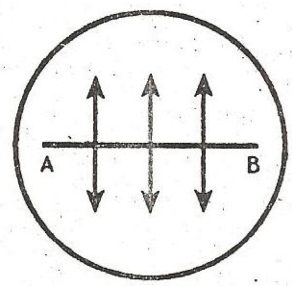
"The force per unit length acing on either side of the imaginary line".

The direction of this force is tangential to the surface and perpendicular to the imaginary line. If 'F' is the force acting on a length 'L', then the surface tension 'T' is given by:

$$\sigma = \frac{F}{L}$$

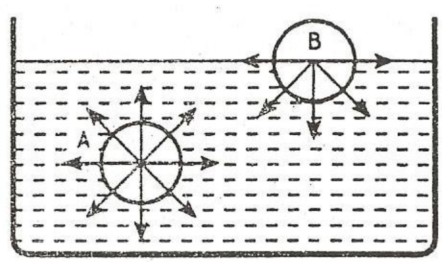
In the system international the unit of surface tension is N/m.

Fig. 10.12: Imaginary line drawn on the liquid. The molecules are pulling away from each other on the two sides of the line.



Consider a molecule 'A' of a liquid lying well inside it and a molecule 'B' on the surface of the liquid as shown in the figure. The molecule 'A' is attracted by all the molecules lying in its closest neighbour called the sphere of influence of radius 10^{-7} cm. the resultant force acting on it is zero.

Fig. 10.13: Molecule A lying in the interior has no resultant force acting on it. The resultant force acting on molecule B lying on the surface is downward.



Now consider the molecule 'B'. This molecule is acted upon by the molecules on the surface and those below the surface. Here the sphere of influence is half of the sphere as shown in figure. Due to the downward forces acting on the surface molecules, the free surface of the liquid behaves like a stretched membrane.

Examples:

a) In general a steel needle is dropped in water will sink because the density of the steel is greater than water, if the needle is slightly oily and then placed horizontally on the surface of water it will float leaving a depression on the surface of water that is because of surface tension will not make to sink in water.

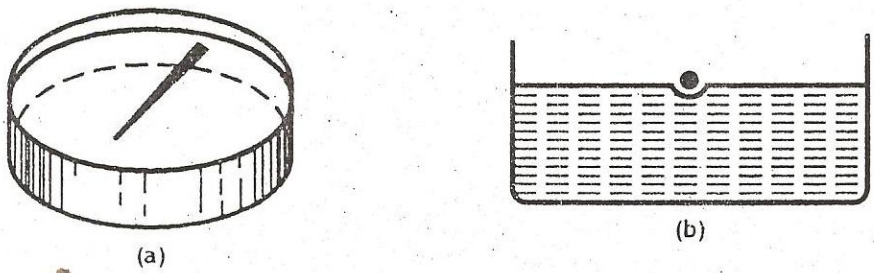


Fig. 10.14

b) A metallic ring is dipped into a soap solution and a light moist loop of cotton thread of any shape is gently placed over the soap film due to surface tension.

c) When a drop of olive oil is dropped in the mixture of spirit and water (the density of mixture being equal to that of the olive oil) the drop of olive oil goes inside the mixture like a spherical droplet, this shape of drop is due to the surface tension.

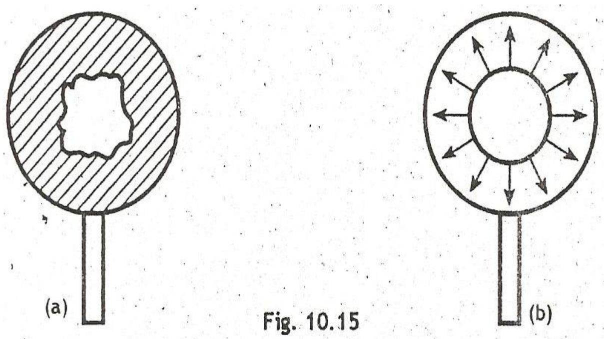


Fig. 10.15

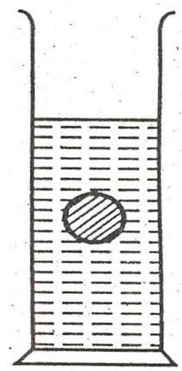


Fig. 10.16

d) You may have observed water droplets falling from a tap; the drops are spherical in shape and are due to the surface tension that minimizes the surface of the drops similarly.

<u>CAPILARITY</u>:-

A Glass tube of small bore is dipped into water; it rises up the tube a few centimeters. As tube becomes narrower, there is more rise up. Adhesion force between water and glass exceeds cohesion force between water molecules, the meniscus causes up and surface tension. This effect is called capillarity or capillary action. The height of column can be calculated by:

$$h = \frac{2T}{\rho r g}$$

UNIT 07 FLUID DYNAMICS

FLUID FRICTION:-

. In our daily life we observe that thin liquids like water, alcohol, sprit etc flow readily where as thick liquids, like coal tar, castor oil, glycerin flow more slowly under similar conditions. The liquids of the second kind are said to be more viscous than the liquids of the first kind.

As there is friction between two solid surfaces tending to oppose their relative motion when one is made to slide over the other, there is also friction between two layers of even the same liquid (in general fluids) when they are in relative motion.

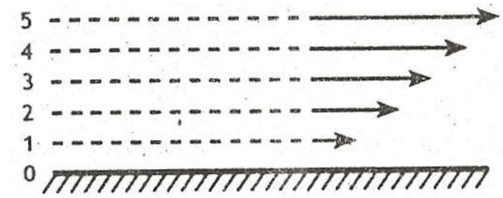


Fig. 10.17: Solid Surface

Consider two layers lying at a distance x from a stationary surface. Let 'A' be the area of the layer moving with a velocityυ. It is found that the backward dragging force F acting on any layer is found to vary as:

$$F \propto A$$

$$F \propto \upsilon$$

$$F \propto \frac{1}{x}$$

$$\therefore \quad F = -\eta \frac{A\upsilon}{x}$$

Where η is a constant depending upon the nature of the liquid and is known as the coefficient of viscosity. Negative sign shows that the dragging force acts in a direction opposite to the flow of the liquid.

The unit of coefficient of viscosity is "poise" after Poisenille. The coefficient of viscosity of liquid is said to be of 1 poise if the backward dragging force required to maintain a relative velocity of 1m/s between two layers each of area $1m^2$ and separated by a distance of 1m is 1N. Another commonly used unit of coefficient of viscosity is centipoises so that:

$$1 \text{ centipoises} = \frac{1}{100}\text{poise}.$$

UNIT 08 ELECTRC FIELDS

CHARGE:

"Charge is a basic property of the elementary particles by which they attract or repel in an electric field."

There are two types of charge, one positive charge and second negative charge.

POSITIVE CHARGE BODY:

"The body which loses electrons considered as positively charged body"

NEGATIVE CHARGE BODY:

"The body which gains electrons considered as negatively charged body. Like charge repel each other and unlike charges attract each other.

Properties of charges

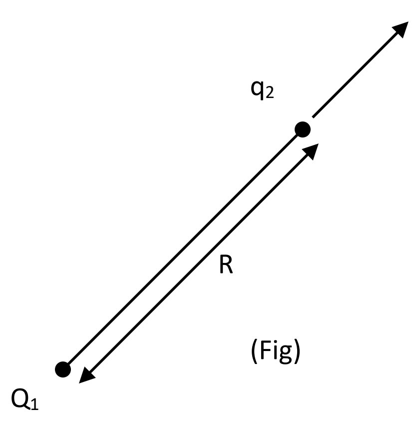
1. Charges is a fundamental property of a material body that determines whether or not it attracts or repels another object.
2. Two distinct types of charge are produced by friction on two distinct materials (glass and plastic).
3. Charges are identical to one another always repel one another.
4. Charges that are not similar to one another always attract one another.
5. The only reliable indicator of charges on a body is repulsion.

COULOMB'S LAW:

It is the law formulated by Coulomb in 1784 about the force of attraction or repulsion between two point charges separated by some distance with the help of torsion balance.

The force of attraction OR Repulsion between two point changes is directly proportional to the product of the magnitude of the charges and inversely proportional to the square of distance between then.

Consider q_1 and q_2 are two point charges separated by distance 'r'



(Fig)

According to the statement of the law.

$F \propto q_1 q_2$

And $F \propto 1/r^2$

OR

$F \propto \frac{q_1 q_2}{r^2}$

OR

$$F = \frac{K q_1 q_2}{r^2} \_\_\_\_\_\_\_\_\_\_ (1)$$

Equation (1) is called Mathematical form of Coulombs law. Where K is proportionality constant whose value depends upon the medium between the two charges and the system of unit used In mks system the value of K is.

$$k = \frac{1}{4\pi \in_0}$$

Vectorial form of Coulomb's Law.

As force is a vector quantity. In order to indicate the direction of force we use a unit vector $\hat{r}_{12}$.

Now coulomb's law becomes.

$$F = \frac{1}{4\pi\epsilon} \frac{q_1 q_2}{r_2}$$

EFFECT OF DIELECTRIC MEDIUM ON COULOMB'S LAW

When free space replaced y some dielectric medium then the Coulomb's force is decreases by some constant factor TR

$$\therefore \vec{F}_{12} \frac{1}{4\pi\varepsilon r} \frac{q_1 q_2}{r^2} \hat{r}_{12} \_\_\_\_\_\_\_\_\_\_ (4)$$

Where $\in_r$ is a relative permittivity of the medium. The unit of charge is coulomb.

ELECTRIC FIELD:

Electric field is a region around a charge body in which another charge experiences an electric force.

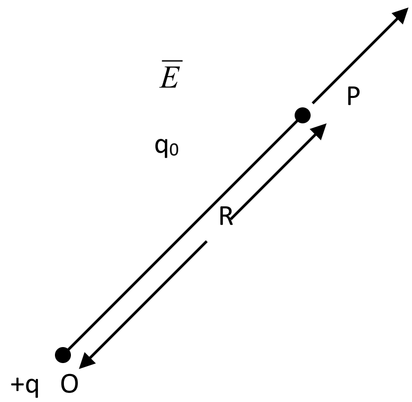
ELECTRIC INTENSITY:

The strength of the electric field is known as electric intensity. The electric intensity of the file at any point is defined as the force experimented by a unit positive charge placed at that point. It is a vector quantity; its direction is the same as that of the electric force.

$$\vec{E} = \frac{\vec{F}}{q}$$

The S.I unit of electric intensity is $\frac{N}{C}$ OR NC^{-1}

Electric intensity near an isolated Point charge q.



Force on q_o due to q is

$$\vec{F}\frac{1}{4\pi\ t0}\frac{qq_0}{r^2}\hat{r}$$

$\therefore$ Electric field intensity is

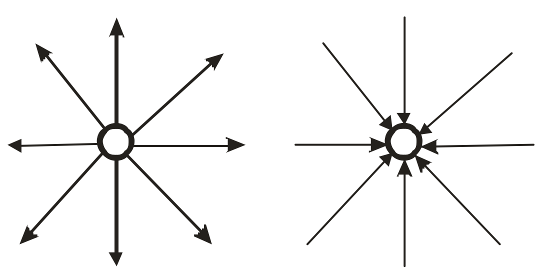
$$\vec{E}\frac{1}{4\pi\ t\ o}\frac{q}{r^2}\hat{r} \qquad (\because \vec{E}=\frac{\vec{F}}{q_0}).$$

ELECTRIC LINES OF FORCE:

These are the lines which originate from positive charge and terminate at negative charge. They are invisible but they can be traced.

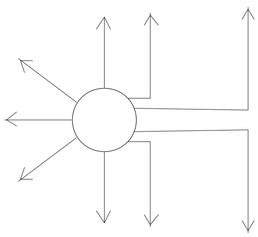
The electric field depends upon the electric lines of force. Electric field and electric lines of force are directly proportional and electric lines measure the electro intensity of the field.

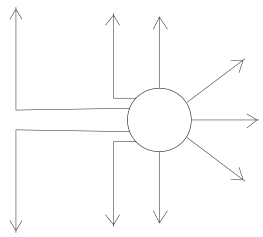
Electric fields due to different charge configurations are visualized by the following gig (I to VI).



(i) (ii)

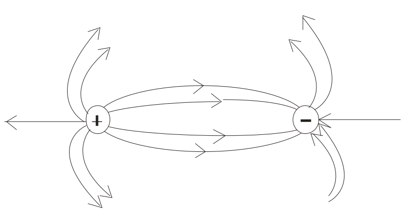
(Around a positive charge) (Around a negative charge)

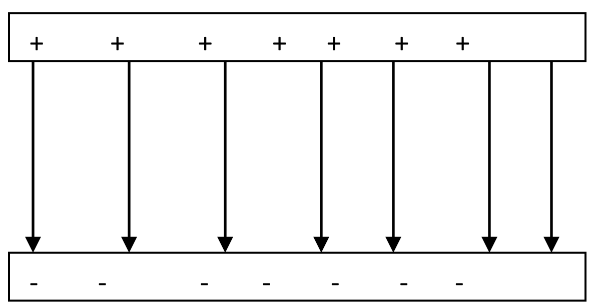




(iii) (IV)

(Around two like charges)





(B/w two oppositely charged plates).

ELECTRIC FLUX

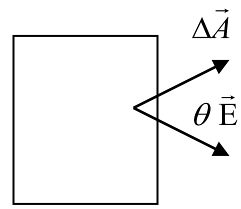
Definition: The electric flux passing through a given area is the total lines of force passing through it perpendicularly. It is represented by ϕ e

Math thematically, it is the dot product of electric intensity and the vector area.

$\Delta\phi e = \vec{E}.\Delta\vec{A}$

OR

$\Delta\phi e = E\Delta A \cos\theta$ _________ (1)



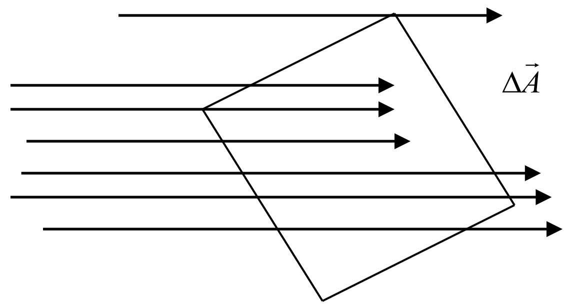
PARTICULAR CASES:

1. When the direction of the vector area is along the electric intensity then

$\Delta\phi$ e = $\vec{E}.\Delta$ EΔ ACos 0

$\Delta\phi$ e = $\vec{E}\Delta A \times 1$

$\Delta\phi$ e = EΔA



This shows that maximum flux is passing through the area.

2. When the direction of the vector area is perpendicular to the electric intensity then

$\Delta\phi$ e = $\vec{E}.\Delta A \Delta =$ EΔ A Cos 90

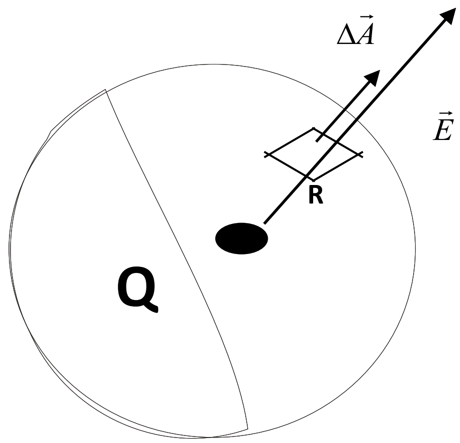
$\Delta\phi$ e = E ΔA (o)

$\Delta\phi$ e = O

This shows that no flux is passing through the area.

ELECTRIC FLUX THROUGH THE SURFACE OF A SPHERE OF RADIUS "r" WHICH HAS POINT CHARGE "q" AT ITS CENTRE.

Consider a small patch area $\Delta\vec{A}$ of the surface of the sphere.



Let $\Delta\phi$ e be the flux through this patch, then

$\Delta\phi$ e = $\vec{E}.\Delta\vec{A} = \Delta A \text{ Cos o} \qquad (\because \theta = 0)$

$\therefore \Delta\phi$ e $= E\Delta A$

Flux through the surface of the sphere is given by

ϕe = $\Sigma \Delta \phi$ e = $\Sigma E\Delta A$

OR

ϕe = $E\Sigma \Delta A$ _________ (i) ($\therefore$ E is constant at any point of the surface)

As $\Sigma \Delta A$ represents the total surface area of the sphere so its value is

$\Sigma \Delta A = 4 \pi r^2$

And E = $\dfrac{1 \qquad 9}{4ƛ \text{ to } r^2}$

Now eq: (I) becomes

$\phi e = \frac{1}{4\pi \text{ to}} \frac{1}{r^2} \times 4r^2$

$\phi e = \frac{q}{t0}$ ____________ (ii)

GAUSS'S LAW:

Statement:

The total flux through any closed is always equal to $\frac{1}{\text{to}}$ times the total chare enclosed by that surface.

Mathematically we can write

$\phi e = \frac{Q}{\text{to}}$

Proof:

Consider a closed surface enclosing a point charge q^1. Imagine a sphere with q_1 at its centre such that its surface lies wholly into. It is clear from the fig. that fluxes through the closed surface same as through the sphere. Thus flux through S due to a point charge will be

$\phi e = \phi_{1e} + \phi_{2e} + \ldots\ldots\ldots\ldots\ldots + \phi_{Ne}$

$\phi e \ \frac{q_1}{t\text{o}} + \frac{q_2}{\text{to}} + \ldots\ldots\ldots\ldots + \frac{qn}{t\text{o}}$

$\phi e = \frac{1}{\text{to}} \times (q_1 + q_2 + q_3 \ldots\ldots\ldots + qn).$

OR

$\phi e = \frac{1}{\text{to}} \times Q$

Which is Gauss's law

ELECTRIC POTENTIAL

Potential difference:

The potential difference between two points is defined as the change in electric potential energy per unit charge between two points, it is represented by Δ V.

OR

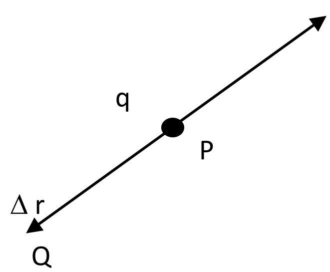
The amount of external work done required in moving a unit positive charge from one point to another against the electric field.

Consider a charge q is placed at point P. let this charge be taken against the electric intensity through a very small distance$\Delta\gamma$.

$$\Delta W = \vec{F}.\Delta \vec{r}$$

$$\Delta W = q\vec{E}.\Delta \vec{r}$$

This work appears as change in electric potential energy Δu.



$$\Delta u = q\,\vec{E}\,.\,\Delta \vec{r}$$

OR

$$u\,Q - up = q\vec{E}\;.\Delta \vec{r}$$

OR

$$\frac{q\,\mathrm{u}}{\mathrm{q}} = \vec{E}.\Delta\,\vec{\mathrm{r}}$$

OR

$$\Delta\,\mathrm{V} = \vec{\mathrm{E}}\,\Delta\,\vec{\mathrm{r}}$$

Unit:

The unit P.D is volt. One volt can be defined as. “P. D is 1 v if the work done per unit charge in moving the test charge (1 coulomb) between these points is 1 joule

ABSOLUTE ELECTRIC POTENTIAL

The absolute potential at a point in an electric field is defined as the work done in moving a unit positive charge from infinity to that point.

ELECTRIC POTENTIAL NEAR AN ISOLATED POINT CHARGE “Q”

Consider two point A and B in a straight line at distances r_A and Rb respectively from a point charge “q” As

E = $\frac{1}{u\pi\,\mathrm{to}} \times \frac{\mathrm{q}}{\mathrm{r}^2}$

The value of electric intensity changes with the distance, there fore AB is divided into large no. of equal steps each. Of which is represented by Δr.

The value of electric intensity change with the distance, there fore, AB is divided into large do of equal steps each of which is represented by Δr.

The work in moving coulomb of charge from A to 1 is

$\Delta W_1 = \vec{F}.\Delta\vec{r} = f\,\Delta r\,\cos 180$

$\Delta W_1 = q_o E\,\Delta r$ __________ (i)

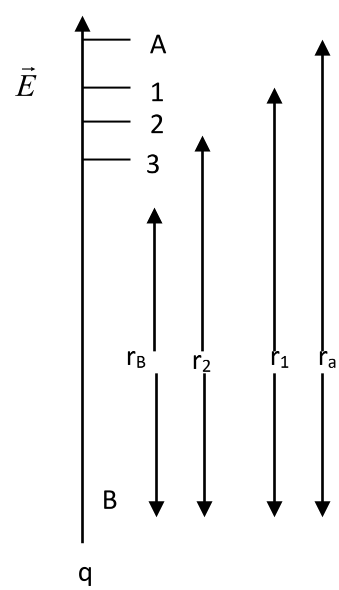
$$V_a - V_1 = \frac{\Delta w_1}{q_o} = -E\Delta r \text{ \_\_\_\_\_\_\_\_\_\_\_ (i)}$$

But Δ r rA – r_1 ___________ (ii)

Let 'r' is the mean distance between r_A and r_1 then

$r = \frac{r_a + r_1}{2}$

$r^2 = (\frac{r_a + r_1}{4})$.



From Equation (ii) $r_A \Delta r + r_1$

$\therefore$ $r^2 = \frac{(2r1 + \Delta 2)}{4} = \frac{4r12 + 4\,r1\,\Delta}{4}$

$r^2 = 4r_1 \frac{(r_1 + \Delta r)^2}{4}$ ($\because$ Δ r is very small)

$r^2 = r_1 r_A$

Noe eg (i) becomes.

$V_A - V_1 = -\frac{Kq}{r_A r_1}(r_A - r_1)$

$$\because \begin{pmatrix} \Delta r = r_A - r_1 \\ r^2 = r_1 r_A \\ E = \frac{kq}{r^2} \end{pmatrix}$$

$V_A = V_1 = + Kq\left(\frac{1}{r_1} - \frac{1}{r_A}\right.$ ____________ (iii)

In the next step .

Δ V_2 = Kq $\left(\frac{1}{r_A}-\frac{1}{r_1}\right)$

OR

Δ V_1 = Kq $\left(\frac{1}{r_N}-\frac{1}{r_B}\right)$

P.d b/w an nd B Δ V =Δ V_1+ Δ V_2 + ………… + Δ V_N.

$\therefore$ P.D b/w A and B = V_B –V_A = Kq ($\frac{1}{r_B}-\frac{1}{r_A}$)

Δ V =Kq $\left(\frac{1}{r_B}-\frac{1}{r_A}\right)$__________$(iv)$

If point A is at infinity, so VA= 0 and $\frac{1}{r_A}=0$

$\therefore$ $V_A=\frac{Kq}{r_A}$ OR V_A $\frac{1}{4\pi to}\frac{q}{r_A}$

Similarly at distance ‘r’ V= $\frac{1}{4\pi to}\frac{q}{r}$________$(v)$

From equation (v) we can cellule absolute potential at any point due to charge (q).

RELATION BETWEEN ELECTRIC FIELD AND POTENTIAL

Consider two points a and b that are separated by a small distance ΔS on a line of force PQ. The field is practically constant over the small distance ΔS.

Work done by the field = ΔW = $\vec{F}.\vec{\Delta S}$

ΔW = F ΔS cos 180 = FΔS (-1)

($\therefore$ Work done by an out side agent P in moving a positive charge against the field is taken) +ve and the work done by the field is taken –ve.

ΔW - FΔS – q_o E ΔS.

OR

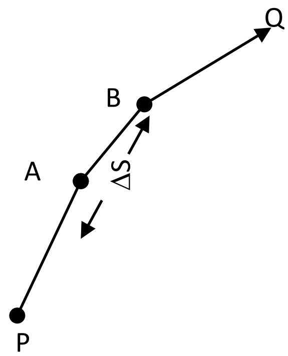
$$\frac{\Delta W}{q_o} = -E\Delta S$$

OR

ΔV = - EΔS

OR

$E = -\frac{\Delta V}{\Delta S}$ ____________ (i)



Component of field along x, y and z- axis is given by.

$E_x = -\frac{\Delta V}{\Delta x}, Ey = -\frac{\Delta y}{\Delta y} \text{ and } Ez = -\frac{\Delta V}{\Delta Z}$

ELECTRON VOLT (Ev):

One electron volt (eV) is the amount of energy acquired OR lost by an electron when it is displaced across two points between which potential differences is one volt.

$1eV = (1.6 \times 10^{-19})$ cx 1 volt

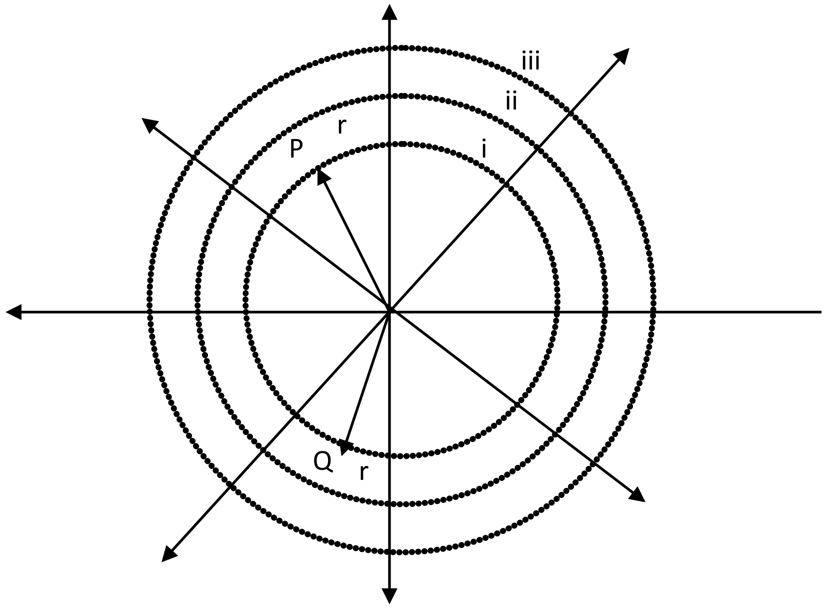
$1eV = 1.6 \times 10^{-19}$ Joules.

Electron volts is a unit of energy, in atomic physics, the energies of accelerated fundamental particles are expressed in term eV.

EQUIPOTENT SURFACES:

In an electric field there are points which have the same value of potential. A surface passing through such points is known as equipotent.

A charge can move upon such a surface without doing any work.



For a point charge the potential at point p is

$$V_1 = \frac{1}{4\pi \, t\, o}\frac{q}{r}$$

Similarly a point Q on the this surface

$$V_2 = \frac{N}{4\pi \, to}\frac{1}{4} - V_1$$

Thus all the points lying on the surface II with the point charge "q" at the centre and "r" as the radius will have the same values of potential... Hence the surface of this sphere will be an equi potential surface.

UNIT 09 CAPACITORS

CAPACITOR:

"A device which is designed to store the charge is called capacitor.

PRINCIPLE:

When an amount of charge is given to one plate of the capacitor another amount of charge is induced due to electric field of this charge or due to force of attraction of this charge by means of electrostatic induction.

CONSTRUCTION:

A simple capacitor consists of two parallel metallic plates having a distance between them. It has insulating material between them, which is dielectric.

WORKING:

When a amount of charge positive is supplied to one plate of the capacitor it produces an electric field due to which negative charge is attracted and they are stored in other plate. In other words a potential difference is developed across the capacitor plate.

Suppose the magnitude of charge which is stored in a capacitor is Q.

A potential difference "V" appears across the plates.

As potential difference is directly proportional to the magnitude of charge stored in a capacitor in a capacitor therefore

$$Q \propto V$$

$$Q = CV$$

Where C is constant of proportionality which is known as capacitance or capacity of a capacitor.

UNIT OF CAPACITANCE:

The unit of capacitance is fared (F). Farad is a big unit there sub multiples unit.

1 microfarad	=	10^{-6} F
1 nanofarad	=	10^{-9} F
1 picofarad	=	10^{-12} F

Factor Affecting the Capacitance: - Capacitance of a capacitor depend upon three factor which are following

1) ***Medium Between the Plates: -*** If an insulator is introduced between the plates, its capacity to store the charge increases.
2) ***Area of the plates: -*** By increase the area of plates of capacitor increase the capacity of capacitor.
3) ***Distance between Plates: -*** The capacity of a capacitor also depend upon the distance between the charges by decreasing the distance, the capacity of capacitor also increase.

CAPACITANCE OF A PARALLEL PLATE CAPACITOR:

Consider a parallel plate capacitor consisting of two parallel metal plates each of area A, separated y small distance "d" consider air or vacuum present^{-1} as a medium between the plate.

Then

$$C = \frac{Q}{V} \_\_\_\_\_\_\_\_ (i)$$

Where

Q = charge on the capacitor

V = potential difference between plates.

But

E= $\frac{V}{d}$ or V = Ex d

And

The magnitude of electric intensity between, two oppositely charged plates is.

E = $\frac{\sigma}{to}$

OR

E = $\frac{Q}{\text{to A}}$ $\left(\because \sigma \frac{\text{Q}}{\text{A}}\right)$

OR

Q = E to A.

By substituting value of Q S v in equation (i), we have

$\theta \frac{E \text{ x } d}{E \text{ t.}}$ C= $\frac{E \text{ x to x A}}{E \text{ x d}} = \frac{A \text{ to}}{\text{d}}$

$$\therefore C = \frac{\text{A to}}{\text{d}} \_\_\_\_\_\_\_\_\_(ii)$$

Equation (ii) shows that the capacitance is directly proportional to the area of plates and inversely proportional to the distance between the plates.

WHEN DIELECTRIC IS SUED:

It an insulating material of relative permittivity ε_r is **present** between the plates, the capacitance of the capacitor is increased by ε_r.

Now equation (ii) becomes.

$$C^1 = \frac{\text{A to tr}}{\text{d}} \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_} (iii)$$

Dividing equation (iii) by equation (ii) we have.

$$TR = \frac{\dot{C}}{C} \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_} (iv)$$

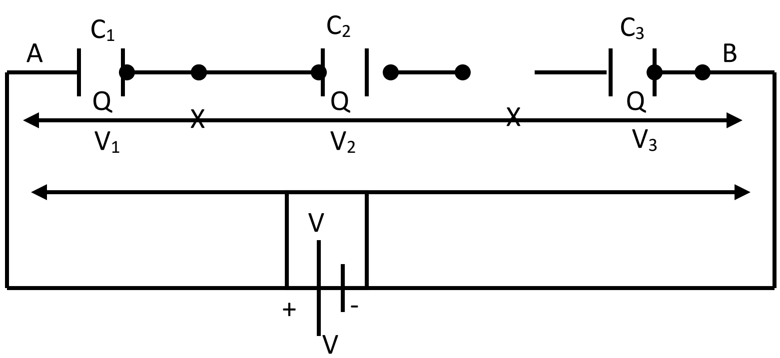
Equation (IV) provides us the definition of relative permittivity.

COMBINATION OF CAPACITOR:

There are many situations in electrical circuit where more than one capacitors are used. The capacitors can either be combined in series or in parallel.

1. **Capacitor in Series:**

Consider three capacitors say C_1, C2 and C_3 are connected in series as shown in fig.



The potential difference V must be equal to the some of the potential drops across C_1, C_2 and C_3

$V = V_1 + V_2 + V_3$ ___________ (i)

But according to the definition.

$V_1 = Q/C$, $V_2 = Q/C_2$, $V_3 = Q/C_3$ and $V = Q/C_e$

Now eq: (i) become

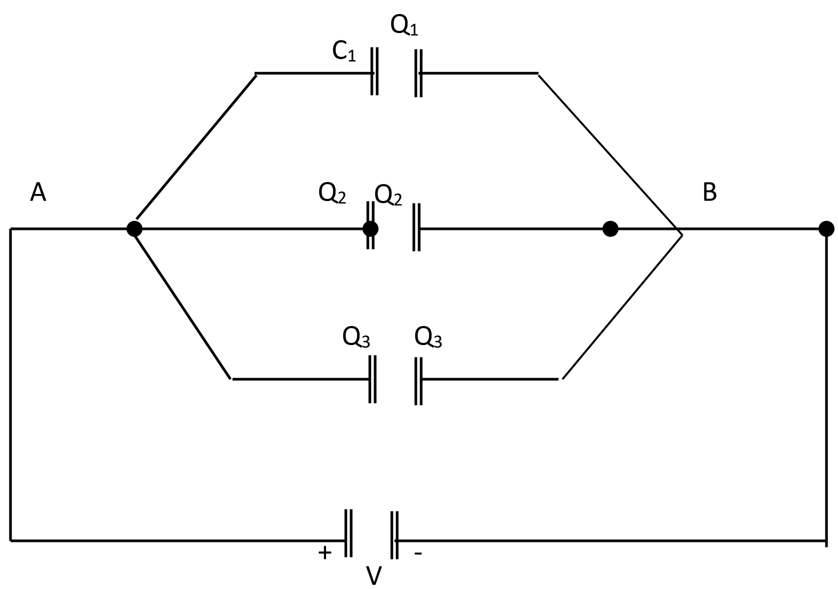
$Q/C_e = Q/C_1 + Q/C_2 + Q/C_3$

OR

$1/Ce = 1/C_1 + 1/C_2 + 1/C_3$

2. CAPACITORS IN PARALLEL:

Consider three capacitors C_1, C_2 and C_3 are connected in parallel between two points A and B as shown in fig.



As each capacitor is between t he same two points, so when potential difference V is applied between A and B, all the capacitors have been equal potential difference across them. On giving Q charge to point charges Q_1, Q2 and Q_3 will acquire different charges Q_1, Q2 and Q_3 respectively.

$Q = Q_1 + Q_2 + Q_3$ _____________ (i)

As Q = CV

$\therefore \quad CeV = C_1V + C_2 V + C_3V$

OR

$Ce = C_1 + C_2 + C_3$

Uses of capacitors:-

Electrical and electronic circuits use capacitors in a broad variety of ways. They are utilized, for instance, in the process of tuning transmitters, receiver and transistor radios. Also they are utilized to run table, ceiling, exhaust fans and many other appliances to keep them running at high efficiency. It is also common to find the capacitors in electronic circuitry of computers and other products.

UNIT 10 D.C CIRCUITS

OHM'S LAW:

"The current passing through a conductor is directly proportional to the potential difference applied across the ends of a conductor provided that temperature and other physical conditions of the conductor remain constant."

Suppose I is a current passing through a conductor and potential difference between the conductor's two ends then according to ohm

$V \propto I$

$V = IR$

Where R is a constant known as resistance of a conductor.

The unit of resistance is ohm (Ω). It is to be noted that R is a physical property of a conductor

As voltage and current are directly proportional then graph between them is a straight line if resistance remains constant.

RESISTIVITY:

The resistance offered by a conductor depends on the following factor.

1. Directly proportional to length of the conductor.
2. Inversely proportional to the area of cross-section A of the conductor.
3. Nature of the material.
4. Temperature of the conductor.

Neglecting the lat two factors for the time being, we can say that

$R \propto L$

$R \propto \frac{1}{A}$

Combining the above

$R \propto \frac{L}{A}$

Or

$R = \frac{eL}{A}$

Where e is proportionality constant known as specific resistance OR resistivity its value depends upon the nature of material.

UNITS:

As e = $\frac{R \times A}{L} = \frac{\Omega \times m^2}{m} = \Omega - m$

E is defined as resistance between two opposite phases of a meter cube.

DEPENDENCE OF RESISTIVITY UPON TEMPERATURE

The resistance offered by a conductor to the flow of electric current is due to the collision of free electron with the atoms of lattice. As the temperature increase, atoms in the lattice vibrate with large amplitudes and chances of the collision between free electrons and atoms increase. Hence the resistance of the conductor increases.

Consider a conductor whose resistance increase from Ro to Rt, when it is heated from O ^{o}c to t ^{o}c respectively.

It has been found from experiments that

$\Delta R \propto Ro$

and

$\Delta R \propto \Delta T$

OR

$\Delta R \propto Ro \Delta T$

$\Delta R \propto Ro \Delta T$ → (i)

OR

$Rt = Ro + \propto Ro\Delta T$ ($\because$ $\Delta R = Rt = Ro$)

$Rt = Ro + \propto Ro\Delta T$

$Rt = Ro (1 + \propto \Delta T)$ → (ii)

from equation (i) we can write → (iii)

$$\alpha = \frac{Rt - Ro}{Ro} \times \frac{1}{\Delta T}$$

Where $\propto$ is proportionality constant and called as temperature coefficient of resistivity and defined as "Increase in resistance per unit resistance per degree rise in temperature".

The unit of $\propto$ is C^{-1} OR K^{-1}

As resistivity is directly proportional to the resistance of metal. We can thus derive that

$$\alpha \frac{\ell t - \ell o}{\ell o \times \Delta T}$$

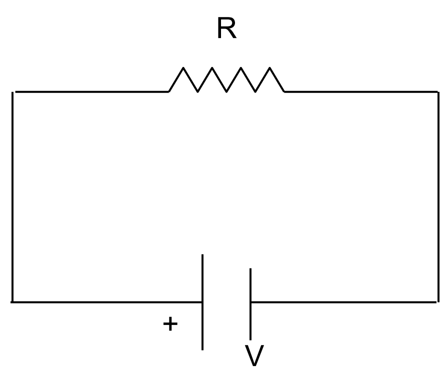
$\propto$ is helpful in differentiating between two metals having the same resistivity.

e.g

Iron and platinum have the same resistivity ($11 \times 10^{-8}\,\Omega$-m) but have different temperature coefficient.

POWER DISSIPATION IN RESISTORS:

Consider a resistor having resistance R is connected with a attery as shown in figure



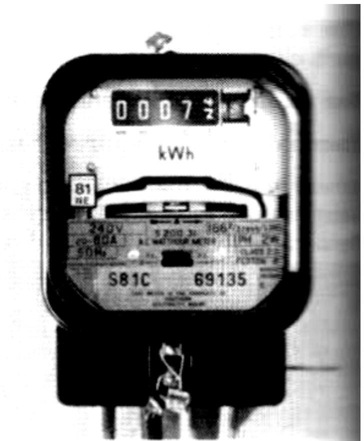


Figure 4 The electricity board's meter measures the energy supplied in kilowatt hours

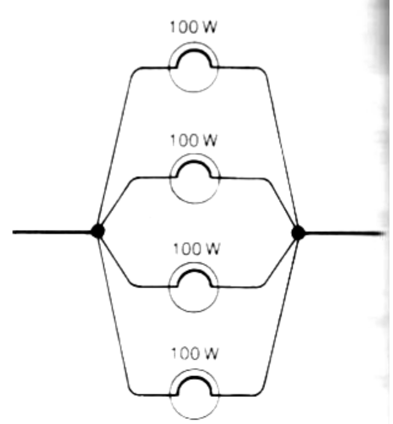


Figure 5

The current flowing is given by

$$I = \frac{Q}{t}$$

There is a lose of P.E 9electrical potential energy) of the charges during their transfer from higher potential terminal A to the Lower potential terminal B. this lose of electrical P.E appears as heat energy in the resistor.

The lose of P.E = Energy dissipated = $V \times Q$

$\therefore W = V \times Q$

$\therefore$ Power dissipated through resistor $= \frac{W}{t} = \frac{V \times Q}{t}$

$P = V \times \frac{Q}{t}$

P = VI → i

As V = IR

And

I = V/R

∴ equation (i) becomes

$P = I^2 R$ → ii

And

$P = \frac{v^2}{R}$ → ii

UNIT:

The unit of power in Watt (W). the power dissipated is one watt (W), if a currnet^{-1} of 1 ampere flows through a resistor against a p.d of 1 volt for one second.

1 KW=10^3 W

1 MW=10^6 W

HEAT ENERGY: If a current^{-1} flows through resistor R for ti8one't' total energy supplied to the resistor is

Heat energy generated = P × t = V × I× t = I^2 Rt = $\frac{V^2}{R} \times t$

UNIT (KWH) When a power of 1kw is maintained through a circuit for one hour, the heat energy generated is 1KWH.

1KWH =1KW × 1 Hour

= 1000 × 36000 $\frac{J}{2} \times S$

= 36 ×10^5 Joules

∴ 1 KWH = 36 × 10^5 joules

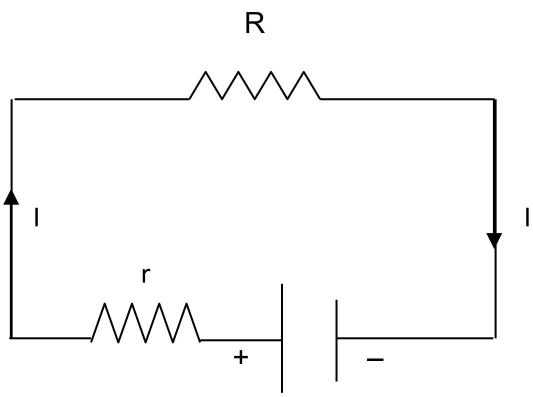
ELECTROMOTIVE FORCE:

To maintain a constant current through a source is required to provide energy equal to that which is dissipated as heat in the resistance. The strength of such a force is known as Electromotive force (ℓ.m.f).

The devices such as dry cell, battery, solar cell and electric generator converts some other form of energy into electrical energy, there by maintaining the required potential difference across a circuit, called the source of electromotive force.

TERMINAL POTENTIAL DIFFERENCE AND INTERNAL RESISTANCE

Consider a battery which is connected with a resistor of resistance R as shown in fig. Let r be the internal resistance of the battery.



The potential difference between points a b of the battery when the circuit is closed is called terminal potential Difference.

The electromotive force of the battery is

E = IR + Ir -1

Where IR=V =Terminal potential

R= Internal resistance

E = V = Ir

OR

V=E-Ir

We can say that the internal resistance of a battery governs the maximum current it can supply.

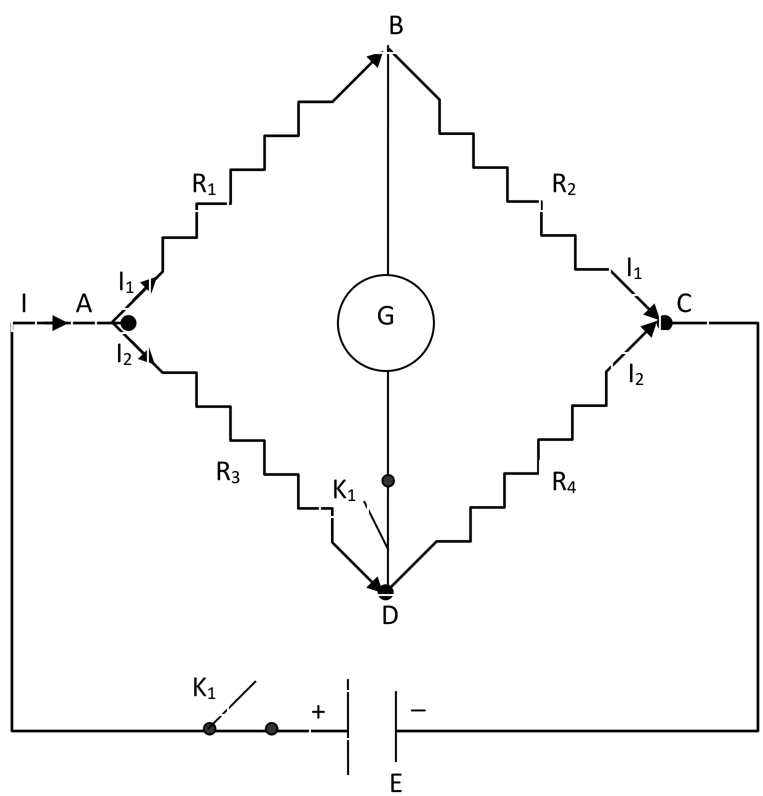
WHEATSTONE BRIDGE

DEFINITION

This is an arrangement of electrical circuit which form the basis of most of the instrument used to determine an unknown resistance. It was devised by Charles Wheat Stone and hence it is known as **Wheatstone bridge.**

Circuit arrangement

A Wheatstone bridge consist of four resistances R_1, R_2, R_3 and R_4 joined together to form a loop ABCDA. A battery of emf E is connected between the point A and C with key K_1 and a sensitive galvanometer with Key K_2 are connected between the point B and D as shown in Figure.



Working

In bridge if the key K_1 is closed first some current flow though the resistances R_1, R_2, R_3 and R_4. If the key K_2 is also closed, then some current passes through the galvanometer and its needle shows deflection. However by adjusting the resistance R_1, R_2, R_3 and R_4 (or at least one of them) condition can be attained for no current flows through the galvanometer. The Wheatstone bridge is then said to be balanced. It this case, the potential at B is equal to at D.

$\therefore$ P.d between AB = P.d between AD

and P.d between BC = P.d between CD

If, I be the current through R_1, R_2 and I_2 to the current through R_3, R_4, then by Ohm's law.

$$I_1R_1 = I_2R_3 \quad \text{(1)}$$

and

$$I_1R_2 = I_2R_4 \quad \text{(2)}$$

Dividing equation (2) by equation (1) we have

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \quad \text{(3)}$$

Equation (3) is the principle of a balanced Wheatstone bridge (no current flows the galvanometer). Under balance co-addition if any three resistances are known the fourth (unknown) can easily be calculated.

To determine unknown resistance X, we conned this resistance in the arm containing resistance R_4. The values of resistance R_1, R_2 and R_3 are so adjusted that the bridge is balanced and no current flows through the galvanometer. In this position.

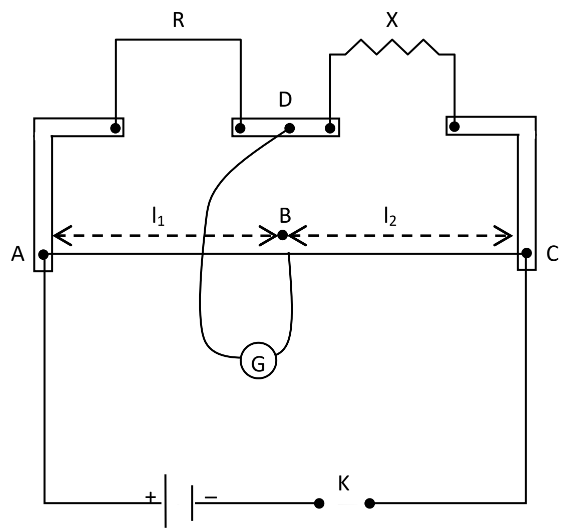
$$\frac{R_1}{R_2} = \frac{R_3}{X} \; OR \quad \boxed{X = \frac{R_3}{R_1} R_2}$$

As the values of R_1, R_2 and R_3 are known, the value of X can be calculated.

METER BRIDGE

A meter bridge is a practical form of Wheatstone bridge and is sued to determine the value of the known resistance .

Construction



It consist thin metallic (constantan) wire AC. It is one meter 100g. It is stretched and fixed to tow thick copper strips A and C, and is placed along a meter scale over a wooden board. A jockey J is connected to the terminal D through a galvanometer G as shown in Figure a resistance box is connected in the gap A and D. the resistance X to be measured is connected between C and D. A cell is connected across A and C through a key K.

Working

To find the value of the unknown resistance, the key K is closed and suitable resistance R is taken out the resistance box. The jockey is moved along the wire till for a certain position, the galvanometer shows zero deflecton. Let B be this position let the length AB(l_1) and BC (l_2) correspond to resistant R_1 and R_2 respectively.

Applying principle of Wheatstone bridge

$$\frac{R1}{R2} = \frac{R3}{R4} OR \frac{R1}{R2} = \frac{R}{X}$$

OR $$\frac{l_1}{l_2} = \frac{R}{X}$$

$$\therefore \quad \boxed{X = \frac{l_2}{l_2} R}$$

using this formula, X can be calculated if R is a known resistance.

POTENTIOMETER

A potentiometer is a device for measuring the potential different (or voltage) between two points of a circuit or the e.m.f. of a source.

Potential divider

Consider a uniform resistance wire AB of length L and resistance R across which is connected a source of e.m.f. E through a key and a rheostat to adjust and maintain a constant current I through it.

The P.D. between A and B is $V_{AB} = IR$ and $V_{AC} = I_{r\chi}$.

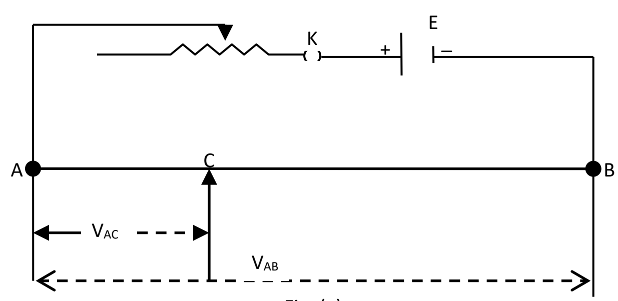


Fig. (a)

As C is a running point, the resistance rx and the P.d. V_{AC} change continuously from O to IR as C moves from A to B. The point C thus divides the P.d V_{AB}, across the wire AB into two part V_{AC} and V_{CB}. There fore this circuit is called potential divider.

Potentiometer

In a potentiometer the length of the wire AB may be 1 meter or 5 or 10 meters. The larger the length, the greater is the accuracy of measurement.

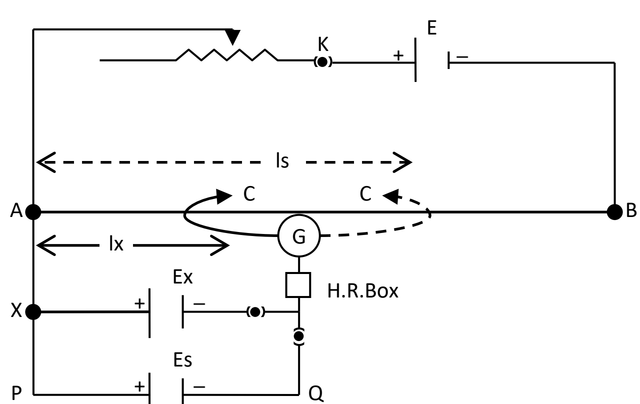


Fig. (b)

Let the positive terminals of a cell of unknown e.m.f. and a standard cell of e.m.f. Es are connected to the terminal A to which the positive terminal of E is connected. The negative terminals of both the cells are joined to the jockey.

Using the two way key first cell Ex only is introduced into the galvanometer branch and the balance point C and length lx are found it. At the balance.

$$Ex = V_{AC} = I_{rx} = I\ell lx$$ $(\because R = \ell l)$

OR

$$Ex = I\ell lx \quad \text{——— (1)}$$

Where ℓ is the resistance per unit length. Then putting Ex out of circuit, cell Es only is introduced and the balance point C^1 and length ls are determined.

$Es = V_{AC} = Irs = I\ell ls$

OR

ℓ

$$Es = I\ell ls \quad \text{——— (2)}$$

Dividing equation (1) by (2), we have

$$\frac{Ex}{Es} = \frac{lx}{ls} \quad \text{——— (3)}$$

The equation (3) gives the ratio of the two e.m.f. in terms of the ratio of their balancing lengths. If Es is known, Ex can easily be calculated.

Unit:11

Oscillation

Periodic motion:

The to and fro motion of body in equal intervals of time is called periodic motion

Time period:

It is the time required by one vibration (t).

Frequency:

(υ) It is the number of vibrations per second completed by the body its units are cycle/sec or hertz.

Vibration:

It is a complete round trip of a vibrating body from and to its extreme position

Displacement:

(x) It is the distance of a vibrating body from the equilibrium or mean position of a body at any instant.

Amplitude:

(x_o) It is the maximum displacement either sides of its equilibrium position.

Phase:

It is defined as the ratio between displacement (x) and amplitude(x_o).

Definition Of S.H.M.

Derivation For The Acceleration Of A Body Vibrating Under Elastic Restoring Force:

It is a type of vibratory motion in which the acceleration is directly proportional to the displacement of body and is always directed to the equilibrium position. i.e

$a \propto -$ displacement

Characteristics Of S.H.M:

(1) The body should exhibit too and fro vibratory motion.
(2) The acceleration of body at any instant should be directly proportional to its displacement i.e $a \propto -x$
(3) The acceleration should always be directed to mean position, for that a negative sign is used.

Acceleration:

Consider a mass (m) attached to one end of an frictionless horizontal surface as shown in fig (a) when mass is pulled towards right through distance(x) fig (b) the applied force at that instant is given by hooks law.

$F \propto x$

$F = Kx \longrightarrow 1$

Where (K) is constant called spring constant or force constant due to elasticity spring opposes the applied force. This opposite force is called restoring force. It is given by

$$F=-kx \longrightarrow 2$$

If the mass is pulled and then released it begins to oscillate about the mean position due to re-solving force and inertia. According to newton's second law of motion.

$F = ma$ 3

Comparing eq 2 and 3

$ma = -kx$

$a = -kx/m \longrightarrow 4$

:- k/m = constant

:- $a \propto -x$

Or $a \propto$ - displacement

This is equation for S.H.M

To Prove That The Projection Of A Particle Moving On A Circumference Of A Circle Executes S.H.M or To Drive Expression For Acceleration:

Let a particle (p) is moving in a circle of radius (r) with constant linear velocity (Vp) and angular velocity (ω) such that.

$V_p = r\omega$

When the particle (p) is moving in a circle it has centripetal acceleration given by.

$a_c = r\omega^2$

This acceleration is directed towards the center of circle resolving it into two triangular components

1. $a_c \cos\emptyset$ (PN) or (OØ)
2. ac sinØ (NO) or (PØ)

As (PN) moves along the diameter therefore the acceleration of projection (Ø) will be

$a = a_c \cos\emptyset$

$= r\omega \times \frac{\text{base}}{\text{Hype}} \longrightarrow 1$

From Fig: $\frac{\text{base}}{\text{Hyp}} = \frac{PN}{OP} = \frac{O\emptyset}{OP} = \frac{x}{r}$

:- $a\ r\omega^2 \times \frac{x}{r}$

$a = \omega^2 x$

or $a = -\omega^2 x \longrightarrow 2$

Negative sign shows that the acceleration is towards the mean position. This shows that (Ø) performs S.H.M

Time Period And Frequency:

It is the time taken by a vibrating body to complete one vibration. In case of projection is equal to the time taken by particle (P) to complete one revolution i.e

$\emptyset = \omega t$

Or $2\pi = \omega T$

$T = 2\pi/\omega \longrightarrow 1$

Acceleration of body attached to elastic spring executing S.H.M acceleration of projection (Ø)

$= -\frac{Kx}{m}$

$= -\omega^2 x$

:- $-\omega^2 x = \frac{-kx}{m}$

Or $\omega = \sqrt{k/m}$

Putting this valve of (ω) in eq 1

$T = 2\pi \sqrt{k/m}$

Because $T = \frac{1}{F}$

:- $f = \frac{1}{T} = \frac{1}{2\pi} x \sqrt{k/m}$

Displacement Of A Body Executing S.H.M:

Set a particle moving in a circle of radius (r)with constant linear and angular velocity. Set (Ø)be the projection of (P) and (x) be its displacement from mean position (O) as shown in fig. It is RT is a rt Δ OQP, where

$\frac{Base}{Hyp} = Cos\emptyset$

$\frac{OQ}{OP} = Cos\emptyset$

$\frac{x}{r} = Cos\emptyset$

$x = r\, Cos\emptyset$

but $r = x_o$ maximum displacement

:- $x = x_o\, Cos\emptyset$

Instantaneous Velocity Of A Body Executing S.H.M:

Let a particle (P) is moving in a circle of radius (r) with constant linear velocity (VP). Resolving (VP) in to two rectangular components (PN) and (MN) i.e

1. $\vec{PN} = \vec{Vp} \sin\emptyset$
2. $\vec{MN} = \vec{Vp} Cos\emptyset$

$\vec{PN}$ is parallel to OØ, So $(\vec{PN})$ represents the velocity of projection (Ø) i.e

$V = V_p \sin\emptyset$

:- $V = r\omega \sin\emptyset \rightarrow 1$ (:- $V_p = r\omega$)

Now $\sin^2\emptyset + \cos^2\emptyset = 1$

Or $\sin^2\emptyset = 1 - \cos^2\emptyset$

Or $\sin\emptyset = \sqrt{1-\cos^2\emptyset} \rightarrow 2$

But in rt $\Delta O\emptyset P$

$\cos\emptyset = \frac{O\emptyset}{OP} = \frac{x}{r}$

Putting the valve of CosØ in eq 2

$\sin\emptyset = \sqrt{1 - \frac{x^2}{r^2}}$

Putting this valve of sinØ in eq 1

$V = r\omega \sqrt{1 - \frac{x^2}{r^2}}$

or

$V = r\omega \frac{\sqrt{r^2 - x^2}}{r}$

$V = \omega \sqrt{r^2 - x^2}$

:- $\omega = \sqrt{\frac{k}{m}}$ and $r = x_o$

:- $V = \sqrt{\frac{k}{m}} \sqrt{{x_0}^2 - x^2}$

This is an expression for instantaneous velocity of a body attached to an elastic spring executing S.H.M.

Maximum Velocity:

The projection (Ø) has maximum

$F = Kx$

At mean Position $F_1 = O$

At displacement (x) $F_2 = kx$

Average force during displacement from mean position to point (A)

$F = \frac{F_1 + F_2}{2}$

$= \frac{O + Kx}{2}$

$= 1\ Kx \rightarrow 1$

$\overline{2}$

Work done is moving the mass (m) through displacement (x) is

Work = Fd

O→A

$= \frac{1}{2} Kx.x$

$= \frac{1}{2} Kx^2$

This work is stored in the mass as its elastic PE of the spring i.e

$$PE = \frac{1}{2} Kx^2 \rightarrow 2$$

Maximum PE:

The PE is maximum at extreme position where $x = x_o$

i.e $$PE = \frac{1}{2} Kx_0^2 \longrightarrow 3$$

Kinetic Energy:

The instantaneous velocity of the mass at a distance (x) from the mean position is

$$V = \sqrt{\frac{k}{m}} \sqrt{x_0^2 - x^2}$$

Velocity when it is at mean position or when its displacement (x) is zero.

X = 0

Putting this valve of (x) In the expression of instantaneous velocity

$$V = \sqrt{\frac{k}{m}}\, x_0^2$$

Instantaneous KE, PE, Maximum KE, PE Of A Body Executing S.H.M And Law Of Conservation Of Energy:

Potential Energy:

Consider a mass (m) attached with an elastic spring is stretched by an amount

(x) against the elastic restring force. According to Hook's Law.

The KE at that instant is

Putting the valve of (v)

$$KE=\frac{1}{2} \left(\sqrt{\frac{k}{m}} \sqrt{x_0^2 - x^2}\right)^2$$

$$KE= \frac{k}{2\,m}(x_o^2 - x^2) \longrightarrow 4$$

Maximum KE:

The KE is maximum at mean position where x = o i.e

$$KE = \sqrt{\frac{k}{m}}\sqrt{x_0^2 - x^2}$$

$$KE = \frac{1}{2} Kx_o^2 \longrightarrow 5$$

Total Energy:

For any displacement (x) the total energy

$$E = PE + KE$$

$$= \frac{1}{2} Kx^2 + \frac{1}{2} K (x_o^2 - x^2)$$

$$= \frac{1}{2} Kx^2 + \frac{1}{2} Kx_o^2 - \frac{1}{2} Kx^2$$

$$E = \frac{1}{2} Kx_o^2$$

By comparing equation 3, 5 and 6 It is found that the total energy is constant everywhere in its path. This is known as law of conservation of energy.

Simple Pendulum

A simple pendulum consist of a small heavy mass suspended by a light string fixed at its upper end consider a simple pendulum having a mass (m) suspended by a Light in extensible string of length (L) fixed at its upper end when the bob mass(m) is disturbed form its mean position (0) it starts vibrating.

Suppose the bob is at the position (A) from (0) and its displacement (0) to (A)= (x) in this position two force are acting on the bob given as under.

1. Weight → w=mg of the bob acting vertically down ward.
2. Tension (T) of the string along (ac)

Now resolve the weight in to two components.

1. mgcosθ → Along the string.
2. mg Sinθ → Perpendicular to the string

Since the bob has no motion along the string so

$$T = mg \cos\theta$$

or

$$T - mg \cos\theta = 0$$

The component mg sinθ is responsible for the moti→n of the bob which brings the bob back to its mean position. So restoring force = F = -mg sinθ According to newton's second law of motion applied force = F = ma

$$ma = -mg \sin\theta$$

$$a = -g \sin\theta \longrightarrow 3$$

From figure Δ OAC

If (θ) is small then, $\sin\theta = \frac{OA}{Oc} = \frac{x}{L}$

Putting in eq 3

$$a = -g\frac{x}{l}$$

$$a = -g\frac{x}{L}$$

Where g/L is constant for the pendulum

:- a = - (constant) x

a α- x

hence the pendulum execute S.H.M

Time Period:

The acceleration of the body (projection) executing S.H.M is given by

$$a = -\omega^2 x \qquad 4$$

The acceleration of simple pendulum is given

By $$a = \frac{-g}{L}x \qquad 5$$

Comparing eq 4 and 5

$$-\omega^2 x = -\frac{g}{L}x$$

The time period of a body (projection) executing S.H.M is given by

$$T = 2\pi/\omega$$

Putting the valve of (ω) from eq 6

$$T = 2\pi/\sqrt{\frac{g}{l}}$$

Or $$T = 2\pi\sqrt{\frac{l}{g}}$$

These relation shows that time period of a simple pendulum does not depend upon the mass of this bob. But depends only upon its length and the value of (g).

Frequency:

Frequency is given by

$$F = 1/T$$

Putting the valve of (T)

$$F = 1/2\pi\sqrt{\frac{l}{g}}$$

$$F = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

Unit:12

Acoustics

Speed Of Sound:

Newton's Formula:

Sound travels in fluid in form of compressions and refractions the speed of such wave depends upon.

1. Compressibility of medium
2. Inertia (density) of medium

This led newton to propose the following formula for speed of sound.

$V=\sqrt{B/e}$

- B Bulk modulus of medium. It is given by

$B = \Delta P/\Delta V/V$

Error In Newton's Formula:

Newton assumed that sound wave propagate that sound wave propagate through fluid under isothermal conditions i.e compression and rarefactions take place at same temperature.

Speed of sound at $o^{o}c$ by newton's formula

$V = 281\ m^{5-1}$

Experimental $V = 332\ m^{5-1}$

Error of about 16%

Laplace correction:

Laplace said that during the propagation of compression end rarefaction of sound waves in air the temperature does not remain constant thus compressions and rarefaction occur adiabatically Laplace pointed out that.

$B = \gamma P$

Thus formula for speed of sound in air is

$V= \sqrt{\gamma P/e}$

Where γ= Cp/Cv

For an ideal gas

$PV = RT$

:- $P = RT/v$

And $\alpha = \frac{mass}{Volume} = \frac{m}{V}$

:- $V = \sqrt{\gamma \times \frac{RT}{V} \times \frac{V}{m}}$

$V = \sqrt{\gamma RT/m}$

Where γ= 1.42 for air

$P = 10139061.6\ D/cm^2$

$R = 8.31$ J/mole K

M = molecular mass

Speed of sound at $O^{o}C$ by this formula

FACTORS EFECTINGSPEED OF SOUND:

1. **Effect Of Pressure:**

Since density is directly proportional to the pressure. Thus the ratio "P/e Remains constant."

Therefore the speed of sound close not changes with the change of pressure.

2. **Effect Of Density:**

At the same temperature and pressure for the gases hawing the same Value of (ɻ).

$V \alpha \sqrt{1/e} \alpha \sqrt{T}$

Or $V \alpha \sqrt{T}$

Velocity of sound in air increases or decreases by 0.61 rn^{5-1} for every degree change in temperature. Thus

$$Vt = V_o t + 0.61t$$

Where→ V_o speed at O^oC

T temperature in C^o

Vt Speed at te^o.

Sound:

It is a sensation perceived by ear it is produced by vibrating bodies it can be classified as.

Musical Note:

It is a sound of regular frequency it is caused by a body producing regular vibrations.

Noise:

It is a sound of irregular frequency it is produce by irregular vibrations of body.

Characteristics Of Musical sound:

1. **Intensity (I):**

It is the energy transferred by sound wave per second per unit area perpendicular to the direction of travel of sound.

$$I = \frac{energy}{Area \times Time}$$

$$I = \frac{1}{Area} \times \frac{energy}{Time}$$

$$I = \frac{1}{Area} \times power$$

$$I = P/A = P/4\pi r^2$$

Its units is $watt/m^2$

Human ear can hear leas intensity $10^{-12}w/m^2$

Loudness:

It is the magnitude of auditory sensation produced by sound it depends upon following factor.

1. **Amplitude:**
   The greater the amplitude of vibrating body the more is loudness.
2. **Surface Area Of Sounding Body:**
   The greater the surface area the more is loudness.
3. **Density of Medium:**
   The great the density the louder is sound.
4. **Sensitivenss of Ear:**
   Different human ear record different loudness are related as

$$L \alpha \log I$$

$$L = K \log I$$

K constant its valve depends upon the units used.

This is called **Weber Fechner Law.**

If I and I_o are the intensity of two sounds. Then the difference in loudness is called intensity Level

Is given by

$$\text{Intensity Level} = K \log I - K \log I_o$$

$$= K (\log I - \log I_o$$

$$= K \log I/I_o$$

Its unit is **Bell** other unit is decibel (d.b)

Intensity in decibel = = $10k.\log I/I_o$

Pitch:

Pitch the characteristics of sound by which a will sound can be distinguished from a grave sound is called pitch of sound.

If is directly proportional to the frequency of sound.

Quality of sound:

It is that characteristic of sound of same loudness and pitch it depends upon the number of over tones in the number of over tone in the sound.

Beats:

Periodic rise and fall in the loudness of sound is called beat. Frequency .the time interval between two successor loud sound is called beat period.

Reason:

The phenomenon of beats takes place when two sound waves of slightly different frequencies but of same amplitude super pose on each other

Explanation:

The formation of beats can show in graphically as consider two tuning forks a and having slightly different frequencies 256 and 252 Hz fig

1. Continuous line of wave form of the note emitted from tuning fork (a) similarly dotted line wave form of the note emitted by the lining fork (b) when both the tuning forks are sounded together the resultant wave form is shown in fig.
2. At some instant (x) the displacement of two waves in the same direction the resultant displacement is large and a loud sound is heard.

After 1/8 second the displacement of the waves due to (a) is opposite to the displacement of the wave due to (b) as a result a minimum displacement is produced hence faint sound or no sound is heard. Another 1/8 second resultant displacement is maximum and a loud sound is heard this means a loud sound is heard four time in each second as the difference of the frequency of the two tuning forks is also 4hz 50 we find that.

No.s of beats = $F_a - F_B$

F_a frequency of tuning fork (A)

F_B frequency of tuning fork (B)

The difference of two frequencies should not more than 16Hz.

Waves:

It is a form of disturbance which travels through medium due to periodic motion of particles of medium about their mean positions.

Wave Motion:

It is a process by which energy is transferred from one point to another.

Types of waves:

Waves may be classified in to three types.

1. **Mechanical waves:**

These are the waves which required a medium for either propagation.

For examples

1. Water waves 2. sound waves 3. seismic waves

2. **Electromagnetic:**

These are the waves which require no medium for their propagation.

For examples

Light , X-ray, Heat waves, Radio waves, micro waves, UV and IR radiations

Matter Waves:

These are the waves which are associated with moving particles they are called de Broglie waves.

Kinds Of Waves

1. **Transverse Waves:**

The waves in which the particles of the medium vibrate along a line perpendicular to the direction of propagation of waves are called transverse waves.

Example:-

Light, Radio waves, Water waves.

2. **Longitudinal Waves:**

The waves in which the particles of the medium vibrate along the direction of propagation of waves are called longitudinal wave.

Examples:

Sound waves are called longitudinal wave in spring.

Travelling Or Progressive Waves:

A wave which transfers energy in moving away from the source of disturbance is called progressive or travelling wave. These are three important terms are used in connection with a wave given as under.

Wave Length:

(λ)The distance covered by the wave during one vibration is called wave length. or the distance between two successive crests or troughs is called wave length.

Velocity:

(V) The distance covered by the wave in one second is called velocity.

Frequency:

(V) The number of vibration made in one second is called frequency.

$$V = V\lambda$$

Analytical Treatment Of Progressive Waves:

Consider a travelling wave whose displacement at any instant (t=0) in y – direction is (A_o)

It is given by relation

$$Y = A_o \sin \omega t$$

If the wave travels in +x direction with speed (V) then after (t) seconds displacement in + x direction. = x = vt

$$t = x/v$$

or

Then initial time = $\frac{x}{V} - t = o$ then at initial time

$$Y = A_o \sin\omega \left(\frac{x}{v} - t\right)$$

$$\text{or } Y = A_o Sin \left(\frac{\omega x}{v} - \omega t\right)$$

$$Y = A_o \sin\left(\frac{\omega x}{V} - \omega t\right)$$

but $\omega = 2Лf$

Or $\frac{\omega}{V} = \frac{2Лf}{V}$

$\frac{\omega}{V} = \frac{2Лf}{V}$

:- $V = f\lambda$

$\frac{F}{V} = 1/\lambda$

:- $\frac{\omega}{V} = \frac{2Л}{\lambda}$

Now $\frac{\omega}{V} = K$ it is called wave number and it is constant

$$Y = A_o \sin(Kx - \omega t)$$

This equation describes a wave travelling along the string in + x direction. If wave travels in – z direction them

$$Y = A_o \text{ Sin } (Kx - \omega t)$$

Stationary Waves:

When two waves of equal amplitude and frequency travelling through the same medium in opposite direction super impose the result is a wave which does not travel in either direction such a wave is called standing wave.

They are transverse in nature energy is not transferred by them they have fixed positions of zero amplitude called **NODES** and maximum amplitude called **ANTINODES** the enclosed area between them is called **LOOP** as shown in fig

Analytical treatment of standing waves:

Consider two waves of same frequency speed and amplitude which are travelling in opposite direction along a string displacement of two such waves in y- direction is given by.

$Y_1 = A_o \sin(kx - wt)$ + x →direction

$Y_2 = A_o \sin(kx + wt)$ + x →direction

Hence the resultant displacement of wave after super position may be written as.

$Y = y_1 + y_2$

$Y = A_o \text{ Sin } (Kx - wt) + A_o \text{ Sin } (kx + wt)$

$Y = A_o \{\text{Sin}(Kx - wt) + \text{Sin } (Kx + wt)$ ⟶ 1

But $\sin(\alpha + \beta) + \text{Sin } (\alpha - \beta) = 2 \text{ Sin } \alpha \text{ Cos}\beta$

:- Sin (kx – wt) + Sun (kx + wt) = 2 Sinkx Coswt

Putting this valve in eq 1 we get

$Y = 2A_o\text{Sin } kx \text{ Cos}wt$

This is the equation of standing wave. This equation states that

Amplitude is maximum = $2A_o$, (ANTINODE)

When Sin kx=1

i.e kx = λ/2 = Sin90 = 1

Or x = λ/2 x 1 k = 2Л/λ

x = λ/2 x λ/2Л

x = λ/2

similarly x =λ, 3λ/2

Fundamental Frequency And Over Tones Of Standing Waves In String:

A Case of one loop: Consider a string of length (L) fixed at both ends as shown in fig Let the string is plucked at its middle. It starts vibrate in one loop with nodes at ends. Such the distance between two nodes is λ_2, so

: = $\lambda^1/2$

Or $\lambda_1 = 2L$

If (V) is the velocity of wave then

$F_1 = V/\lambda_1$

$F_1 = V/2L$ 1

F_1 is the lowest frequency of standing wave and is called **Fundamental Frequency Or First Harmonic**

B Case of two loop: when the string is plucked at $1/4^{th}$ of its length, then it vibrates in two loops.

Now

$L = \lambda_2/2 + \lambda_2/2$

$L = 2 (\lambda_2/2)$

:- $\lambda_2 = 2L/2$

Then $F_2 = V/\lambda_2$

$F_2 = V \times 2/2L$

$F_2 = 2 \times V/2L$ but f_1 V/2L

:- $F_2 = 2f_1$

F_2 called Fist over tone or second harmonic.

C Case of n-loop:

When wire vibrates in one loop $f_1 = V/2L$

two loops $f_2 = 2f_1$

three loops $f_3 = 3f_1$

n-loops $f_n = nf_1$

in general

$fn = nf_1$

Fundamental Frequency:

It is the lowest frequency of standing waves.

Overtone:

It is the integral multiple of fundamental frequency.

Application:

It is used to verify all the lams of transverse vibr**ating of string.**

Law Of Transverse Vibration Of string:

1. **Law of length:**

Frequency is inversely proportion to the vibrating length of string.

i.e.

$\nu \propto 1/L$

If(t) and (m) constant keeping the stretching force constant force and by changing the frequency of tuning fork. it can be verified.

2. **Law of tension:**

Frequency is directly proportional to the square root of tension.

i.e

$\nu \propto \sqrt{T}$ if (L)and (m) are constant. It is verified by keeping (L) constant, changing the suspended mass i.e tension frequency.

3. **Law Of Mass:**

Frequency is inversely proportional to the square root of mass per Unit length of string.

i.e

$\nu \propto 1/\sqrt{m}$ if (L) and (T) are constant. It is verified by using different wires.

Doppler's Effect:

The apparent change in frequency (or pitch) due to relative motion between the source of sound and observer is called Doppler's effect

When an observer is standing on a railway plat form the pitch of whistle of an approaching engine heard to be higher pitch but when they become lower.

These are four possible case of the Doppler's effect.

When Observer Move Toward The Stationary Source Of Sound:

Suppose (v) is the velocity of sound in the medium and source emits a sound of frequency (V) and wave length(λ). It both the source and the observer are stationary.

$V = \lambda\nu$

$\lambda = V/\nu$

$\nu = V/\lambda$

It the observer moves towards the source with a velocity (Vo). The relative velocity of the wave and the observer is increased to ($V+V_o$) then the changed frequency will be.

$$\nu' = \frac{V + V_o}{\lambda} \rightarrow 1$$

putting the valve of (λ)

$\nu' = (V+V_o)\nu$

$$\overline{}$$
V

When Observer Move Away From The Stationary Source Of Sound:

Suppose observer moves away from the stationary source with the same velocity (Vo) then the relative velocity of the wave and observer is (V-Vo) then the change frequency as heard by observer will be.

$$\nu' = \frac{(V-V_o)\nu}{V}$$

So the pitch of sound decreases.

When Source Of Sound Moves Towards The Stationary Observer:

Suppose the source of sound moves towards the stationary observer with velocity (Vs) then in one second

$$\lambda' = \frac{V-V_s}{\nu}$$

The compression of waves is due to the fact that same number of waves is contained in a shortly space depending upon the velocity of the source.
The wave length for the observer is then.

$$\nu' = \frac{V}{\lambda'}$$

$$\nu' = V\left(\frac{\nu}{V-V_s}\right)$$

$$\nu' = \nu\left(\frac{V}{V-V_s}\right)$$

When Source Of Sound Moves Away From The Stationary Observer:

Suppose the source of sound moves away from the stationary observer with velocity (Vs) the modified frequency for observer will be

So the pitch of sound decreases

$$\nu' = \nu\left(\frac{V}{V+V_s}\right)$$

If both are moving away from each other than frequency heard is

$$\nu' = \nu\left(\frac{\mathrm{V-Vo}}{V+V_s}\right)$$

If source of sound and observer are approaching towards each other then the frequency heard is

$$\nu' = \nu\left(\frac{\mathrm{V+Vo}}{V-V_s}\right)$$

Application:

1. used in police radar to cheek the spaced of vehicle
2. tracking satellites
3. To measure speed of stars relating to earth

Shock Waves And Sonic Boom:

If the speed of source is equal to the speed of wave the wave pile up and from a plane wave front that extends perpendicular to the direction of motion of source as shown in fig.

When the source such as aero plane moves at a speed greater than speed of sound (Vs>V) the spherical wave front pile up on one another along the side of aero plane the different wave crest.

Overlap one another forming a very dense region of sound energy this moving region of high energy is called shock wave.

Generally it is of the form of cone shaped wave front. The angle b/w the direction of travel and any tangent line to the cone is given by

$$\sin\theta = V/V_s$$

The ratio V/Vs is called Mach number when shock wave passes observer it is heard a loud sound called sonic boom.

Unit. 13
Physical optics

PARTICLE THEORY OF LIGHT

One of the early theories of light, about 400BC suggested that

"Particles were emitted from the eye when an object was seen."

In 1660 Newton proposed a theory according to which

(a) Particles or corpuscles are emitted from a luminous object with speed of light.
(b) The corpuscules have an attraction with molecules of the medium so it can travels faster in denser medium as compare to its speed in vacuum i.e. $V_d > V_r$.

HUYGEN'S WAVE THEORY

In 1676 christen Huygen (1629 – 1695) proposed his wave theory of light according to which

(a) Light is emitted from a luminous body in the form of wave like water or sound wave.
(b) The speed of light is greater in rare medium and slower in denser medium $V_d < V_r$.

OBJECTION ON HUYGEN'S WAVE THEORY

Newton did not accept this theory.

According to Newton

(a) A medium is necessary for wave propagation but light can travel in vacuum.
(b) Sound and water waves bends (Diffraction) around obstacles, but light do not appear to do so.

The experimental evidence of the wave theory in Huygen's time was very small & the theory (wave) was dropped for more than a century.

In 1801 Thomas Young experimentally showed that light has interference phenomena.

In 1880 Foucault proved that velocity of light is more in rare medium than in denser medium.

In 1873 Maxwell proposed a theory according to which

"Light is electromagnetic wave which can travel with or without medium."

The experimental evidence during the nineteenth century led to the general acceptance of wave nature of light.

QUANTUM THEORY OF LIGHT

In the beginning of 20th century many phenomena were discovered which could not be explained on the basis of wave theory.

In 1900 Max Planck's proposed Quantum theory. According to Quantum theory

"Light is collection of small particles are called "PHOTON."

In 1905 Einstein successfully explained the photoelectric effect on the basis of quantum theory.

In 1923 compton confirmed photon nature of light by explaining his effect.

In 1913 Bohr also confirmed the photon theory of light

CONCLUSION

The above discussion about the nature of light shows that

“Light has dual nature some time it behaves like a wave and some time it behaves like a particle (photon).”

WAVE FRONTS

Wave front is defined as

“The locus of all those points in a medium which are vibrating in phase”.

Ray can be defined as

“A line normal to the wave front, indicating the direction of motion of the wave”.

TYPES OF WAVE FRONT

Wave fronts are of two types

(i) SPHERICAL WAVE FRONT
(ii) PLANE WAVE FRONT

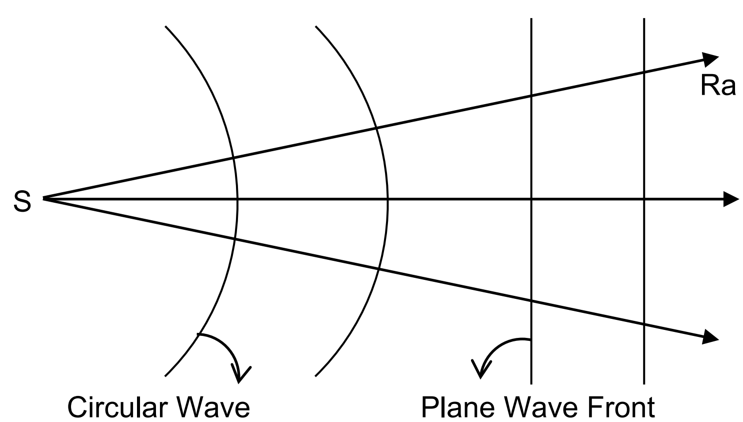
(i) SPHERICAL WAVE FRONT

If the wave is propagated outward in all possible direction from a point source, the resulting wave fronts are spheres and known as spherical wave front. These wave front are formed near the source.

(ii) PLANE WAVE FRONT

If the wave is propagated in a single direction, then wave front is known as plane wave front.

At a very large distance from the source the spherical wave fronts have very small curvature then a small portion of spherical wave front can be treated as plane wave front.

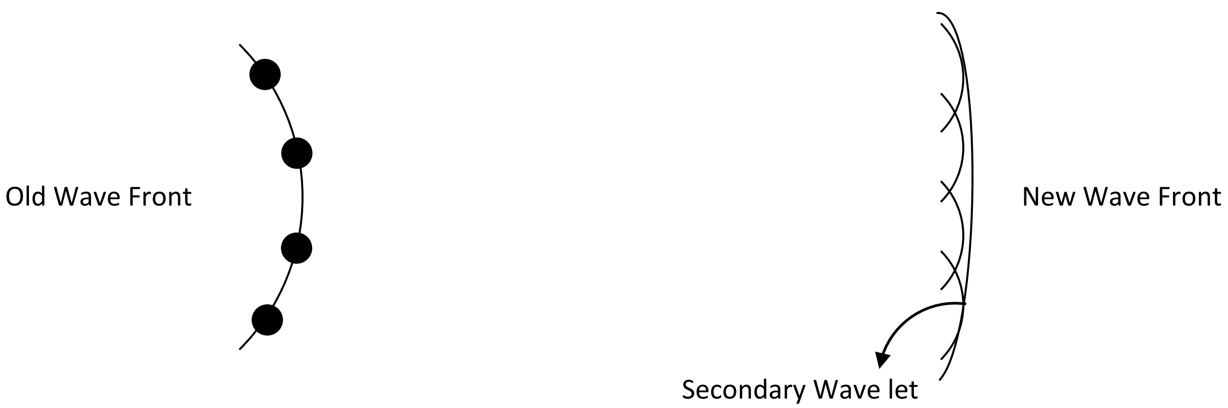


HUGYEN’S PRINCIPLE

Huygen’s principle is a geometrical method for finding the positive of new wave front at any time ‘t’. It has two parts.

(a) Each point on a wave front can be considered as source for secondary (Spherical) wave lets.

(b) The new wave front after time ’t’ can be obtained by drawing a plane tangent line to the secondary wavelets.



INTERFERENCE OF LIGHT

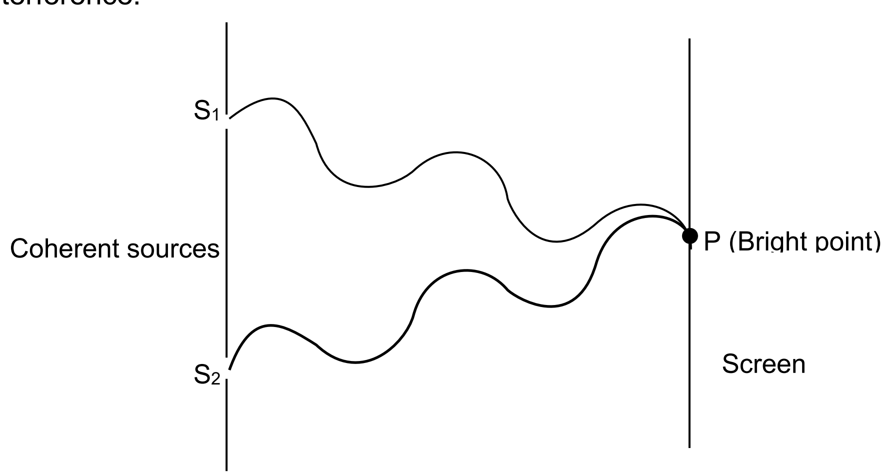
When two sources of light (coherent) are super imposed upon each other than the distribution of energy is not uniform in the surroundings, such non – uniform distribution of energy is called INTERFERENCE OF LIGHT.

TYPES:

It is of two types:

(i) CONSTRUCTION INTERFERENCE
(ii) DISTRUCTIVE INTERFERENCE

If two rays arrived at a point in phase they reinforce each other effect and detector shows a maximum effect or bright point in this region. Such region is called an interference maxima and interference is known as constructive interference.



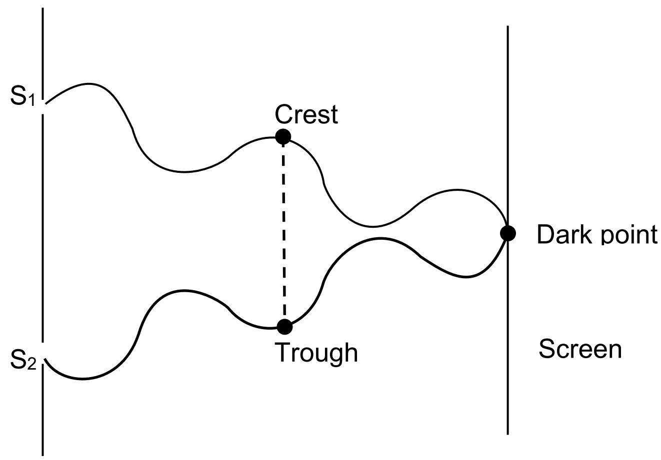
CONDITION:

Path difference = O, X, 2X, ……………….. mX.

Phase difference = ϕ = 0, 2π, 4π, ………………, $2m\pi$

DESTRUCTIVE INTERFERENCE:

If two light rays arrived at a point out of phase; theory cancel each other effect and detector shows a minimum effect or dark point in this region. Such region is called region of interference minima and interference is called **DESTRUCTIVE INTERFERENCE.**



CONDITIONS:

Path difference = $\lambda/2$, $3\lambda/2$, $5\lambda/2$, …………

Phase angle = π, 3π, 5π, ………………

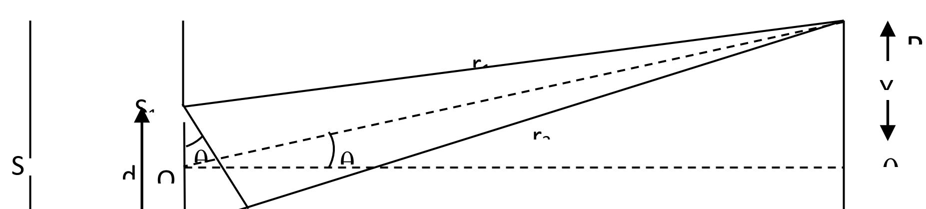
Q. What are the conditions for interference?

For Interference:

1. The two sources should be narrow.
2. The separation between slits should be very small.
3. The amplitude, wave length and frequency of the waves emitted by these sources should be the same.

YOUNG'S DOUBLE SLIT INTERFERENCE.

In 1801 Thomas Young a British physicist gave experimental evidence for Huygens wave theory. His experiment called young's double slit experiment. The quantitative description of young's experiment can be obtained with the help of fig (i).



Light waves with wave length λ are incident on the pair of narrow slits S_1 and S_2 which are separated by distanced. The interference pattern is observed on screen which is placed at a perpendicular distance L from the screen containing slits S_1 and S_2. "The light intensity on screen at the point" is the resultant of the light coming from both slits. The path difference between the two rays arriving at P is.

$$S_2 p - S_1 p = d \sin \theta \rightarrow \text{(i)}$$

This path difference with determine whether or not the two waves are in phase when they arrive at the point P. If the path difference either zero or integral multiple of wave length of the light use the waves are in phase and constructive interference results. There fore constructive interference.

$$d \sin \theta = m \lambda \rightarrow \text{(ii)}$$

Where m = O, ±1, ±2, ±3,……..

The central bright fringe at θ = O (m = O) is called zeroth order maxima. The first maximum on either side, where m = $\pm$ 1 is called first order maximum and so forth.

In between the bright fringes there are the dark fringes given by.

$$d \sin \theta = (m + ½) \lambda \text{ ---(iii)}$$

Where m = 0, $\pm$ 1, $\pm$ 2, $\pm$ 3,………

POSITION OF BRIGHT AND DARK FRINGES

In exp. L is of the order of 1m while d is a fraction of millimeter (d $\angle\angle\angle$ L) under these condition θ is small.

$$\therefore \tan \theta \cong \sin \theta$$

Mathematically we can write

$\Delta X = y2 - y1$

$\Delta X = 2\frac{\lambda L}{d} - \frac{\lambda L}{d}$

$$\boxed{\Delta X = \frac{\lambda L}{d}} \longrightarrow \text{(vii)}$$

From fig (i) in triangle OPQ

Sinθ = Tanθ = $\frac{Y}{L}$

OR

d Sinθ = $\frac{Yd}{L}$ ⟶ (iv)

For computing the position of a nth bright fringe we substitute Y = Ym and comparing eq (ii) and eq (iv).

$$M\lambda = \frac{Ym \times d}{L}$$

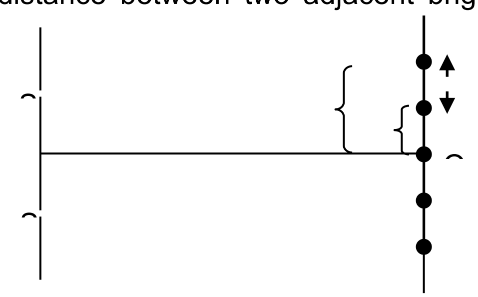
Where Ym be the distance of the center of the nth bright band from the center of the central band at θ = O.

$$\boxed{Ym = \frac{\lambda L}{d} \times m} \longrightarrow \text{(v)}$$

Similarly by comparing equation (iii) and (iv) we find that the dark fringes are located at

$$\boxed{Yd = \frac{\lambda L}{d}\left(m + \frac{1}{2}\right)} \longrightarrow \text{(vi)}$$

From eq (v) we can calculate the distance between two adjacent bright and dark fringes. This distance is knows as fringe spacing.



Mathematically we can write

ΔX = y2 – y1

ΔX = $2\frac{\lambda L}{d} - \frac{\lambda L}{d}$

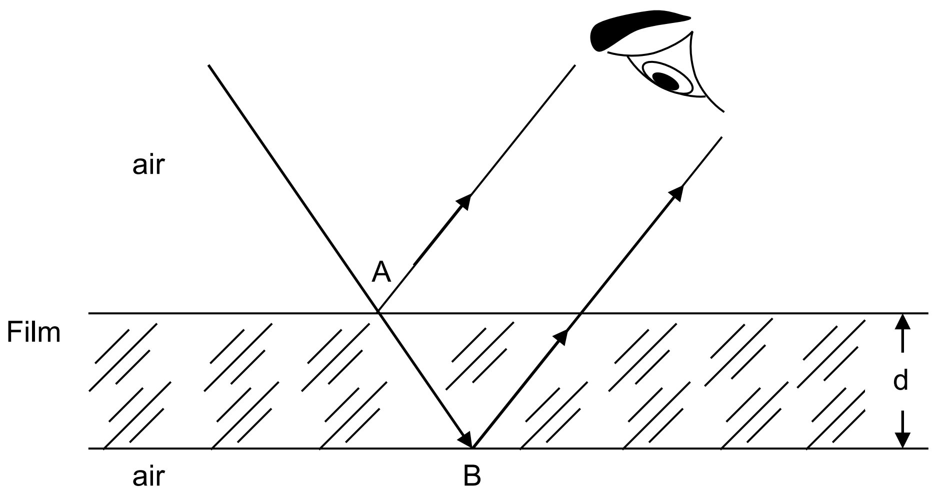
$$\boxed{\Delta X = \frac{\lambda L}{d}} \longrightarrow \text{(vii)}$$

From eq (vii) we can obtain the wave length of light.

INTERFERENCE IN THIN FILMS

The colour of soap bubbles, oil slicks and other thin films are the result of interference. The wave reflected from the upper and the lower surface of the film interfere to in give the interference fringes.

Suppose we consider a thin film of thickness d. From a point P of the extended source of light, the rays after reflection from the upper surfaced at A and from the lower surface at B reach the eye E. The bottom ray has a longer path which is simply 2d in the medium (film) which will be equivalent to 2dn where n is the refractive index of the medium.



The top wave has suffered a phase change of π or a path deference of $\frac{\lambda}{2}$ (This is because when reflection occurs from an interface beyond which the medium has a lower index of refraction, the reflected wave undergoes no phase change and when the medium beyond the interface has a higher index, there is a phase change of π).

Under these conditions, the maximum intensity will be received at the eye.

If

$$2dn = m\lambda + \lambda/2 = \left(m + \frac{1}{2}\right)\lambda$$

$$2dn = \left(m + \frac{1}{2}\right)\lambda \longrightarrow \text{(i)}$$

where

m = 0, 1, 2, ………

Also the condition for minimum intensity (destructive interference) is

$2dn = m\lambda$ ⟶ (ii)

where

m = 0, 1, 2, ……………

- **NEWTONS RINGS**

Newton discovered an example of interference which is known as Newton’s rings. In this case a lines L is place on a glass plate H.

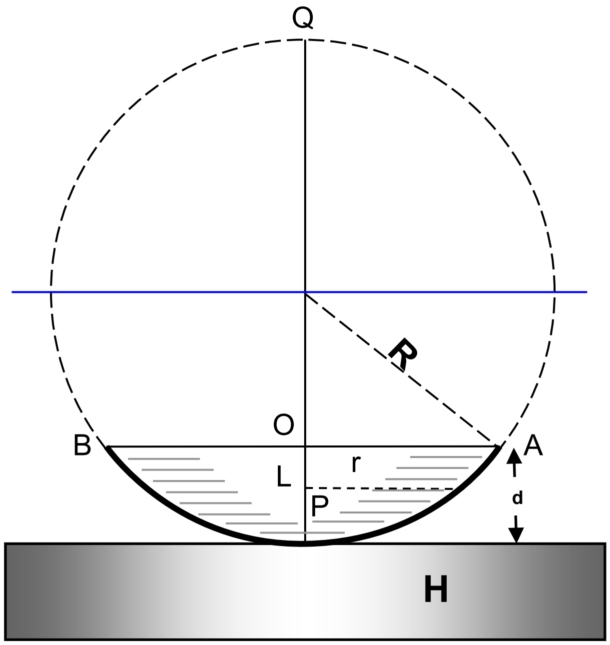


FIGURE (1)

If the radius of curvature of lens R is very large as compared with the radius r. The point of content gives a dark circle due to zero path difference at this point, and 180^{O} change in phase in the light externally reflected at the lower surface. The photograph of Newton’s ring shows a series of dark and bright rings. These are due to constructive and destructive interference.

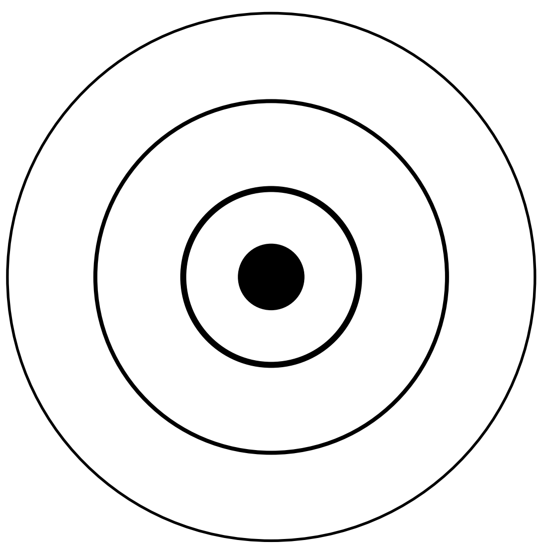


FIGURE (2)

According to well known theorem concerning the segments of chord in a circle.

$$|\overrightarrow{PO}| \times |\overrightarrow{OQ}| = |\overrightarrow{OA}| \times |\overrightarrow{OB}| \longrightarrow \text{(i)}$$

But

$$|\overrightarrow{OA}| = |\overrightarrow{OB}| = r$$

$$|\overrightarrow{PO}| = d$$

and

$$|\overrightarrow{PQ}| = |\overrightarrow{PO}| + |\overrightarrow{OQ}|$$

$$|\overrightarrow{OQ}| = |\overrightarrow{PQ}| - |\overrightarrow{PO}| = 2R - d$$

$$\boxed{|\overrightarrow{OQ}| = 2R - d}$$

Now equation (i) becomes

D x (2R – d) = r x r

$r^2 = 2Rd - d^2$

As d^2 is negligible small

$$r2 = 2Rd$$

$$r = \sqrt{2Rd} \longrightarrow \text{(ii)}$$

The path difference for constructive interference in this film is

2nd = (m + ½)λ

Suppose = n = 1 (air)

$$2d = \left(m + \frac{1}{2}\right)\lambda \longrightarrow \text{(iii)}$$

For Nth bright ring (note that N = m + 1)

$$2dn = \left[(N-1) + \tfrac{1}{2}\right]\lambda = \left(N - \tfrac{1}{2}\right)\lambda$$

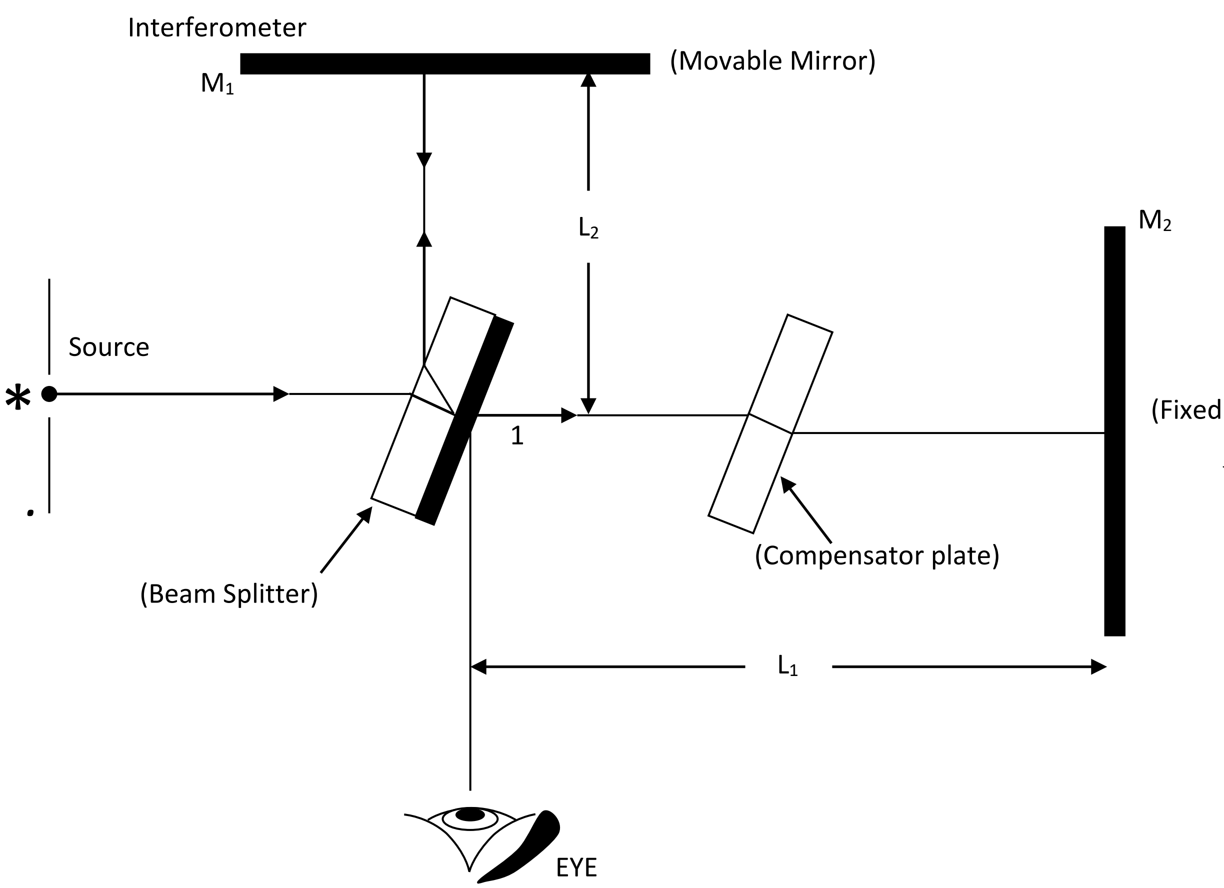
Now equation (ii) becomes

$$\boxed{rn = \sqrt{R\left(N - \tfrac{1}{2}\right)\lambda}} \longrightarrow \text{(iv)}$$

MICHELSON INTERFEROMETER

Interferometers are precision instruments used for comparison of wave length of light, measurement of thin plates and small and precise displacement the basis of the construction of these precise instruments is the interference of the light beams reflected from mirrors.

Following is a schematic diagram of the interferometer.



A beam of light provided by a monochromatic source is split into two rays by a partially silvered mirror M inclined at 45 with respect to the incident beam. One ray is reflected vertically upward toward mirror M_1 while the second ray is Transmitted horizontally through M toward mirror M2 Hence the two ray travel separate paths L_1 and L_2. After Reflecting from mirrors M_1 and M_2 the two rays are recombine to produce an interference pattern which can be seen through telescope. If a dark fringe appears at the centre of the pattern. If the mirror M_1 is moved a distance $\frac{\lambda}{4}$ the path difference changes by $\frac{\lambda}{2}$ The two rays will now

Interfere constructively, giving a bright fringe. As M_1 is moved and, an additional distance of $\frac{\lambda}{4}$ (total distance $\frac{\lambda}{2}$), a dark fringe will appear once again. Thus we see that successive dark and bright fringes are formed each time M_1 is moved a distance $\frac{\lambda}{4}$. The wavelength of the light is them measured by counting the number of fringe shifts for a given displacement

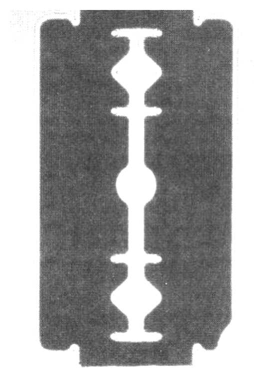
of M_1 . If we have dark fringe, ray in the beginning, the next dark fringe will appear by moving the mirror M_1 to a distance of $\frac{\lambda}{2}$ every time, thus if the displacement is represented by X then.

$$X = m\frac{\lambda}{2} \longrightarrow \text{(i)}$$

OR

$$\lambda = \frac{2X}{m} \longrightarrow \text{(ii)}$$

DIFFRACTION



In daily practice we notice that light casts geometric shadow when blocked by opaque objects. Under certain conditions one notice that light can encroach into the geometrical shadow and thus banks around the edges of the object .This phenomenon where the light deviates from its rectilinear propagation or bends round the adages is called diffraction.

CONDITION:

The condition for diffraction is that the size of the obstructing object most be comparable with the wave length of incident light.

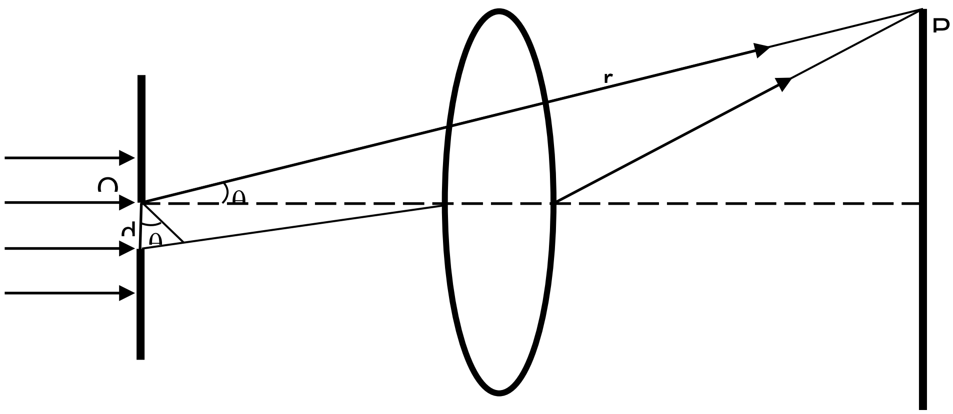
DIFFERENCE BETWEEN INTERFERENCE AND DIFFRACTION

INTERFERENCE	DIFFRACTION
1. Interference is the result of interference between the light rays coming from two difference sources originating from the same source. 2. Points of minima or points of maximum intensity are uniform.	1. Diffraction results due to the interaction of light waves coming from different parts of the same wave front. 2. Point of minimum or points of maximum intensity are not be the same intensity. 3. Diffraction fringes are not be the same width.

3. Interference fringes may or may not be often same width.	

- **DIFFRACTION BY A SINGLE SLIT:**

A slit is a rectangular aperture of width much smaller than its length. Let us consider an aperture A of width d illuminated by a parallel beam of light of wavelength λ .



It is clear from figure that the path difference between two diffracted rays is d sin θ. It was found experimentally that for constructive or bright fringe.

$$d \, \text{Sin}\theta = \left(m + \frac{1}{2}\right)\lambda$$

and for dark fringe

$$d \, \text{Sin}\theta = m\lambda$$

where $m = \pm 1, \pm 2, \pm 3, \ldots\ldots\ldots\ldots\ldots$

Q. What is diffraction Grating? How it is used to find the unknown wavelength of light.

DIFFRACTION GRATING

DEFINITION:

It is an optical device which is used to measure the unknown wavelength of radiation except x – rays.

PRINCIPLE:

The principle of diffraction grating is based on both diffraction and interference.

CONSTRUCTION:

It consist a plane glass plate on which many thousand equally spaced parallel lines are ruled with the help of a fine diamond point. A fine grating consist 6000 lines (slits) per centimeter.

GRATING ELEMENT (d):

It is the distance between two adjacent slits.

Mathematically it is given by

$$\boxed{d = a + b}$$

Where a = distance between two adjacent slits.

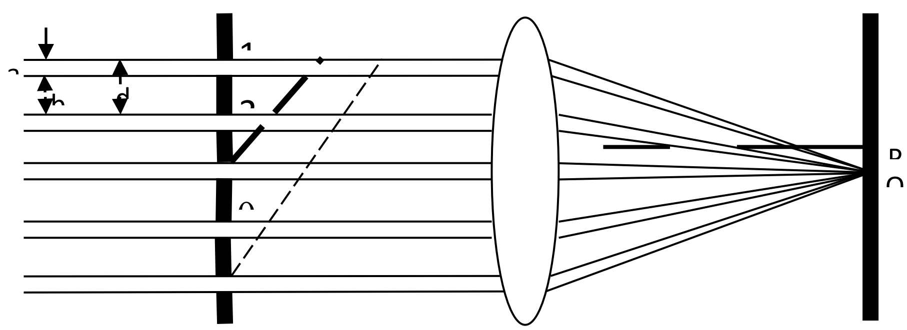
b = slit width.

The formula for 'd' is

$$d = \frac{Widthofslit}{Numberofslits}$$

WORKING:

A parallel beam of light is made to fall normally on the grating, diffraction takes place. Diffraction pattern is brought on the screen with the help of a double convex lens. The rays coming from slit no. 1 and 2 will interfere constructively only when the path difference between then is



$$\boxed{dSin\theta = m\lambda} \quad \text{(i)} \quad (m = 0, \pm1, \pm2, \pm3, \ldots\ldots\ldots\ldots)$$

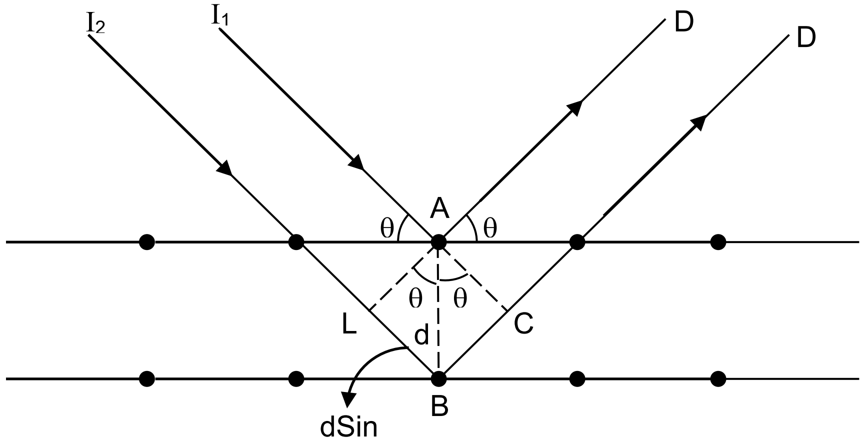
Equation (i) is called Grating equation.

USES:

Diffraction grating is used to find out an unknown wave length of light. It is placed on the turntable of the spectrometer, Light is incident on the grating, diffraction is take place and angle of different order of spectrum is measured with the help of telescope and so unknown wave length is found out because now θ, m and d are known to us.

- **DERIVATION OF BRAGG'S LAWS:**

A set of parallel lattice planes having spacing d between each other. Let us consider the elastic sheltering of two incident X-rays. $I_1 A$ and $I_2 B$ each of length λ from the two atoms A and B in the two neighboring planes



It is clear from figure that the path difference b/w two ray = LB + BC

$= d\sin\theta + d\sin\theta$

$= 2d\sin\theta$

For constructive interference

$$2d\sin\theta = m\lambda \longrightarrow \quad \text{(i)}$$

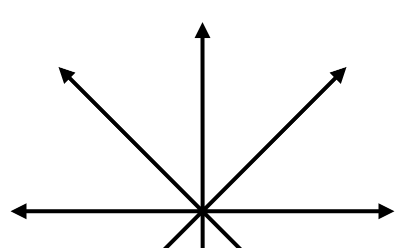
Equation (i) is known as Bragg's law.

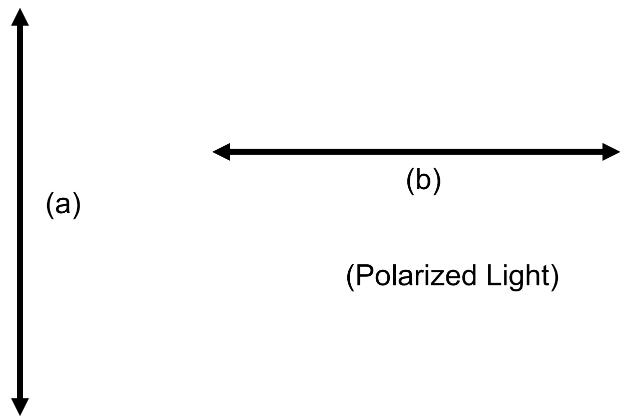
- **<u>Importance of Bragg's Law.</u>**

It can be used to determine the unknown wave length of X-ray. It is easy to conclude from the Bragg relation that of x-rays of unknown λ fall, on a crystal of known lattice spacing d, then by detecting diffracted x-rays along the angle θ one can determine the value of λ.

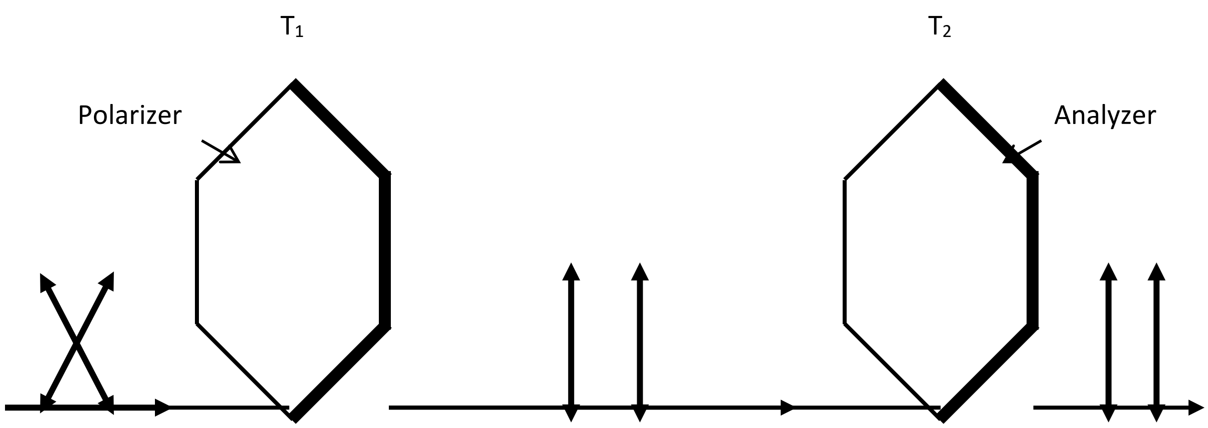
POLARIZATION OF LIGHT WAVES

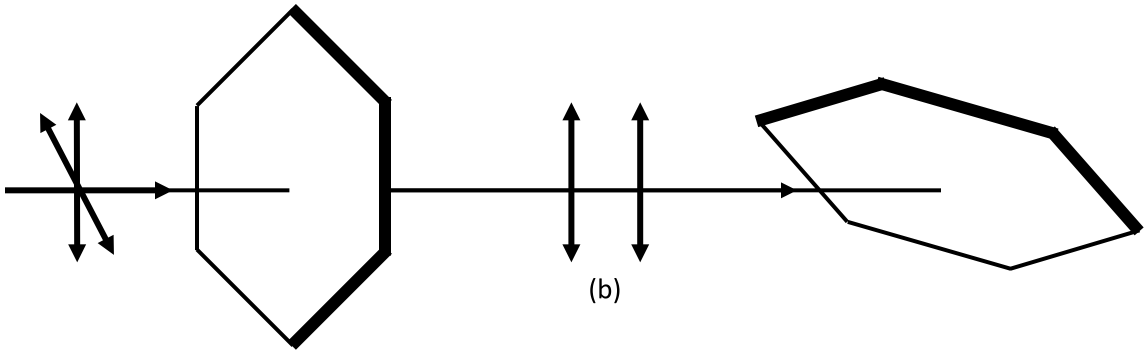
A beam of light from the normal source contains large number of waves which is known as un-polarized light. The direction of whose vibrations is completely different. The beam of light is said to be polarized, if un polarized beam passes through a polarizing sheet known as Polaroid.





The plane polarized light can be obtained by passing the light through a tourmaline crystal. This crystal has the property of selectively absorbing one of the two rectangular components of ordinary light. When a pencil of un-polarized light is sent through a tourmaline crystal like T_1, the transmitted light is found to be polarized. Such a device is known as polarizer. This can be verified by a.





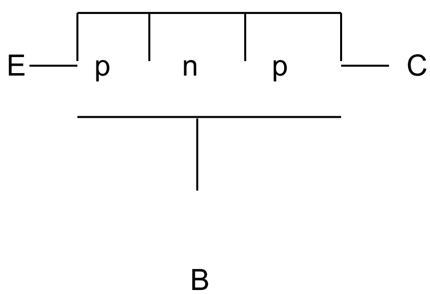
(b)

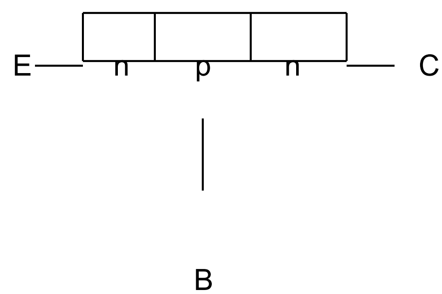
Second crystal T_2. When T_1 and T_2 are parallel to each other, the light transmitted by the first crystal is also transmitted by the second. When the second crystal is rotated through 90^o degrees, no light gets through. A device like T_2 is known as analyzer.

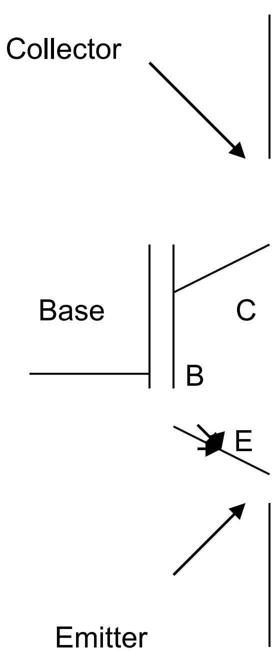
Unit 14 # Communication

Transistor:

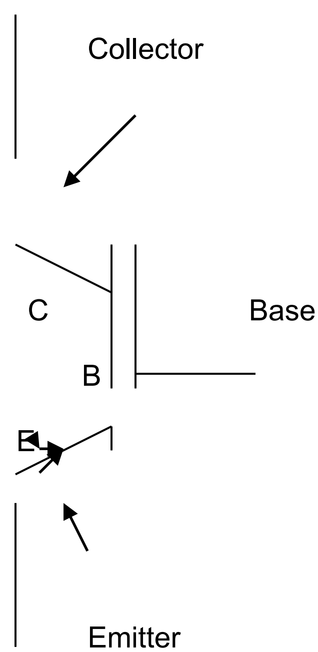
The transistor is a three layer semiconductor device consisting on either two n and one p-type layers. The former is called an n p n transistor, while the later is called a p n p transistor. The outer layers of the transistor are heavily doped semiconductor materials having width greater then that of the sandwich p **–** or **–** n-type material. The doping of the sandwiched layer is also less then that of the outer layer (typically 10:10^5 or less). This lower doping level decreases the conductively or increases the resistance of this material.







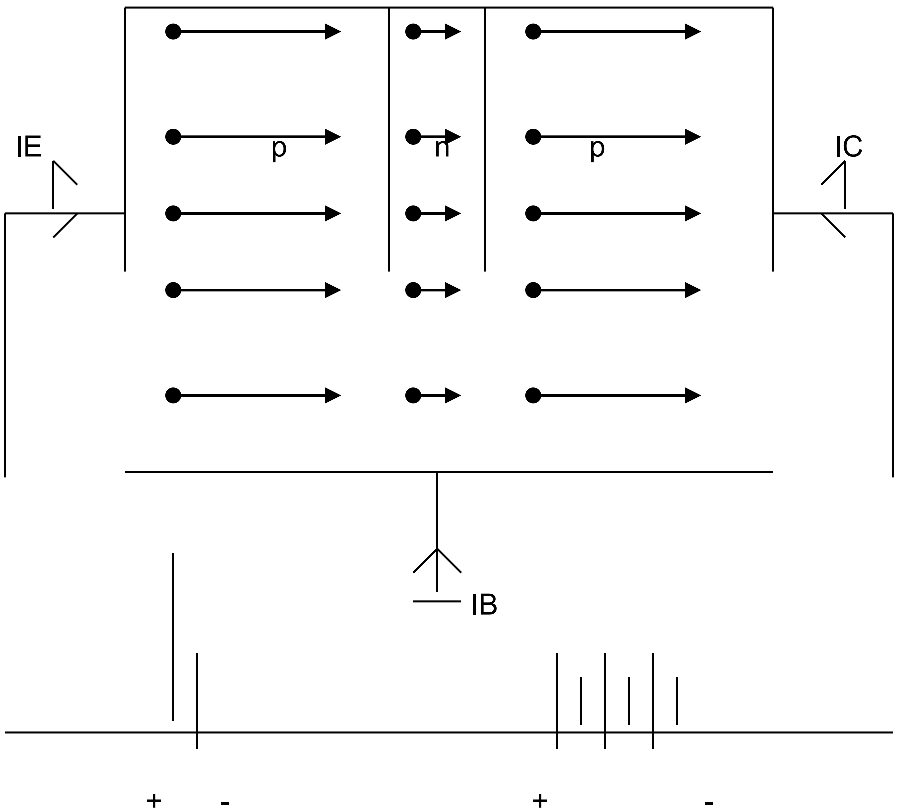
n-p-n

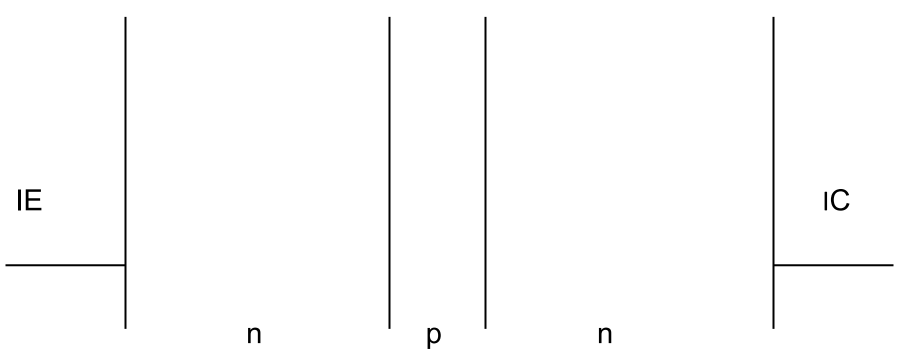


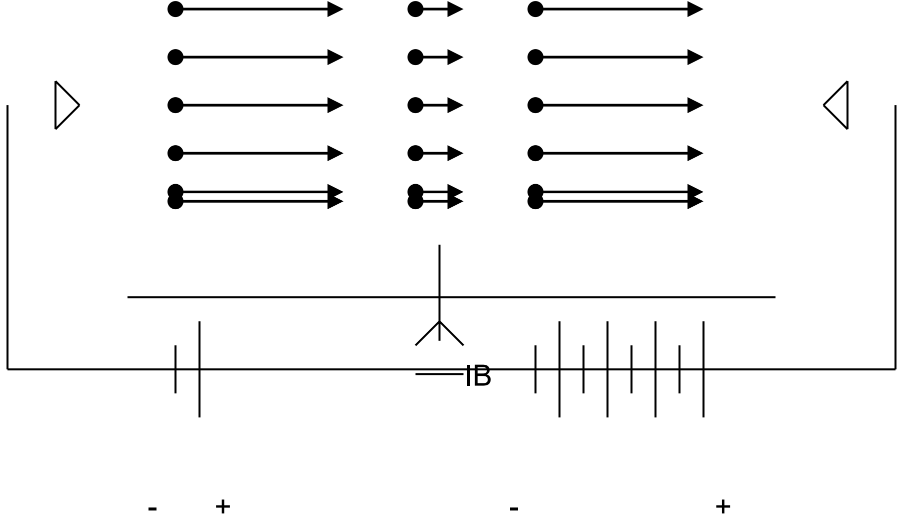
p-n-p

- **Transistor Operation:**

The action of both n-p=n and p n p transistor is very similar, the essential difference being that in the p n p type the main current is carried out by holes where as for the n p n transistor the main current is carried out by electrons. Because of this difference the polarities of the biasing voltages are different for each type.

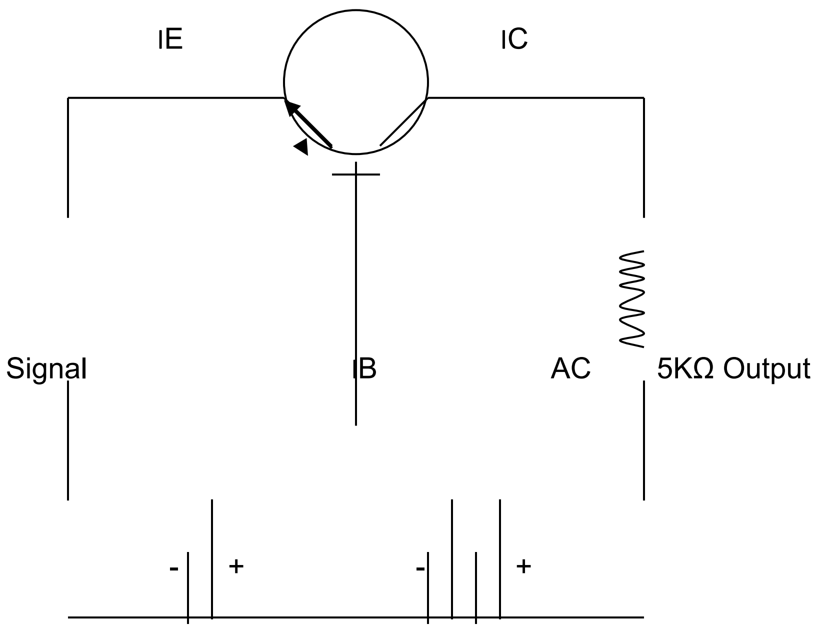






- **Transistor As An Amplifier.**

  The process to convert low voltage (or current) signals to high (or current) is known as amplification. We can amplify both the current as well as voltage by a transistor. The following is a circuit for an a.c amplifier with a pnp transistor.



A small a.c. power sources connected to the input produces changes in the base current. This produces much larger changes in the collector current part of the a.c current which goes through the capacitor C_O to the output load. Another friction of it flow through Rc . Depending upon the combined impedance Rc and out put load, large voltage may be produced at output. Hence transistor operates here both as a current and a voltage amplifier.

The Characteristics of a transistor can be studied by Using suitable electric circuit. This device can be used for both current and voltage amplification and also for oscillators. It has replaced vaccum tubes due to its compactness, reliability long life and low power consumption.